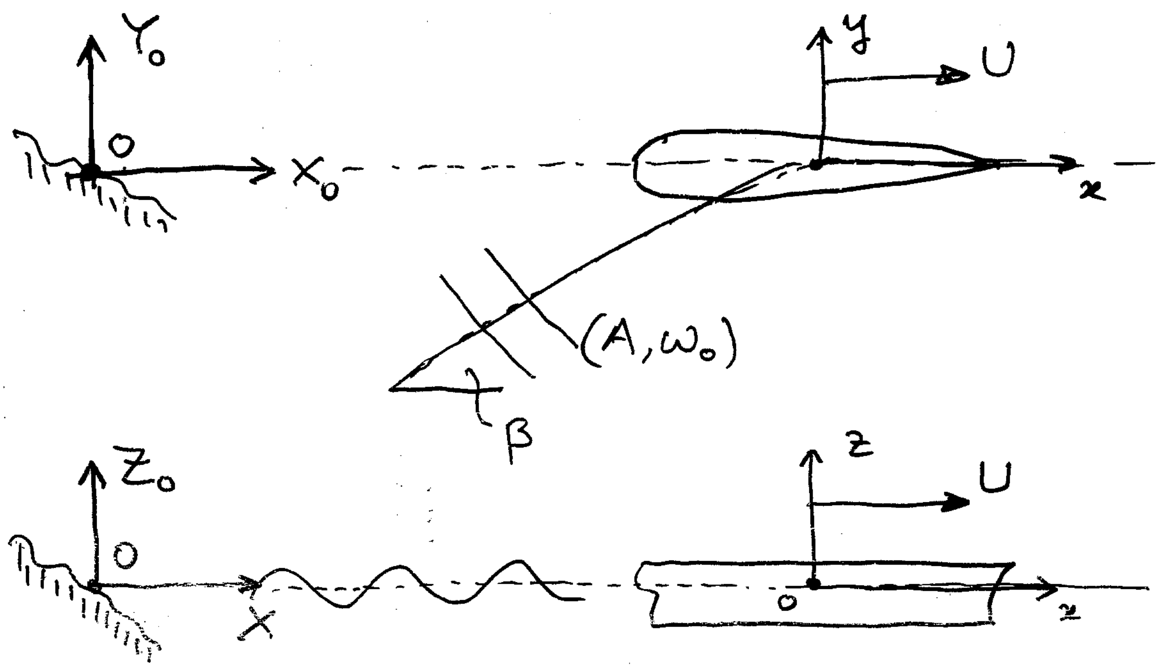
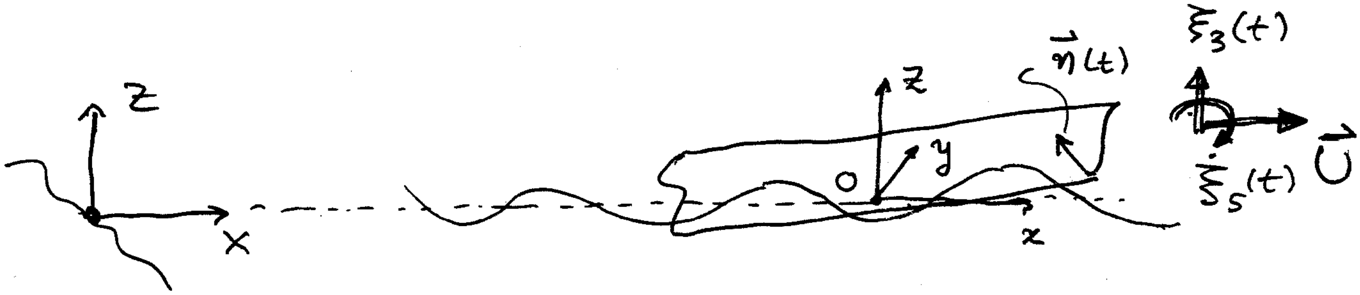
SHIP MOTIONS WITH FORWARD SPEED



- SHIP ADVANCES IN THE POSITIVE X-DIRECTION WITH CONSTANT SPEED U
- REGULAR WAVES WITH ABSOLUTE FREQUENCY ω_0 AND DIRECTION β ARE INCIDENT UPON THE SHIP
- THE SHIP UNDERGOES OSCILLATORY MOTIONS IN ALL SIX DEGREES OF FREEDOM $\xi_j(t)$, $j = 1, \ldots, 6$
- THE SHIP SEAKEEPING PROBLEM WILL BE TREATED IN THE FREQUENCY DOMAIN UNDER THE ASSUMPTION OF LINEARITY.

- ## SHIP-HULL BOUNDARY CONDITION



THE RELEVANT RIGID BODY VELOCITIES FOR A SHIP TRANSLATING WITH A CONSTANT FORWARD VELOCITY AND HEAVING AND PITCHING IN HEAD WAVES ARE:

$$\vec{U} = U\vec{i} \; ; \quad \text{FORWARD SPEED}$$

$$\vec{\dot{\xi}}_3(t) = \dot{\xi}_3(t)\vec{k} \; ; \quad \text{HEAVE VELOCITY (POSITIVE UP)}$$

$$\vec{\dot{\xi}}_5(t) = \dot{\xi}_5(t)\vec{j} \; ; \quad \text{PITCH ANGULAR VELOCITY}$$

(POSITIVE, BOW DOWN)

LET $\vec{n}(t)$ THE TIME-DEPENDENT UNIT NORMAL VECTOR TO THE INSTANTANEOUS POSITION OF THE SHIP HULL. LET $\vec{n}_0$ BE ITS VALUE WHEN THE SHIP IS AT REST.

- THE BODY BOUNDARY CONDITION WILL BE DERIVED FOR SMALL HEAVE & PITCH MOTIONS AND WILL BE STATED ON THE MEAN POSITION OF THE SHIP AT REST.

THE NONLINEAR BOUNDARY CONDITION ON THE EXACT POSITION OF THE SHIP HULL IS:

$$\frac{\partial \phi}{\partial n} = \vec{V} \cdot \vec{n}$$

WHERE

$$\phi = \phi_3 + \phi_5$$

$$\vec{V} \cong U\vec{i} + \vec{k}\,(\dot{\xi}_3 - x\,\dot{\xi}_5)$$

- THE UNSTEADY VELOCITY POTENTIAL ϕ HAS BEEN WRITTEN AS THE LINEAR SUPERPOSITION OF THE HEAVE AND PITCH COMPONENTS. THE TOTAL RIGID-BODY VELOCITY IS THE SUM OF THE FORWARD VELOCITY AND THE LINEAR VERTICAL VELOCITY DUE TO THE SHIP HEAVE AND PITCH

- THE NORMAL VECTOR $\vec{n}(t)$ IS DEFINED BY

$$\vec{n}(t) = \vec{n}_0 + \vec{\xi}_5 \times \vec{n}_0$$

WHERE $\vec{\xi}_5(t)$ IS THE SHIP PITCH ROTATION ANGLE, AND:

$$\vec{n}_0 \equiv (n_1, n_2, n_3).$$

KEEPING ONLY THE UNSTEADY COMPONENTS:

$$\vec{V} \cdot \vec{n} = U \xi_5 n_3 + (\dot{\xi}_3 - x \dot{\xi}_5) n_3 = \frac{\partial \phi}{\partial n}$$

- NOTE THAT THE STEADY COMPONENT $U n_1$ HAS ALREADY BEEN ACCONTED FOR IN THE STATEMENT OF THE STEADY FLOW. LET

$$\phi = \mathbb{R}e \left(\varphi e^{i\omega t} \right), \quad \varphi = \Xi_3 \chi_3 + \Xi_5 \chi_5$$

WHERE φ_3 AND φ_5 ARE THE UNIT-AMPLITUDE HEAVE & PITCH COMPLEX VELOCITY POTENTIALS IN TIME-HARMONIC FLOW. ALSO:

$$\xi_3(t) = \mathbb{R}e \left\{ \Xi_3(\omega) e^{i\omega t} \right\}, \quad \xi_5(t) = \mathbb{R}e \left\{ \Xi_5 e^{i\omega t} \right\}$$

IT FOLLOWS THAT:

- $\dfrac{\partial \chi_3}{\partial n} = i\omega n_3 \quad ; \text{ on } \bar{S}_B$

- $\dfrac{\partial \chi_5}{\partial n} = -i\omega x n_3 + U n_3 \quad ; \text{ on } \bar{S}_B$

- NOTE THE FORWARD-SPEED EFFECT IN THE PITCH BOUNDARY CONDITION AND NO SUCH EFFECT IN HEAVE

SHIP-HULL BOUNDARY CONDITIONS

- DIFFRACTION PROBLEM: $\varphi = \varphi_7$

$$\frac{\partial \varphi_7}{\partial n} = -\frac{\partial \varphi_I}{\partial n} \quad , \quad \frac{\partial}{\partial n} \equiv \vec{n} \cdot \nabla$$

WHERE:

$$\varphi_I = \frac{igA}{\omega_0} e^{kz - ikx\cos\beta - iky\sin\beta}$$

- RADIATION PROBLEM : $\varphi = \varphi_3 + \varphi_5$

WE WILL CONSIDER THE SPECIAL BUT IMPORTANT CASE OF HEAVE & PITCH WHICH ARE COUPLED AND IMPORTANT MODES TO STUDY IN THE SHIP SEAKEEPING PROBLEM.

- HEAVE AND PITCH ARE THE ONLY MODES OF INTEREST IN HEAD WAVES ($\beta = 180°$) WHEN ALL OTHER MODES OF MOTION (ROLL-SWAY-YAW) ARE IDENTICALLY ZERO FOR A SHIP SYMMETRIC PORT-STARBOARD. SURGE IS NONZERO BUT GENERALLY SMALL FOR SLENDER SHIPS AND IN AMBIENT WAVES OF SMALL STEEPNESS.

- BERNOULLI EQUATION

THE LINEAR HYDRODYNAMIC DISTURBANCE PRESSURE DUE TO UNSTEADY FLOW DISTURBANCES IS GIVEN RELATIVE TO THE SHIP FRAME:

$$p = -\rho \left(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x} \right) \phi$$

WHERE ϕ IS THE RESPECTIVE REAL POTENTIAL

RADIATION PROBLEM

$$\phi = Re \{ \varphi e^{i\omega t} \}$$

$$\varphi = \Xi_3 \varphi_3 + \Xi_5 \varphi_5$$

$$p = Re \{ \mathbb{P} e^{i\omega t} \}$$

$$\mathbb{P} = -\rho \left(i\omega - U \frac{\partial}{\partial x} \right) \left(\Xi_3 \varphi_3 + \Xi_5 \varphi_5 \right)$$

WHERE THE COMPLEX VELOCITY POTENTIALS SATISFY THE 2D BOUNDARY VALUE PROBLEMS DERIVED EARLIER FOR SLENDER SHIPS.

- THE HYDRODYNAMIC PRESSURE WILL BE INTEGRATED OVER THE SHIP HULL TO OBTAIN THE ADDED-MASS AND DAMPING COEFFICIENTS NEXT:

$$F_i(t) = \iint_{\bar{S}_B} p\, n_i\, ds, \quad i = 3,5$$

THE EXPRESSIONS DERIVED BELOW EXTEND ALMOST TRIVIALLY TO ALL OTHER MODES OF MOTION. IN GENERAL

$$n_i \equiv \vec{n} = (n_1, n_2, n_3)\ ; \quad i = 1,2,3$$

$$n_i \equiv \vec{x} \times \vec{n} = (n_4, n_5, n_6)\ ; \quad i = 4,5,6$$

SO FOR HEAVE:

$$n_i = n_3$$

AND PITCH:

$$n_i = n_5 = -x n_3 + z n_1$$

$$\simeq -x n_3 \quad (n_1 << n_3)$$

EXPRESSING F_i IN TERMS OF THE HEAVE & PITCH ADDED MASS & DAMPING COEFFICIENTS WHEN THE SHIP IS FORCED TO OSCILLATE IN CALM WATER, WE OBTAIN WHEN ACCOUNTING FOR CROSS-COUPLING EFFECTS:

$$F_i^H(t) = - \sum_{j=3,5} \left[A_{ij} \frac{d^2 \xi_j}{dt^2} + B_{ij} \frac{d \xi_j}{dt} \right]$$

HYDROSTATIC RESTORING EFFECTS ARE UNDERSTOOD TO BE ADDED TO F_i^H

INTRODUCING COMPLEX NOTATION:

$$F_i^H(t) = \mathrm{Re}\{\mathbb{F}_i e^{i\omega t}\}$$

$$\mathbb{F}_i(\omega) = [\omega^2 A_{ij}(\omega) - i\omega B_{ij}(\omega)]\,\Xi_j$$

WHERE THE SUMMATION NOTATION OVER j IS UNDERSTOOD HEREAFTER.

INTRODUCING THE DEFINITION OF $\mathbb{F}_i$ IN TERMS OF THE HYDRODYNAMIC PRESSURE WE OBTAIN AFTER SOME SIMPLE ALGEBRA:

$$\mathbb{P} = \mathbb{P}_j \Xi_j \equiv \mathbb{P}_3 \Xi_3 + \mathbb{P}_5 \Xi_5$$

$$\mathbb{F}_i = \left(\iint_{\bar{S}_B} \mathbb{P}_j n_i \, ds \right) \Xi_j$$

AND:

- $$\omega^2 A_{ij}(\omega) - i\omega B_{ij}(\omega) = \iint_{\bar{S}_B} \mathbb{P}_i n_j \, ds$$

WHERE FROM BERNOULLI:

$$\mathbb{P}_3 = \left(i\omega - U\frac{\partial}{\partial x} \right) \varphi_3 = \left(i\omega - U\frac{\partial}{\partial x} \right)(i\omega x_3)$$

$$\mathbb{P}_5 = \left(i\omega - U\frac{\partial}{\partial x} \right) \varphi_5 = \left(i\omega - U\frac{\partial}{\partial x} \right)(-i\omega x x_3 + U x_3).-$$

STRIP THEORY

- STRIP THEORY IS A POPULAR APPROXIMATION OF THE 3-D NEUMANN-KELVIN FORMUCATION FOR SHIPS WHICH ARE SLENDER AS IS MOST OFTEN THE CASE WHEN VESSELS ARE EXPECTED TO CRUISE AT SIGNIFICANT FORWARD SPEEDS.

  THE PRINCIPAL ASSUMPTION IS:

$$\frac{B}{L}, \frac{T}{L} = O(\varepsilon) \quad , \quad \varepsilon \ll 1$$

  WHERE B: SHIP MAXIMUM BEAM

  T: SHIP MAXIMUM DRAFT

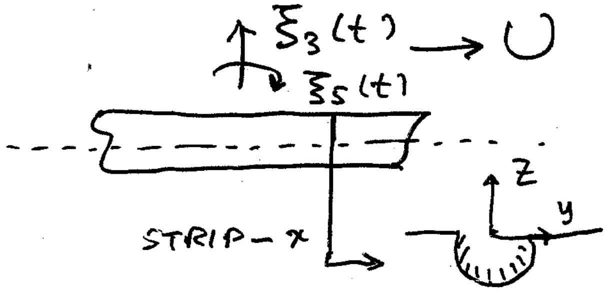
  L: SHIP WATELINE LENGTH.

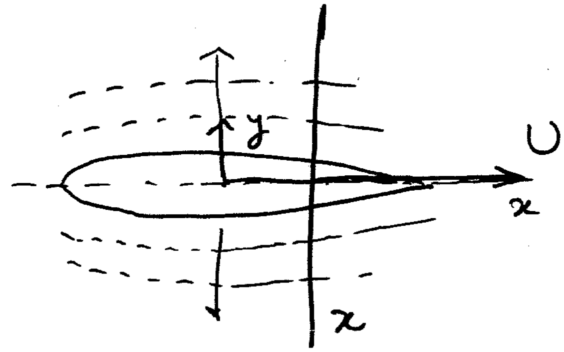
- THE PRINCIPAL ASSUMPTION OF STRIP THEORY IS THAT CERTAIN COMPONENTS OF THE RADIATION AND DIFFRACTION POTENTIALS VARY SLOWLY ALONG THE SHIP LENGTH LEADING TO A SIMPLIFICATION OF THE N-K FORMULATION

- IN HEAD OR BOW WAVES WHERE HEAVE AND PITCH ATTAIN THEIR MAXIMUM VALUES, THE ENCOUNTER FREQUENCY ω IS USUALLY HIGH.

RADIATION PROBLEM

THE SHIP IS FORCED TO OSCILLATE IN HEAVE & PITCH IN CALM WATER WHILE ADVANCING AT A SPEED U.





- DUE TO SLENDERNESS THE VARIATION OF THE FLOW IN THE X-DIRECTION IS MORE GRADUAL THAN ITS VARIATION AROUND A SHIP SECTION. SO

$$\frac{\partial}{\partial x}\phi \ll \frac{\partial \phi}{\partial z}, \frac{\partial \phi}{\partial z}$$

WHERE $\phi = \phi_3 + \phi_5$. THUS THE 3D LAPLACE EQUATION SIMPLIFIES INTO A 2D FORM FOR THE HEAVE & PITCH POTENTIALS. (THE SAME ARGUMENT APPLIES TO ROLL-SWAY-YAW). THUS

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi_j \cong 0, \quad \text{IN FLUID}$$
$$j = 2, \dots, 6$$

- THIS 2D EQUATION APPLIES FOR EACH "STRIP" LOCATED AT STATION-x.

- THE SHIP-HULL CONDITION AT STATION-X FOR THE HEAVE & PITCH POTENTIALS REMAINS THE SAME:

  - $$\phi = \mathbb{R}e\left\{ \varphi e^{i\omega t} \right\}$$

  - $$\varphi = \Xi_3 \chi_3 + \Xi_5 \chi_5$$

  - $$\frac{\partial \chi_3}{\partial n} = i\omega n_3 \quad ; \text{ on } \bar{S}_B$$

  - $$\frac{\partial \chi_5}{\partial n} = -i\omega x n_3 + U n_3 \; ; \text{ on } \bar{S}_B$$

WHERE NOW:

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \chi_j = 0, \quad \text{IN FLUID DOMAIN}$$

- DEFINE THE NORMALIZED POTENTIAL ψ_3:

$$\frac{\partial \psi_3}{\partial n} = n_3 \quad ; \text{ on } \bar{S}_B$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi_3 = 0, \quad \text{IN FLUID}$$

- THERE REMAINS TO SIMPLIFY THE 3D N-K FREE-SURFACE CONDITION:

- THE SOLUTION OF THE 2D BVP FOR ψ_3 ALONG 30 – 40 STATIONS USED TO DESCRIBE THE HULL FORM OF MOST SHIPS CAN BE CARRIED OUT VERY EFFICIENTLY BY STANDARD 2D PANEL METHODS.

- IN TERMS OR $\psi_3(y,z;x)$ THE HEAVE & PITCH POTENTIALS FOLLOW IN THE FORM:

$$\varphi = \Xi_3 \chi_3 + \Xi_5 \chi_5$$

$$\chi_3 = i\omega \psi_3$$

$$\chi_5 = (-i\omega x + U)\psi_3$$

$$\Phi = \mathrm{Re}\{\varphi e^{i\omega t}\} = \phi_3 + \phi_5$$

- THE 2D HEAVE ADDED-MASS AND DAMPING COEFFICIENTS DUE TO A SECTION OSCILLATING VERTICALLY ARE DEFINED BY THE FAMILIAR EXPRESSIONS

STATION-X: $$a_{33}(\omega) - \frac{i}{\omega} b_{33}(\omega) = \rho \int_{C(x)} \psi_3 n_3 \, dl$$

WHERE

$$\omega = \left| \omega_0 - U \frac{\omega_0^2}{g} \cos\beta \right|$$

IS THE ENCOUNTER FREQUENCY. –

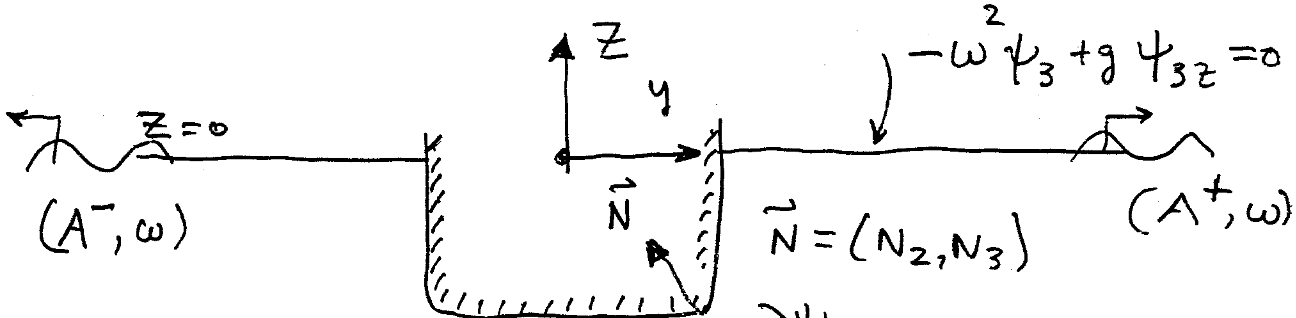
$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2 \varphi + g\,\varphi_z = 0, \quad z=0$$

- THE SHIP SLENDERNESS AND THE CLAIM THAT ω IS USUALLY LARGE IN HEAD OF BOW WAVES IS USED TO SIMPLIFY THE ABOVE EQUATION AS FOLLOWS

$$-\omega^2 \varphi + g\varphi_z = 0, \quad z=0$$

BY ASSUMING THAT $\omega \gg \left|U\frac{\partial}{\partial x}\right|$. A FORMAL PROOF IS LENGTHY AND TECHNICAL.

IT FOLLOWS THAT THE NORMALIZED POTENTIAL ψ_3 ASLO SATISFIES THE ABOVE 2D FS CONDITION AND IS THUS THE SOLUTION OF THE 2D BOUNDARY VALUE PROBLEM STATED BELOW AT STATION-x:



$$-\omega^2\psi_3 + g\,\psi_{3z} = 0$$

$$\vec{N} = (N_2, N_3)$$

$$\frac{\partial \psi_3}{\partial N} = N_3$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi_3 = 0$$

AS $y \to \infty$; $\psi_3 \sim \frac{igA^\pm}{\omega} e^{kz \mp iky + i\omega t}$.—

UPON INTEGRATION ALONG THE SHIP LENGTH AND OVER EACH CROSS SECTION AT STATION $-x$ THE 3D ADDED-MASS AND DAMPING COEFFICIENTS FOR HEAVE AND PITCH TAKE THE FORM:

$$A_{33} = \int_L dx \; a_{33}(x)$$

$$B_{33} = \int_L dx \; b_{33}(x)$$

$$A_{35} = -\int_L dx \; x \, a_{33} - \frac{U}{\omega^2} B_{33}$$

$$A_{53} = -\int_L dx \; x \, a_{33} + \frac{U}{\omega^2} B_{33}$$

$$B_{35} = -\int_L dx \; x \, b_{33} + U A_{33}$$

$$B_{53} = -\int_L dx \; x \, b_{33} - U A_{33}$$

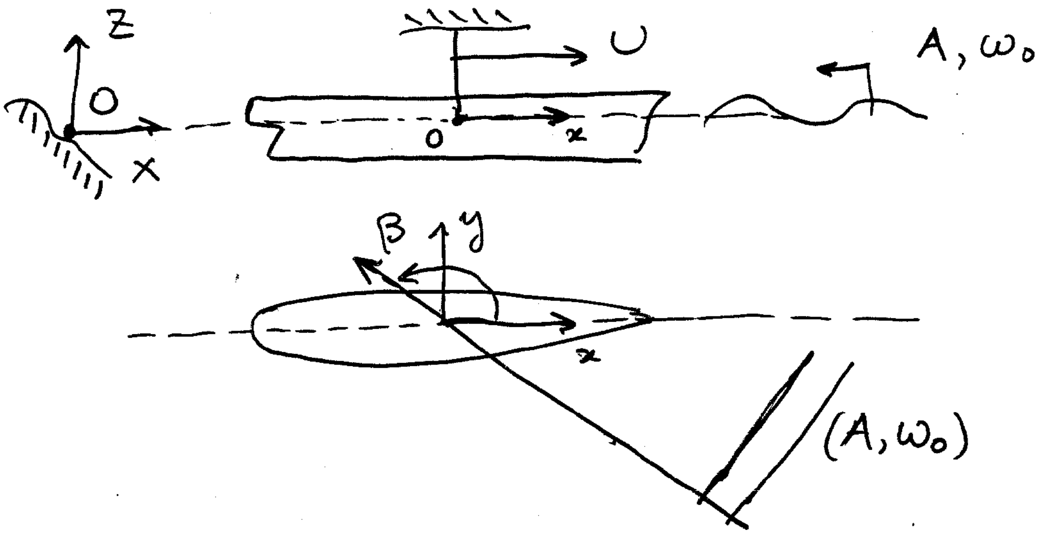
$$A_{55} = \int_L dx \; x^2 a_{33} + \frac{U^2}{\omega^2} A_{33}$$

$$B_{55} = \int_L dx \; x^2 b_{33} + \frac{U^2}{\omega^2} B_{33}$$

WHERE THE 2D ADDED-MASS AND DAMPING COEFFICIENTS WERE DEFINED ABOVE:

$$a_{33} - \frac{i}{\omega} b_{33} = \rho \int_{C(x)} \psi_3 \, n_3 \, dl$$

DIFFRACTION PROBLEM



- WE WILL CONSIDER HEAVE & PITCH IN OBLIQUE WAVES. NOTE THAT IN OBLIQUE WAVES THE SHIP ALSO UNDERGOES ROLL-SWAY-YAW MOTIONS WHICH FOR A SYMMETRIC VESSEL AND ACCORDING TO LINEAR THEORY ARE DECOUPLED FROM HEAVE AND PITCH

- RELATIVE TO THE SHIP FRAME THE TOTAL POTENTIAL IS:

$$\Phi = \Phi_I + \Phi_D = \mathrm{Re}\left\{ (\varphi_I + \varphi_D)\, e^{i\omega t} \right\}$$

WHERE ω IS THE ENCOUNTER FREQUENCY, AND:

$$\varphi_I = \frac{igA}{\omega_0}\, e^{kz - ikx\cos\beta - iky\sin\beta}, \qquad k = \omega_0^2/g.$$

DEFINE THE DIFFRACTION POTENTIAL AS FOLLOWS:

$$\varphi_D = \frac{igA}{\omega_0} e^{-ikx\cos\beta} \psi_7(y, z; x)$$

IN WORDS, FACTOR-OUT THE OSCILLATORY VARIATION $e^{-ikx\cos\beta}$ OUT OF THE SCATTERING POTENTIAL.

- THE SHIP SLENDERNESS APPROXIMATION NOW JUSTIFIES THAT:

$$\frac{\partial \psi_7}{\partial x} << \frac{\partial \psi_7}{\partial y}, \frac{\partial \psi_7}{\partial z}$$

NOTE THAT THIS IS NOT AN ACCURATE APPROXIMATION FOR φ_D WHEN $K = 2\pi/\lambda$ IS A LARGE QUANTITY OR WHEN THE AMBIENT WAVELENGTH λ IS SMALL

- SUBSTITUTING IN THE 3D LAPLACE EQUATION AND IGNORING THE $\partial\psi_7/\partial x$, $\partial^2\psi_7/\partial x^2$ TERMS WE OBTAIN

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - k^2\cos^2\beta \right) \psi_7 \cong 0$$

- THIS IS THE MODIFIED 2D HELMHOLTZ EQUATION IN MOST CASES THE $k^2\cos^2\beta$ TERM IS NOT IMPORTANT FOR REASONS TO BE DISCUSSED AND THE 2D LAPLACE EQ. FOLLOWS.—

- THE BODY BOUNDARY CONDITION SIMPLY STATES:

$$\frac{\partial \varphi_D}{\partial n} = - \frac{\partial \varphi_I}{\partial n}$$

WITH:

$$\frac{\partial}{\partial n} \equiv n_1 \frac{\partial}{\partial x} + n_2 \frac{\partial}{\partial y} + n_3 \frac{\partial}{\partial z}$$

DUE TO SLENDERNESS:

$$n_1 << n_2, n_3$$

INVOKING THE SLENDERNESS APPROXIMATIONS FOR THE $\partial/\partial x$ DERIVATIVES OF ψ_7, THE BODY BOUNDARY CONDITION SIMPLIFIES TO:

$$\frac{\partial \psi_7}{\partial N} = - \frac{\partial}{\partial N} \left(e^{kz - iky\sin\beta} \right) ; \text{ ON } C(x)$$

WITH:

$$\frac{\partial}{\partial N} = N_2 \frac{\partial}{\partial y} + N_3 \frac{\partial}{\partial z}$$

THE POTENTIAL ψ_7 SATISFIES THE 2D LAPLACE EQUATION, AFTER DROPPING THE $k^2\cos^2\beta$ TERM, OR

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi_7 = 0$$

AND THE FREE-SURFACE CONDITION DERIVED NEXT:

THE 3D LINEAR FREE SURFACE CONDITION FOR φ_D IS:

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2 \varphi_D + g\frac{\partial \varphi_D}{\partial z} = 0 \quad ; \quad z=0$$

THE SAME CONDITION IS SATISFIED BY φ_I:

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2 \varphi_I + g\frac{\partial \varphi_I}{\partial z} = 0 \quad ; \quad z=0$$

VERIFY IDENTICALLY FOR φ_I THAT:

$$i\omega - U\frac{\partial}{\partial x} \equiv i\omega_0$$

SO THE FORWARD-SPEED EFFECT DISAPPEARS FROM THE FREE SURFACE CONDITION OF φ_I. THE SAME WILL BE SHOWN TO BE APPROXIMATELY TRUE FOR THE DIFFRACTION POTENTIAL φ_D:

$$\varphi_D = \frac{igA}{\omega_0} e^{-ikx\cos\beta} \psi_7(y,z;x)$$

$$\frac{\partial \varphi_D}{\partial x} = \frac{igA}{\omega_0}\left\{(-ik\cos\beta)\psi_7(y,z;x) + \frac{\partial \psi_7}{\partial x}\right\} e^{-ikx\cos\beta}$$

DUE TO SLENDERNESS: $\left|\frac{\partial \psi_7}{\partial x}\right| \ll |k\cos\beta|\,|\psi_7|$

THEREFORE:

$$\frac{\partial \varphi_D}{\partial x} \simeq -ik\cos\beta\, \varphi_D$$

AND FROM THE FREE-SURFACE CONDITION

$$\left(i\omega - U\frac{\partial}{\partial x} \right) \varphi_D \simeq i\omega_0 \varphi_D$$

WHERE:

$$\omega = \omega_0 - U k \cos\beta$$

SO THE FREE-SURFACE CONDITION FOR φ_D BECOMES TO LEADING ORDER IN SLENDERNESS:

$$-\omega_0^2 \varphi_D + g\frac{\partial \varphi_D}{\partial z} = 0, \quad z = 0$$

- SO IN THE STATEMENT OF THE DIFFRACTION PROBLEM FORWARD-SPEED EFFECTS ARE ABSENT! THE SAME IS NOT TRUE IN THE RADIATION PROBLEM EVEN FOR SLENDER SHIPS

- THIS "INDEPENDENCE" OF THE DIFFRACTION PROBLEM ON FORWARD SPEED EFFECTS IS VERIFIED VERY WELL IN EXPERIMENTS AND IN FULLY THREE-DIMENSIONAL SOLUTIONS.

IN SUMMARY, THE BOUNDARY-VALUE PROBLEM SATISFIED BY THE DIFFRACTION POTENTIAL AROUND A SLENDER SHIP TAKES THE FORM:

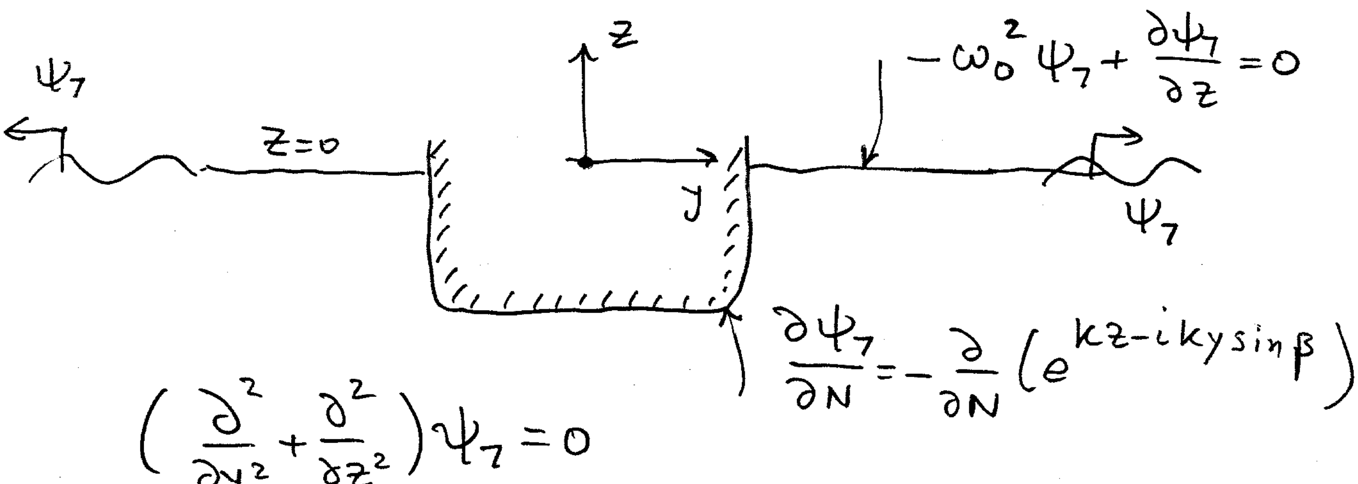
- $$\varphi_D = \frac{igA}{\omega_0} e^{-ikx\cos\beta} \psi_7(y,z;x)$$

- $$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi_7 = 0, \quad \text{IN FLUID}$$

- $$-\omega_0^2 \psi_7 + g\frac{\partial \psi_7}{\partial z} = 0, \quad \text{ON } z=0$$

- $$\frac{\partial \psi_7}{\partial N} = -\frac{\partial}{\partial N}\left(e^{kz-iky\sin\beta}\right)$$

OR IN GRAPHICAL FORM AT STATION-x:



- AS $y \to \pm\infty$ $$\psi_7 \sim \frac{igA_7^{\pm}}{\omega_0} e^{kz \mp iky}$$

DIFFRACTION PROBLEM

THE HYDRODYNAMIC PRESSURE IN THE DIFFRACTION PROBLEM TAKES THE FORM:

$$p = \mathrm{Re}\{ \mathbb{P} e^{i\omega t} \}$$

$$\mathbb{P} = -\rho \left(i\omega - U \frac{\partial}{\partial x} \right) \frac{igA}{\omega_0} e^{-ikx\cos\beta} \times (\psi_I + \psi_7)$$

$$\cong \rho g A \, e^{-ikx\cos\beta} (\psi_I + \psi_7)$$

WHERE $\partial\psi_7/\partial x$ DERIVATIVES ARE DROPPED IN FAVOR OF $k\cos\beta$, DUE TO SLENDERNESS.

THE HEAVE AND PITCH EXCITING FORCES FOLLOW SIMPLY BY PRESSURE INTEGRATION

$$X_i(t) = \mathrm{Re}\{ \mathbb{X}_i e^{i\omega t} \} \; ; \; i = 3, 5$$

$$\mathbb{X}_3 = \rho g A \int_L dx \, e^{-ikx\cos\beta} \int_{C(x)} (\psi_I + \psi_7) \, n_3 \, dl$$

$$\mathbb{X}_5 = \rho g A \int_L dx \, (-x) \, e^{-ikx\cos\beta} \int_{C(x)} (\psi_I + \psi_7) \, n_3 \, dl$$

WHERE THE APPROXIMATION $n_5 = -x n_3$ WAS USED.

- IN SUMMARY, A VERSION OF STRIP THEORY WAS DERIVED FOR SLENDER SHIPS WHICH HAS BEEN FOUND TO BE THE MOST RATIONAL AND ACCURATE RELATIVE TO OTHER ALTERNATIVES. THIS IS PARTICULARLY TRUE FOR THE TREATMENT OF THE DIFFRACTION PROBLEM.

- THE COUPLED HEAVE & PITCH EQUATIONS OF MOTION TAKE THE FAMILIAR FORM

$$\left[-\omega^2 (A_{ij} + M_{ij}) + i\omega B_{ij} + C_{ij}\right] \Xi_j = \mathbb{X}_i(\omega_0)$$

$$i, j = 3, 5$$

WHERE IN PRACTICE:

$$\omega = \left| \omega_0 - U \frac{\omega_0^2}{g} \cos\beta \right|$$

AND THE $A_{ij}(\omega)$, $B_{ij}(\omega)$ ARE FUNCTIONS OF ω DEFINED ABOVE AS INTEGRALS OF THEIR 2D COUNTERPARTS $a_{33}(\omega)$ & $b_{33}(\omega)$.

- THE HEAVE & PITCH EXCITING FORCE AMPLITUDES $\mathbb{X}_i(\omega_0)$ ARE FUNCTIONS OF ω_0, HENCE NOT DEPENDENT ON U.

- ALL HYDRODYNAMIC EFFECTS OSCILLATE AT ω!