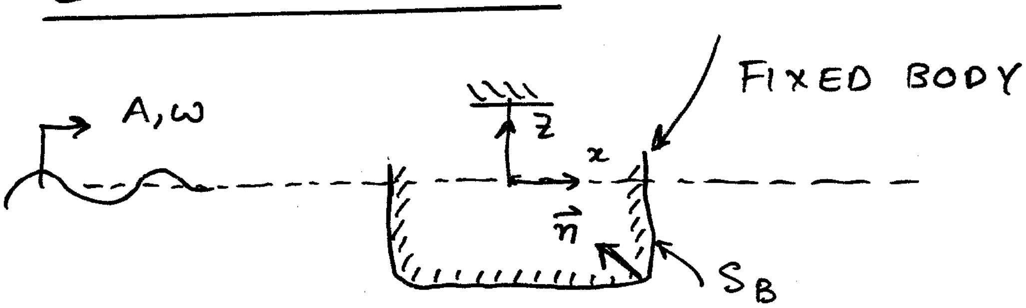
EXCITING FORCES AND HYDRODYNAMIC COEFFICIENTS

- THERE REMAINS TO DERIVE THE BOUNDARY VALUE PROBLEMS LEADING TO THE DEFINITION OF THE EXCITING FORCES $\mathbb{X}_j(\omega)$ AND THE IMPEDANCE HYDRODYNAMIC COEFFICIENTS $A_{ij}(\omega)$, $B_{ij}(\omega)$; $i,j = 1,3,4$

- THE STEPS FOLLOWED IN TWO DIMENSIONS AND THE RELATIONS THAT FOLLOW EXTEND ALMOST WITH NO CHANGE IN THREE DIMENSIONS.

EXCITING FORCES



$$\phi_I = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} \right\}, \qquad k = \omega^2/g$$

$$\zeta_I = A\cos(\omega t - kx)$$
$$= A\cos\omega t \quad , \quad x=0$$
$$= A\,\mathrm{Re}\{e^{i\omega t}\}$$

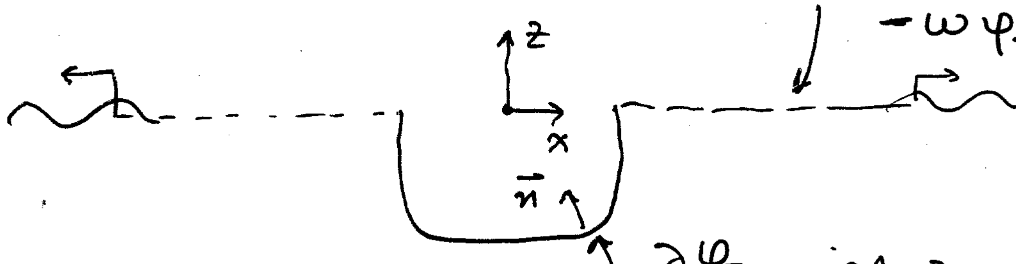
PLANE PROGRESSIVE WAVES WITH POTENTIAL Φ_I ARE INCIDENT TOWARDS A FIXED 2D BODY. THE CORRESPONDING WAVE ELEVATION IS DEFINED ABOVE AND HAS A ZERO PHASE RELATIVE TO $X=0$.

- ALL OTHER SEAKEEPING QUANTITIES HAVE A PHASE MEASURED RELATIVE TO THAT OF ζ AT $X=0$. —

LET ψ BE THE "DIFFRACTION" VELOCITY POTENTIAL DEFINED AS FOLLOWS

$$\begin{cases} \psi = \mathrm{Re}\{\varphi_7 e^{i\omega t}\} \\ \frac{\partial}{\partial n}(\Phi_I + \psi) = 0, \quad \text{on } S_B \\ (-\omega^2 + g\,\partial/\partial z)\varphi_7 = 0, \quad \text{on } z=0 \end{cases}$$

THE BOUNDARY-VALUE PROBLEM SATISFIED BY φ_7 BECOMES:



$$-\omega^2 \varphi_7 + \frac{\partial \varphi_7}{\partial z} = 0$$

$$\frac{\partial \varphi_7}{\partial n} = -\frac{igA}{\omega} \frac{\partial}{\partial n} \left(e^{kz - ikx} \right)$$

AT INFINITY, φ_7 REPRESENTS THE WAVES SCATTERED BY THE BODY WHICH MUST BE OUTGOING. SO AS $|x| \to \infty$

$$\varphi_7 \propto \begin{cases} \dfrac{igA^+}{\omega} e^{kz - ikx}, & x \to +\infty \\[2ex] \dfrac{igA^-}{\omega} e^{kz + ikx}, & x \to -\infty \end{cases}$$

WHERE $A^\pm$ ARE UNKNOWN COMPLEX AMPLITUDES

- THE SOLUTION OF THE BVP FOR φ_7 DEFINED ABOVE AND SIMILAR PROBLEMS DERIVED BELOW IS CARRIED OUT ROUTINELY WITH PANEL METHODS, DISCUSSED LATER.

FOLLOWING THE SOLUTION FOR φ_7 THE HYDRODYNAMIC PRESSURE FOLLOWS FROM THE LINEAR BERNOULLI EQUATION:

$$p = -\rho \frac{\partial}{\partial t}(\Phi_I + \Psi)$$

$$= -\rho \,\mathrm{Re}\left\{ i\omega (\varphi_I + \varphi_7) e^{i\omega t} \right\}$$

$$= \rho g A \,\mathrm{Re}\left\{ (\chi_I + \chi_7)\, e^{i\omega t} \right\}$$

WHERE THE DEFINITIONS:

$$\varphi_I \equiv \frac{igA}{\omega}\chi_I \,, \quad \varphi_7 = \frac{igA}{\omega}\chi_7$$

WERE USED, WITH $\chi_I = e^{kz - ikx}$.

- BY DEFINITION:

$$\vec{F} = \iint_{S_B} p\, \vec{n}\, ds \,, \quad \vec{n} = (n_1, n_3)$$

WITH

$$F_1 = \iint_{S_B} p\, n_1\, ds = \mathrm{Re}\left\{ X_1(\omega)\, e^{i\omega t} \right\}$$

$$F_3 = \iint_{S_B} p\, n_3\, ds = \mathrm{Re}\left\{ X_3(\omega)\, e^{i\omega t} \right\}$$

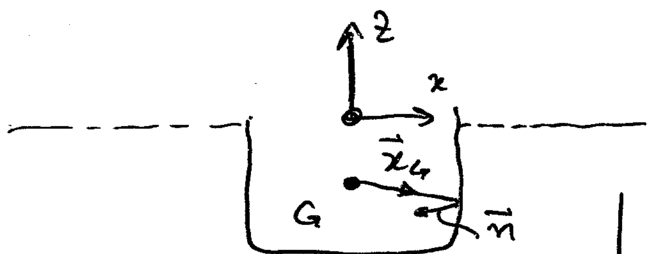
IT FOLLOWS THAT:

- $$\mathbb{X}_1(\omega) = \rho g A \iint_{S_B} (\chi_I + \chi_7)\, n_1\, ds$$

- $$\mathbb{X}_3(\omega) = \rho g A \iint_{S_B} (\chi_I + \chi_7)\, n_3\, ds$$

WITH: $$\chi_I = e^{kz - ikx}$$

THE ROLL MOMENT ABOUT THE CENTER OF GRAVITY G IS DEFINED IN AN ANALOGOUS MANNER



$$|\vec{x}_G \times \vec{n}| = n_4$$

$$\vec{M} = \iint_{S_B} (\vec{x}_G \times \vec{n})\, p\, ds$$

IT FOLLOWS THAT THE COMPLEX ROLL MOMENT BECOMES:

- $$\mathbb{X}_4(\omega) = \rho g A \iint_{S_B} (\chi_I + \chi_7)\, n_4\, ds$$

THE COMPLEX WAVE EXCITING FORCES $\mathbb{X}_j(\omega)$; $j=1,3,4$ CAN BE DECOMPOSED IN AN OBVIOUS MANNER AS FOLLOWS:

$$\mathbb{X}_j(\omega) = \mathbb{X}_j^{FK}(\omega) + \mathbb{X}_j^{DIF}(\omega)$$

WHERE

$$\mathbb{X}_j^{FK} = \rho g A \iint_{S_B} \chi_I \, n_j \, ds \quad , \quad j=1,3,4$$

$$\mathbb{X}_j^{DIF} = \rho g A \iint_{S_B} \chi_7 \, n_j \, ds \quad , \quad j=1,3,4$$

- THE FROUDE-KRYLOV COMPONENT IS ONLY A FUNCTION OF THE AMBIENT WAVE POTENTIAL AND VERY EASY TO EVALUATE

- THE DIFFRACTION COMPONENT IS A FUNCTION OF THE DIFFRACTION POTENTIAL AND REQUIRES THE SOLUTION OF A BOUNDARY VALUE PROBLEM.—

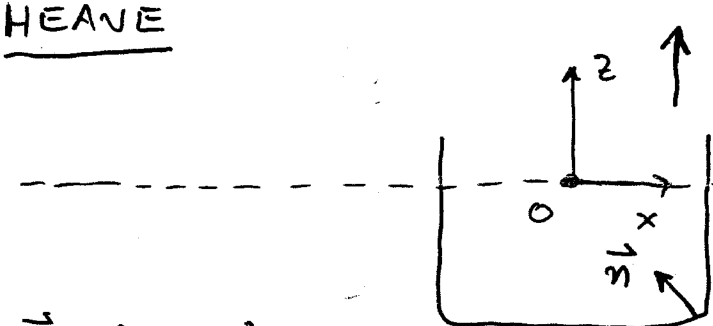
- FURTHER PROPERTIES OF $\mathbb{X}_j^{DIF}$ ARE DISCUSSED LATER.—

ADDED MASS AND DAMPING COEFFICIENTS

THE PRINCIPAL STEPS FOR THE EVALUATION OF $A_{ij}(\omega)$ ARE PRESENTED FOR HEAVE. THE IDEAS EXTEND TRIVIALLY TO SURGE AND ROLL.—

HEAVE

$$\xi_3(t) = \mathrm{Re}\{\Xi_3 e^{i\omega t}\}$$



$\vec{n} = (n_1, n_3)$

$$\dot{\xi}_3 = \mathrm{Re}\{i\omega \Xi_3 e^{i\omega t}\}$$

$$\ddot{\xi}_3 = \mathrm{Re}\{-\omega^2 \Xi_3 e^{i\omega t}\}$$

DUE TO THE FORCED OSCILLATION OF THE BODY WITH DISPLACEMENT $\xi_3(t)$ A FLUID DISTURBANCE WILL RESULT WITH VELOCITY POTENTIAL

$$\Phi_3 = \mathrm{Re}\{\varphi_3 e^{i\omega t}\}$$

LET $\quad \varphi_3 = \Xi_3 \psi_3$;

ON $z=0$; $\quad -\omega^2 \psi_3 + g\,\psi_{3z} = 0$. —

- THE NORMAL COMPONENT OF THE BODY VELOCITY IN THE DIRECTION OF THE UNIT NORMAL VECTOR $\vec{n}$ DUE TO THE HEAVE VELOCITY $\dot{\xi}_3 \vec{k}$ IS SIMPLY:

$$V_{3n} = \dot{\xi}_3 \; \vec{k} \cdot \vec{n} = \dot{\xi}_3 \, n_3$$

WHERE $\vec{k}$ IS THE UNIT VECTOR IN DIRECTION z.

- THE ZERO RELATIVE NORMAL FLUX CONDITION ON THE BODY BOUNDARY REQUIRES THAT:

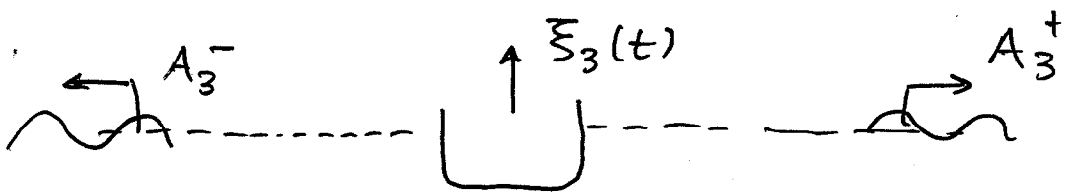
$$\frac{\partial \phi_3}{\partial n} = V_{3n} = \dot{\xi}_3 \, n_3$$

EXPRESSING IN COMPLEX FORM, WE OBTAIN:

$$\frac{\partial \psi_3}{\partial n} = i\omega n_3 \;, \quad \text{on } \overline{S}_B$$

ENFORCED ON THE MEAN POSITION OF THE BODY BOUNDARY DUE TO LINEARITY. —

DUE TO THE FORCED OSCILLATION OF THE BODY IN HEAVE, THE RESULTING WAVE DISTURBANCE WILL REPRESENT OUTGOING WAVES AT INFINITY



SO:

$$\varphi_3 \sim \begin{cases} \dfrac{ig A_3^+}{\omega} e^{kz - ikx + i\omega t}, & x \to +\infty \\ \dfrac{ig A_3^-}{\omega} e^{kz + ikx + i\omega t}, & x \to -\infty \end{cases}$$

WHERE $A_3^{\pm}$ ARE THE COMPLEX AMPLITUDES OF THE WAVES RADIATED AWAY. FOR A SYMMETRIC BODY AND FOR THE HEAVE MOTION

$$|A_3^+| = |A_3^-| .$$

THIS COMPLETES THE STATEMENT OF THE BOUNDARY VALUE PROBLEM SATISFIED BY ψ_3 WHICH IS ALSO SOLVED BY PANEL METHODS.

ASSUMING THAT THE SOLUTION FOR φ_3 HAS BEEN CARRIED OUT, THE HYDRODYNAMIC PRESSURE FOLLOWS AGAIN FROM BERNOULLI:

$$p_3 = -\rho \frac{\partial \phi_3}{\partial t} = -\rho \, \mathrm{Re}\left\{ i\omega \varphi_3 e^{i\omega t} \right\}$$

THE FORCE ACTING ON THE BODY IN THE HEAVE DIRECTION FOLLOWS:

$$F_3 = \iint_{S_B} p_3 n_3 \, ds = \mathrm{Re}\left\{ \mathbb{F}_3 e^{i\omega t} \right\}$$

WITH:

$$\mathbb{F}_3 = -\rho \iint_{S_B} i\omega \varphi_3 n_3 \, ds$$

$$= -i\omega\rho \, \Xi_3 \iint_{S_B} \psi_3 n_3 \, ds$$

- WHEN STATING NEWTON'S LAW, $F_3(t)$ WAS ALSO DEFINED IN TERMS OF THE ADDED MASS AND DAMPING COEFFICIENTS:

$$F_3(t) = -A_{33}(\omega) \frac{d^2 \xi_3}{dt^2} - B_{33}(\omega) \frac{d\xi_3}{dt}$$

A DEFINITION VALID ONLY FOR HARMONIC MOTION.

EXPRESSING IN COMPLEX FORM:

$$F_3(t) = -\mathrm{Re}\left\{\left[-\omega^2 A_{33}(\omega)\,\Xi_3 + i\omega\, B_{33}(\omega)\,\Xi_3\right] \times e^{i\omega t}\right\}.$$

EQUATING THE TWO DEFINITIONS OF $F_3(t)$ AND KEEPING THE COMPLEX TERMS:

$$\omega^2 A_{33}(\omega) - i\omega\, B_{33}(\omega) = -i\omega\rho \iint_{S_B} \psi_3\, n_3\, ds$$

THIS EQUATION DEFINES THE HEAVE ADDED MASS AND DAMPING COEFFICIENTS IN TERMS OF ψ_3, OBTAINED FROM THE SOLUTION OF THE FREE-SURFACE BOUNDARY-VALUE PROBLEM

NOTING THAT: $\dfrac{\partial \psi_3}{\partial n} = i\omega n_3$, on S_B

WE MAY ALSO USE:

$$\omega^2 A_{33}(\omega) - i\omega\, B_{33}(\omega) = -\rho \iint_{S_B} \psi_3 \frac{\partial \psi_3}{\partial n}\, ds$$

THIS DEFINITION EXTENDS TO ALL OTHER MODES OF MOTION TRIVIALLY AND FROM TWO TO THREE DIMENSIONS:

- $$\omega^2 A_{ij}(\omega) - i\omega B_{ij}(\omega) = -\rho \iint_{S_B} \psi_i \frac{\partial \psi_j}{\partial n} ds$$

WHERE FOR SURGE AND HEAVE:

$$\frac{\partial \psi_j}{\partial n} = i\omega n_j \;;\quad j = 1,3$$

AND FOR THE ROLL MOTION ABOUT THE ORIGIN:

$$\frac{\partial \psi_4}{\partial n} = i\omega n_4, \quad n_4 = |\vec{X} \times \vec{n}| = z n_1 - x n_3 \,. -$$

(VERIFY)

ASSUMING THAT ψ_i IS AVAILABLE ON S_B:

- $$A_{ij}(\omega) = \mathcal{R}e\left\{-\frac{\rho}{\omega^2} \iint_{S_B} \psi_i \frac{\partial \psi_j}{\partial n} ds\right\}$$

- $$B_{ij}(\omega) = \mathcal{I}m\left\{-\frac{\rho}{\omega} \iint_{S_B} \psi_i \frac{\partial \psi_j}{\partial n} ds\right\}$$

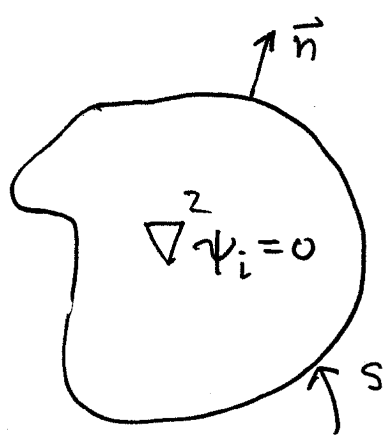
SYMMETRY - RECIPROCITY RELATIONS

IT WILL BE SHOWN THAT:

$$A_{ij}(\omega) = A_{ji}(\omega)$$
$$B_{ij}(\omega) = B_{ji}(\omega)$$

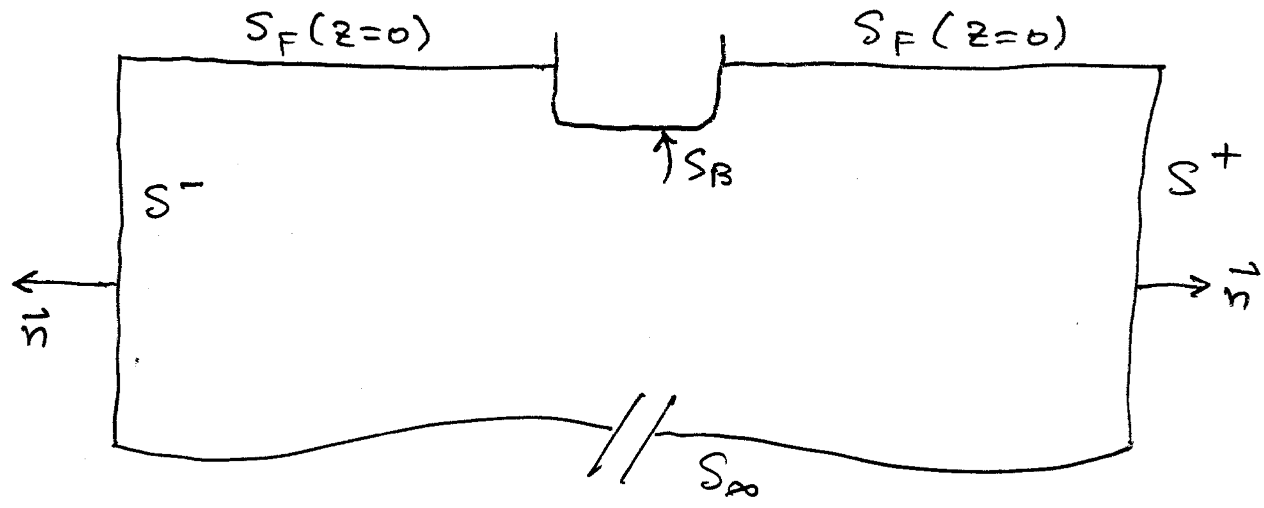
ALONG THE SAME LINES IT WILL BE SHOWN THAT THE EXCITING FORCE $\mathbb{X}_j$ CAN BE EXPRESSED IN TERMS OF ψ_j CIRCUMVENTING THE SOLUTION FOR THE DIFFRACTION POTENTIAL

THE CORE RESULT NEEDED FOR THE PROOF OF THE ABOVE PROPERTIES IS GREEN'S THM:



$$\oiint_S \left(\psi_1 \frac{\partial \psi_2}{\partial n} - \psi_2 \frac{\partial \psi_1}{\partial n} \right) ds = 0$$

IN THE SURFACE WAVE-BODY PROBLEM DEFINE THE CLOSED SURFACE S AS FOLLOWS:



LET φ_j BE RADIATION OR DIFFRACTION POTENTIALS. OVER THE BOUNDARIES $S^\pm$ WE HAVE:

$$S^+: \quad \varphi_j \sim \frac{igA_j^+}{\omega} e^{kz-ikx}$$

$$\frac{\partial \varphi_j}{\partial n} = \frac{\partial \varphi_j}{\partial x} \sim -ik\varphi_j$$

$$S^-: \quad \varphi_j \sim \frac{igA_j^-}{\omega} e^{kz+ikx}$$

$$\frac{\partial \varphi_j}{\partial n} = -\frac{\partial \varphi_j}{\partial x} \sim -ik\varphi_j$$

ON S_F: $\dfrac{\partial \varphi_j}{\partial z} = K\varphi_j \,, \quad \dfrac{\partial \varphi_j}{\partial n} = \dfrac{\partial \varphi_j}{\partial z}$

ON S_∞: $|\varphi_j|, |\nabla \varphi_j| \to 0$. —

APPLYING GREEN'S IDENTITY TO ANY PAIR OF THE RADIATION POTENTIALS ψ_i, ψ_j:

$$\iint_{S_B} \left[\psi_i \frac{\partial\psi_j}{\partial n} - \psi_j \frac{\partial\psi_i}{\partial n}\right] ds =$$

$$- \iint_{S_F} \left[\overbrace{\psi_i \underbrace{\frac{\partial\psi_j}{\partial z}}_{= K\psi_j} - \psi_j \underbrace{\frac{\partial\psi_i}{\partial z}}_{= K\psi_i}}^{=0} \right] ds$$

$$- \iint_{S^+} \left[\underbrace{\psi_i \underbrace{\frac{\partial\psi_j}{\partial x}}_{\sim -ik\psi_j} - \psi_j \underbrace{\frac{\partial\psi_i}{\partial x}}_{\sim -ik\psi_i}}_{=0} \right] ds$$

$$+ \iint_{S^-} \left[\underbrace{\psi_i \underbrace{\frac{\partial\psi_j}{\partial x}}_{\sim ik\psi_j} - \psi_j \underbrace{\frac{\partial\psi_i}{\partial x}}_{\sim ik\psi_i}}_{=0} \right] ds$$

$$= 0$$

IT FOLLOWS THAT:

$$\iint_{S_B} \psi_i \frac{\partial\psi_j}{\partial n} ds = \iint_{S_B} \psi_j \frac{\partial\psi_i}{\partial n} ds \quad \text{OR}$$

$$A_{ij}(\omega) = A_{ji}(\omega), \quad B_{ij}(\omega) = B_{ji}(\omega).$$

HASKIND RELATIONS OF EXCITING FORCES

$$\mathbb{X}_i(\omega) = -i\omega\rho \iint_{S_B} (\varphi_I + \varphi_7)\, n_i \, ds$$

$$= -\rho \iint_{S_B} (\varphi_I + \varphi_7) \frac{\partial \varphi_i}{\partial n} \, ds$$

WHERE THE RADIATION VELOCITY POTENTIAL φ_i IS KNOWN TO SATISFY:

$$\frac{\partial \varphi_i}{\partial n} = i\omega n_i \, , \quad \text{on } S_B$$

AND

$$\frac{\partial \varphi_7}{\partial n} = - \frac{\partial \varphi_I}{\partial n} \, , \quad \text{on } S_B$$

BOTH φ_i AND φ_7 SATISFY THE CONDITION OF OUTGOING WAVES AT INFINITY. BY VIRTUE OF GREEN'S THM:

$$\iint_{S_B} \varphi_7 \frac{\partial \varphi_i}{\partial n} \, ds = \iint_{S_B} \varphi_i \frac{\partial \varphi_7}{\partial n} \, ds = - \iint_{S_B} \varphi_i \frac{\partial \varphi_I}{\partial n} \, ds$$

THE HASKIND EXPRESSION FOR THE EXCITING FORCE FOLLOWS:

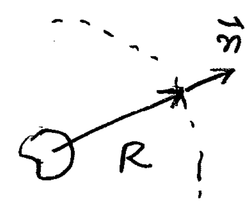
$$\mathbb{X}_i(\omega) = -\rho \iint_{S_B} \left[\varphi_I \frac{\partial \varphi_i}{\partial n} - \varphi_i \frac{\partial \varphi_I}{\partial n} \right] ds$$

COMMENTS:

- THE SYMMETRY OF THE $A_{ij}(\omega)$, $B_{ij}(\omega)$ MATRICES APPLIES IN 2D AND 3D. THE APPLICATION OF GREEN'S THM IN 3D IS VERY SIMILAR USING THE FAR-FIELD REPRESENTATION FOR THE POTENTIAL φ_j

$$\varphi_j \sim \frac{A_j(\theta)}{\sqrt{R}}\, e^{kz - ikR} + O(1/R^{3/2})$$

$$\frac{\partial \varphi_j}{\partial n} = \frac{\partial \varphi_j}{\partial R} \sim -ik\varphi_j + O(1/R^{3/2})$$



WHERE R IS A RADIUS FROM THE BODY OUT TO INFINITY AND THE $R^{-1/2}$ DECAY ARISES FROM THE ENERGY CONSERVATION PRINCIPLE. DETAILS OF THE 3D PROOF MAY BE FOUND IN MEI AND W&L

- THE USE OF THE HASKIND RELATIONS FOR THE EXCITING FORCES DOES NOT REQUIRE THE SOLUTION OF THE DIFFRACTION PROBLEM. THIS IS CONVENIENT AND OFTEN MORE ACCURATE.

- THE HASKIND RELATIONS TAKE OTHER FORMS WHICH WILL NOT BE PRESENTED HERE BUT ARE DETAILED IN W&L.

THE ONES THAT ARE USED IN PRACTICE RELATE THE EXCITING FORCES TO THE DAMPING COEFFICIENTS.

THEY TAKE THE FORM:

2D:
$$B_{ii} = \frac{|\mathbb{X}_i|^2}{2\rho g V_g}, \quad V_g = \frac{g}{2\omega}, \text{ DEEP WATER}$$

3D: (AXISYMMETRIC BODIES)
$$B_{33} = \frac{K}{4\rho g V_g} |\mathbb{X}_3|^2 \text{ — HEAVE}$$

$$B_{22} = \frac{K}{8\rho g V_g} |\mathbb{X}_2|^2 \text{ — SWAY}$$

SO KNOWLEDGE OF $\mathbb{X}_i(\omega)$ ALLOWS THE DIRECT EVALUATION OF THE DIAGONAL DAMPING COEFFICIENTS. THESE EXPRESSIONS ARE USEFUL IN DERIVING THEORETICAL RESULTS IN WAVE-BODY INTERACTIONS TO BE DISCUSSED LATER.

THE TWO-DIMENSIONAL THEORY OF WAVE-BODY INTERACTIONS IN THE FREQUENCY DOMAIN EXTENDS TO THREE DIMENSIONS VERY DIRECTLY WITH LITTLE DIFFICULTY

THE STATEMENT OF THE 6 DOF SEAKEEPING PROBLEM IS:

$$\sum_{j=1}^{6} \left[-\omega^2 (M_{ij} + A_{ij}) + i\omega B_{ij} + C_{ij} \right] \Xi_j = \mathbb{X}_i \, , \quad i = 1, \dots, 6$$

WHERE:

M_{ij} : BODY INERTIA MATRIX INCLUDING MOMENTS OF INERTIA FOR ROTATIONAL MODES. FOR DETAILS REFER TO MH

$A_{ij}(\omega)$: ADDED MASS MATRIX

$B_{ij}(\omega)$: DAMPING MATRIX

C_{ij} : HYDROSTATIC AND STATIC INERTIA RESTORING MATRIX. FOR DETAILS REFER TO MH.

$\mathbb{X}_i(\omega)$: WAVE EXCITING FORCES AND MOMENTS.

AT ZERO SPEED THE DEFINITIONS OF THE ADDED-MASS, DAMPING MATRICES AND EXCITING FORCES ARE IDENTICAL TO THOSE IN TWO DIMENSIONS.

THE BOUNDARY VALUE PROBLEMS SATISFIED BY THE RADIATION POTENTIALS φ_j, $j=1,\dots,6$ AND THE DIFFRACTION POTENTIAL φ_7 ARE AS FOLLOWS:

<u>FREE-SURFACE CONDITION</u>

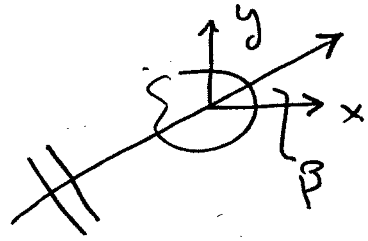
$$-\omega^2 \varphi_j + g \frac{\partial \varphi_j}{\partial z} = 0, \qquad z = 0$$

$$j = 1, \dots, 7$$

<u>BODY-BOUNDARY CONDITIONS</u>

$$\frac{\partial \varphi_7}{\partial n} = -\frac{\partial \varphi_I}{\partial n}, \qquad \text{on } S_B$$

$$\varphi_I = \frac{igA}{\omega} e^{kz - ikx\cos\beta - iky\sin\beta + i\omega t}$$



$i=1$: SURGE

$i=2$: SWAY

$i=3$: HEAVE

$i=4$: ROLL

$i=5$: PITCH

$i=6$: YAW

$$\frac{\partial \varphi_j}{\partial n} = i\omega n_j \, , \quad j=1,\ldots,6$$

$$n_j = \left\{ \begin{array}{ll} n_j & , \quad j=1,2,3 \\ (\vec{X}\times\vec{n})_{j+3} & , \quad j=4,5,6 \end{array} \right\}$$

AT LARGE DISTANCES FROM THE BODY THE VELOCITY POTENTIALS SATISFY THE RADIATION CONDITION:

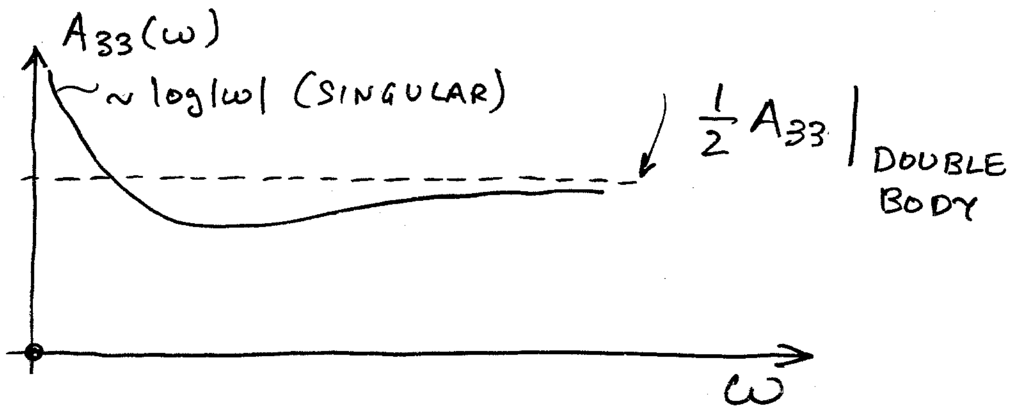
$$\varphi_j(R,\theta) \sim \frac{A_j(\theta)}{\sqrt{R}} e^{kz-ikR} + O(1/R^{3/2})$$

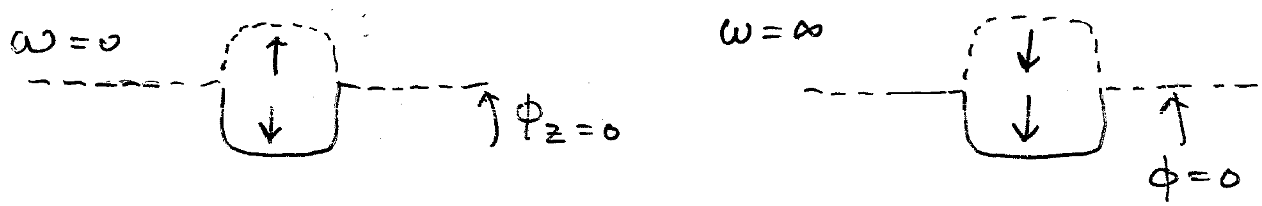
WITH $k=\omega^2/g$.

THIS RADIATION CONDITION IS ESSENTIAL FOR THE FORMULATION AND SOLUTION OF THE BOUNDARY VALUE PROBLEMS FOR φ_j USING PANEL METHODS WHICH ARE THE STANDARD SOLUTION TECHNIQUE AT ZERO AND FORWARD SPEED.—

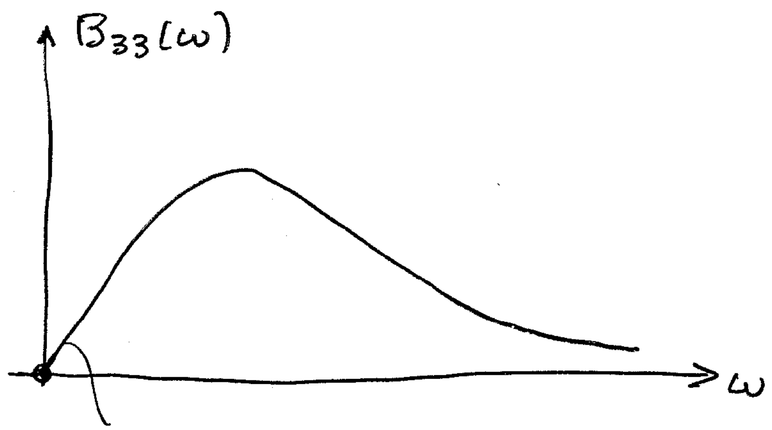
QUALITATIVE BEHAVIOUR OF THE FORCES, COEFFICIENTS AND MOTIONS OF FLOATING BODIES

2D HEAVE



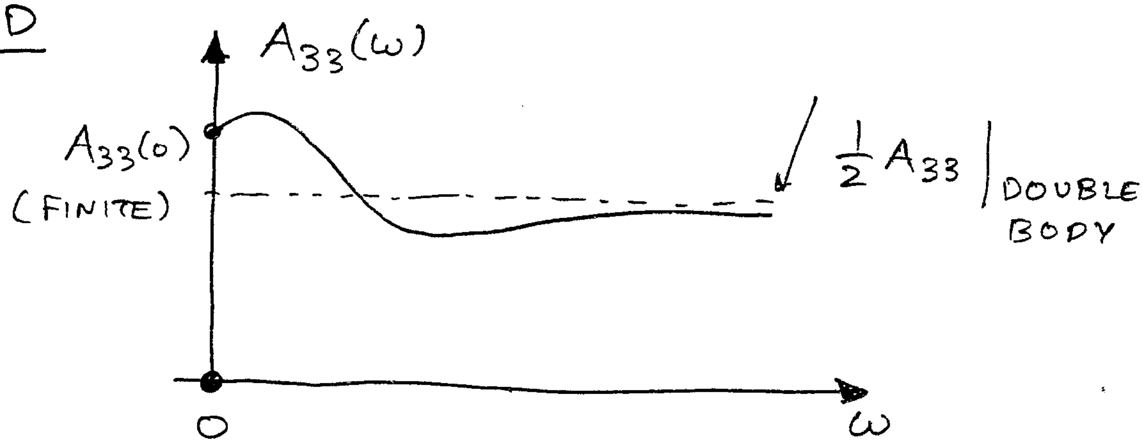


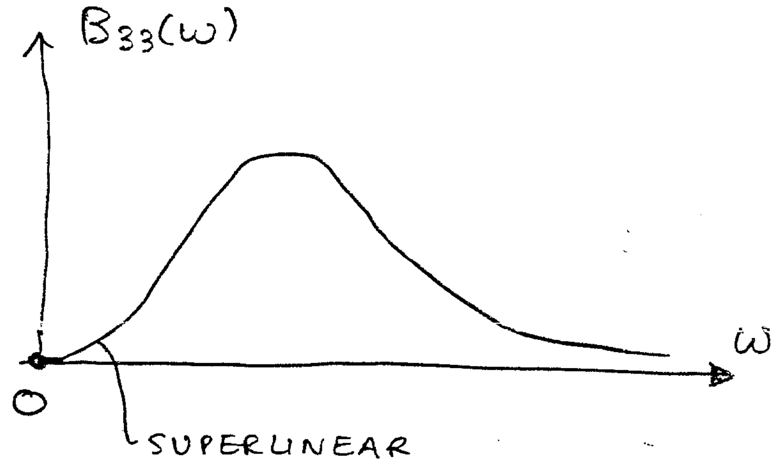
$$-\omega^2 \phi + g \phi_z = 0 \quad \begin{cases} \phi_z = 0, & \omega = 0 \\ \phi = 0, & \omega \to \infty \end{cases}$$



$B_{33}(\omega) \sim \omega$, AT LOW ω

3D

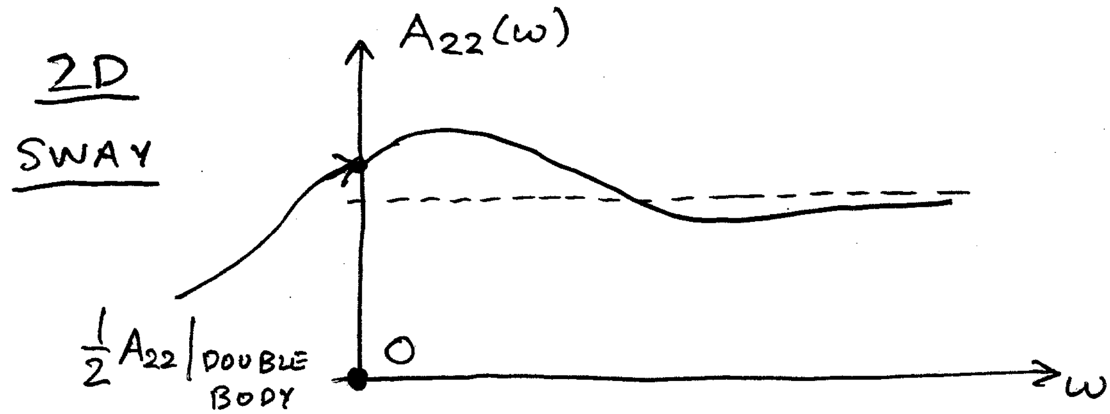


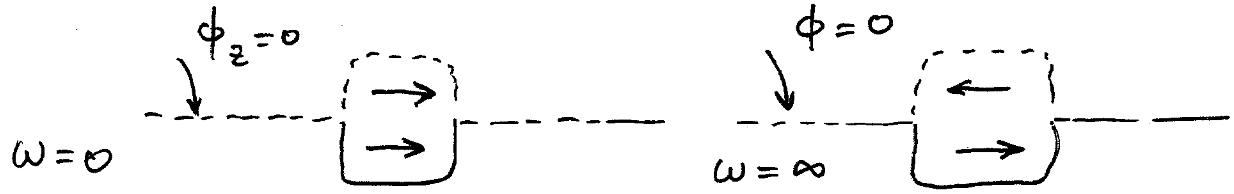


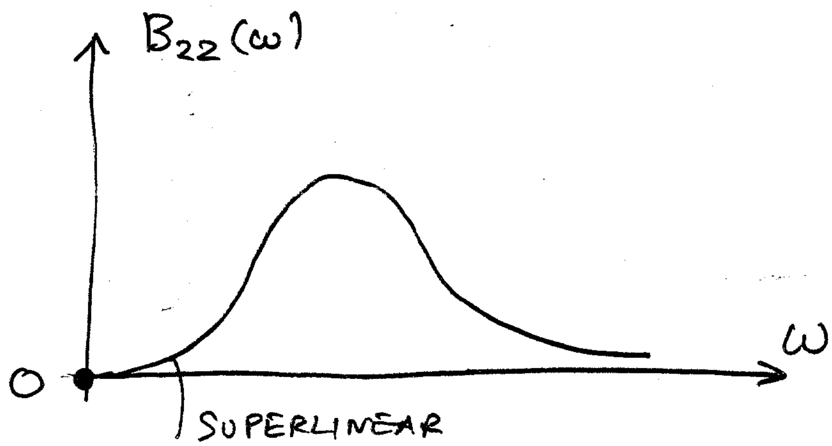
COMMENTS:

- THE 2D HEAVE ADDED MASS IS SINGULAR AT LOW FREQUENCIES. IT IS FINITE IN 3D
- THE 2D HEAVE DAMPING COEFFICIENT IS DECAYING TO ZERO LINEARLY IN 2D AND SUPERLINEARLY IN 3D. A TWO-DIMENSIONAL SECTION IS A BETTER WAVEMAKER THAN A THREE-DIMENSIONAL ONE

2D SWAY



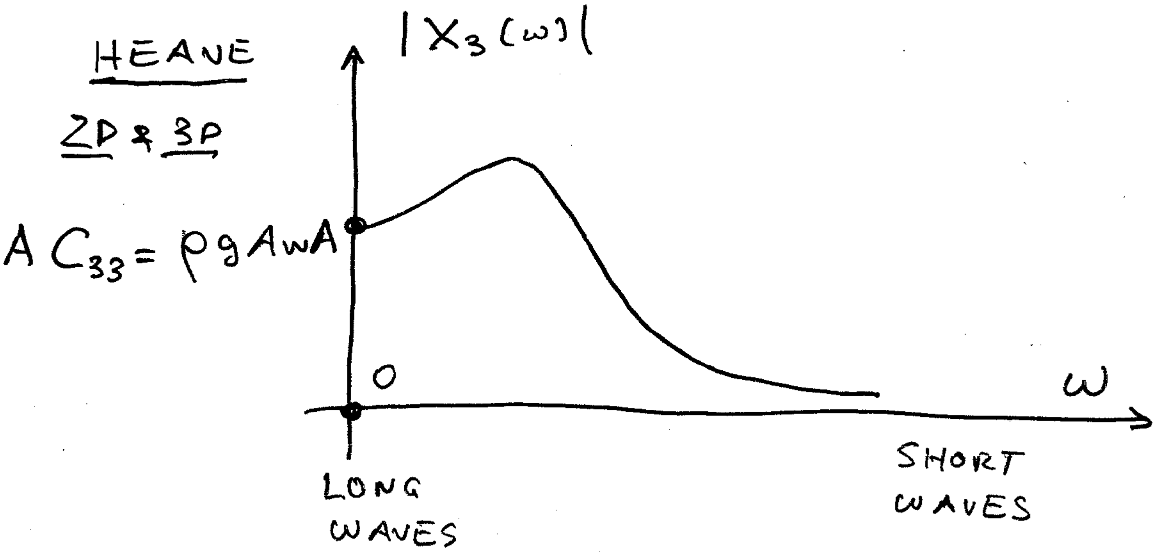


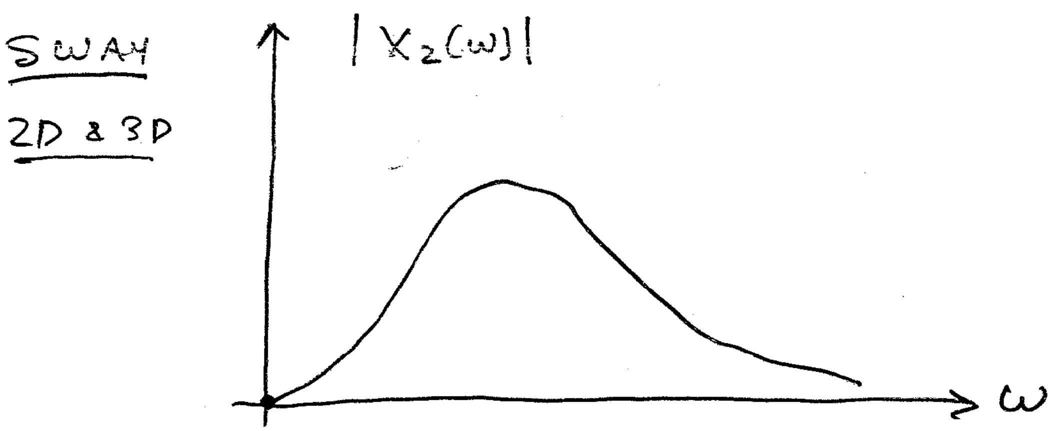


- A 2D SECTION OSCILLATING IN SWAY IS LESS EFFECTIVE A WAVEMAKER AT LOW FREQUENCIES THAN THE SAME SECTION OSCILLATING IN HEAVE

- THE ZERO-FREQUENCY LIMIT OF THE SWAY ADDED MASS IS FINITE AND SIMILAR TO THE INFINITE FREQUENCY LIMIT OF THE HEAVE ADDED MASS.

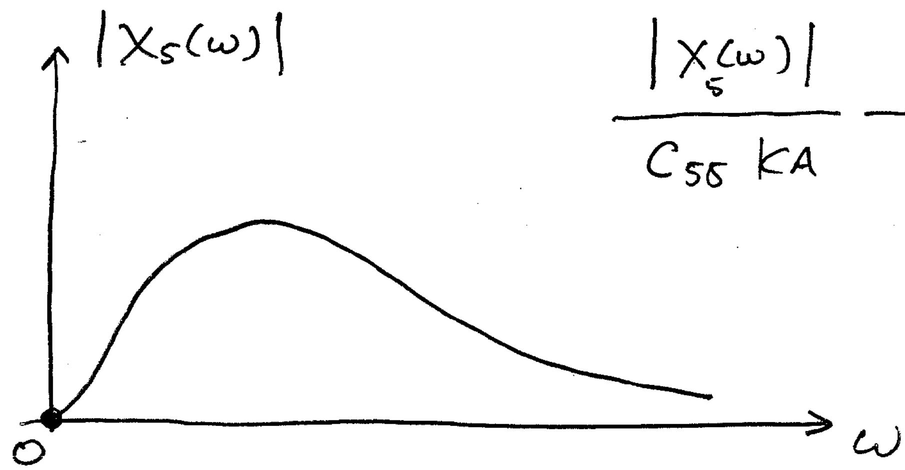
EXCITING FORCES





- IN LONG WAVES THE HEAVE EXCITING FORCE TENDS TO THE HEAVE RESTORING COEFFICIENT TIMES THE AMBIENT WAVE AMPLITUDE
  THE FREE SURFACE BEHAVES LIKE A FLAT SURFACE MOVING UP & DOWN
- IN LONG WAVES THE SWAY EXCITING FORCE TENDS TO ZERO. PROOF WILL FOLLOW
- IN SHORT WAVES ALL FORCES TEND TO ZERO.

PITCH OR ROLL EXCITING MOMENT



$$\frac{|X_5(\omega)|}{C_{55}\, KA} \longrightarrow 1$$

- PITCH EXCITING MOMENT (SAME APPLIES TO ROLL) TENDS TO ZERO. LONG WAVES HAVE A SMALL SLOPE WHICH IS PROPORTIONAL TO KA, WHERE K IS THE WAVENUMBER AND A IS THE WAVE AMPLITUDE

- PROVE THAT TO LEADING ORDER FOR $KA \to 0$:

$$|X_5(\omega)| \sim KA\, C_{55}$$

WHERE C_{55} IS THE PITCH (C_{44} FOR ROLL) HYDROSTATIC RESTORING COEFFICIENT. [NB: VERY LONG WAVES LOOK LIKE A FLAT SURFACE INCLINED @ KA].

BODY MOTIONS IN REGULAR WAVES

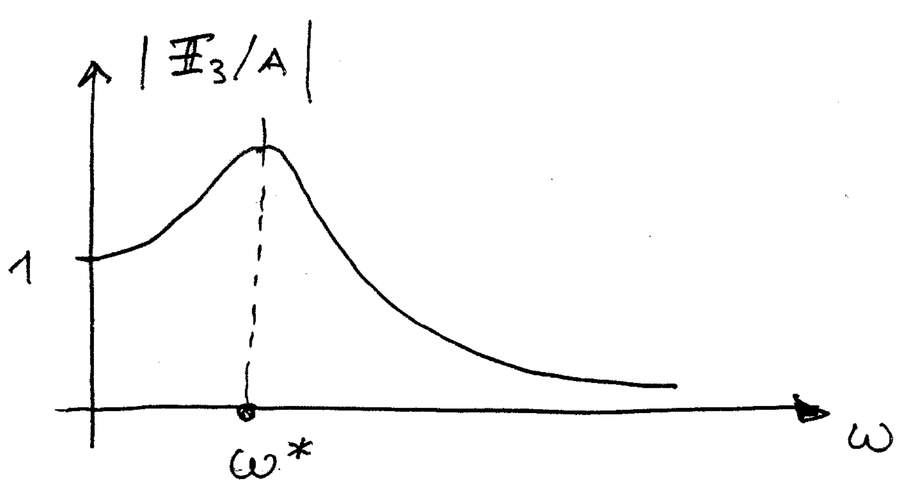
HEAVE

$$\Xi_3 = \frac{\mathbb{X}_3(\omega)}{-\omega^2(A_{33}+M)+i\omega B_{33}+C_{33}}$$

RESONANCE:

$$\omega^2 = \frac{C_{33}}{M+A_{33}} = \frac{\rho g A_w}{M+A_{33}(\omega)}$$

- IN PRINCIPLE THE ABOVE EQUATION IS NONLINEAR FOR ω. WILL BE APPROXIMATED BELOW



AT RESONANCE:

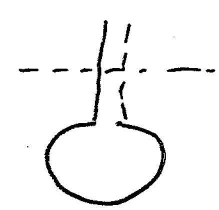
$$\Xi_3 = \frac{\mathbb{X}_3(\omega^*)}{i\omega^* B_{33}(\omega^*)}$$

INVOKING THE RELATION BETWEEN THE DAMPING COEFFICIENT AND THE EXCITING FORCE IN 3D:

$$\left| \frac{\Xi_3}{A} \right| = \frac{|\mathbb{X}_3(\omega)|}{\omega \frac{k}{4\rho g V_g} |\mathbb{X}_3|^2} , \quad V_g = \frac{g}{2\omega}$$

$$= \frac{2\rho g}{\omega^3 |\mathbb{X}_3(\omega)|} , \quad \text{AT RESONANCE}$$

- THIS COUNTER-INTUITIVE RESULT SHOWS THAT FOR A BODY UNDERGOING A PURE HEAVE OSCILLATION, THE MODULUS OF THE HEAVE RESPONSE AT RESONANCE IS INVERSELY PROPORTIONAL TO THE MODULUS OF THE HEAVE EXCITING FORCE

- SWATH VESSELS OF THE FORM  MAY BENEFIT FROM THIS PROPERTY

- VISCOUS EFFECTS NOT DISCUSSED HERE MAY AFFECT HEAVE RESPONSE AT RESONANCE

- STUDY AS AN EXERCISE THE BEHAVIOUR OF THE SWAY RESPONSE.

OFTEN IT IS USEFUL TO ESTIMATE THE HEAVE AND PITCH RESONANT FREQUENCIES IN TERMS OF THE PRINCIPAL DIMENSIONS OF A BODY.

<u>2D</u> : THE HEAVE ADDED MASS A_{33} IS OFTEN NOT TOO FAR FROM ITS INFINITE FREQUENCY LIMIT, OR:

$$A_{33} \cong A_{33}(\infty) = \frac{1}{2} \rho \forall$$

FOR A BOX-LIKE SECTION WITH BEAM B AND DRAFT T :

$$A_{33} \cong \frac{1}{2} \rho BT = \frac{1}{2} M$$

$$\omega^2 = \frac{\rho g B}{\rho BT + \frac{1}{2} \rho BT} = \frac{3}{2} \frac{g}{T}$$

$$\text{OR} \quad \omega^* \cong \left(\frac{3}{2} \frac{g}{T} \right)^{1/2}$$

- DERIVE THE CORRESPONDING RESULT FOR ROLL

<u>3D</u> : THE EXTENSION TO A 3D BODY OF GENERAL SHAPE IS EASY IN PRINCIPLE

- FOR A SHIP LIKE BODY WITH LENGTH LARGE RELATIVE TO ITS BEAM, B, WE MAY APPROXIMATE ITS SHAPE AS A BARGE WITH LENGTH L, BEAM B AND DRAFT T. IN THIS CASE THE 2D ANALYSIS IS VERY ACCURATE

- PROVE THAT THE SAME RESONANT FREQUENCY ω^* IS APPLICABLE TO THE PITCH MOTION OF THE BARGE AS WELL. SO HEAVE & PITCH RESONATE AT THE SAME FREQUENCY. THIS RESULT IS VERY WELL VERIFIED BY 3D COMPUTATIONS

- SIMILAR ARGUMENTS APPLY TO THE ROLL MOTION OF A SHIP APPROXIMATED AS A BOX-LIKE BODY. THE RESONANT FREQUENCY FROM A 2D ANALYSIS IS VERY CLOSE TO THE ROLL RESONANT FREQUENCY FOR A SLENDER SHIP.