SOLUTION OF WAVE-BODY INTERACTION PROBLEMS

- TWO TYPES OF WAVE BODY INTERACTION PROBLEMS ARE ENCOUNTERED FREQUENTLY IN APPLICATIONS AND SOLVED BY THE METHODS DESCRIBED IN THIS SECTION

  - ZERO-SPEED LINEAR WAVE BODY INTERACTIONS IN THE FREQUENCY DOMAIN IN 2D AND 3D

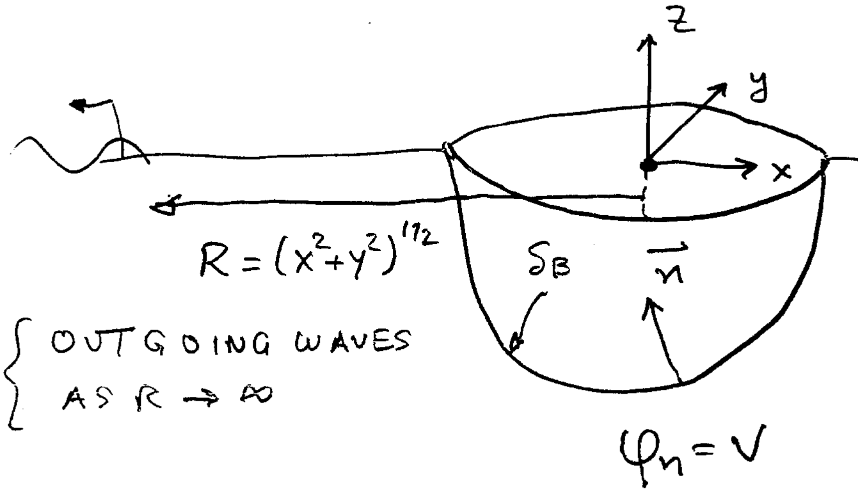
  - FORWARD-SPEED SEAKEEPING PROBLEMS IN THE FREQUENCY OR TIME DOMAIN IN THREE DIMENSIONS (LINEAR & NONLINEAR)

- A CONSENSUS HAS BEEN REACHED OVER THE PAST TWO DECADES THAT THE MOST EFFICIENT AND ROBUST SOLUTION METHODS ARE BASED ON <u>GREEN'S THEOREM</u> USING EITHER A WAVE-SOURCE POTENTIAL OR THE RANKINE SOURCE AS THE GREEN FUNCTION. –

- THE NUMERICAL SOLUTION OF THE RESULTING INTEGRAL EQUATIONS IN PRACTICE IS IN ALMOST ALL CASES CARRIED OUT BY <u>PANEL METHODS</u>. –

FREQUENCY-DOMAIN RADIATION-DIFFRACTION. $U=0$

BOUNDARY-VALUE PROBLEM:



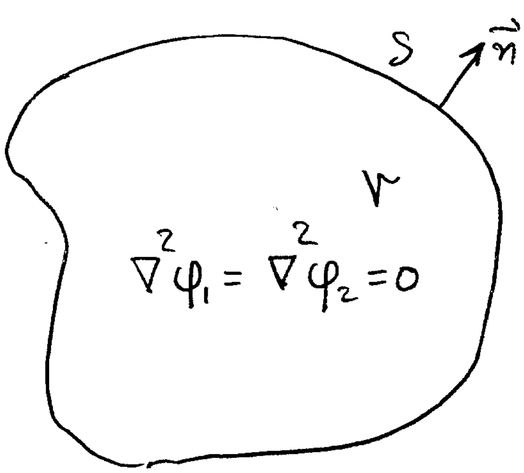
$z=0$

$$\varphi_z - \frac{\omega^2}{g}\varphi = 0$$

$$\underbrace{\varphi_{xx}+\varphi_{yy}+\varphi_{zz}=0}_{\nabla^2\varphi=0}$$

$$V = \begin{cases} n_i, \; i=1,\dots,6; & \text{RADIATION} \\ -\dfrac{\partial \varphi_I}{\partial n}; & \text{DIFFRACTION} \end{cases}$$

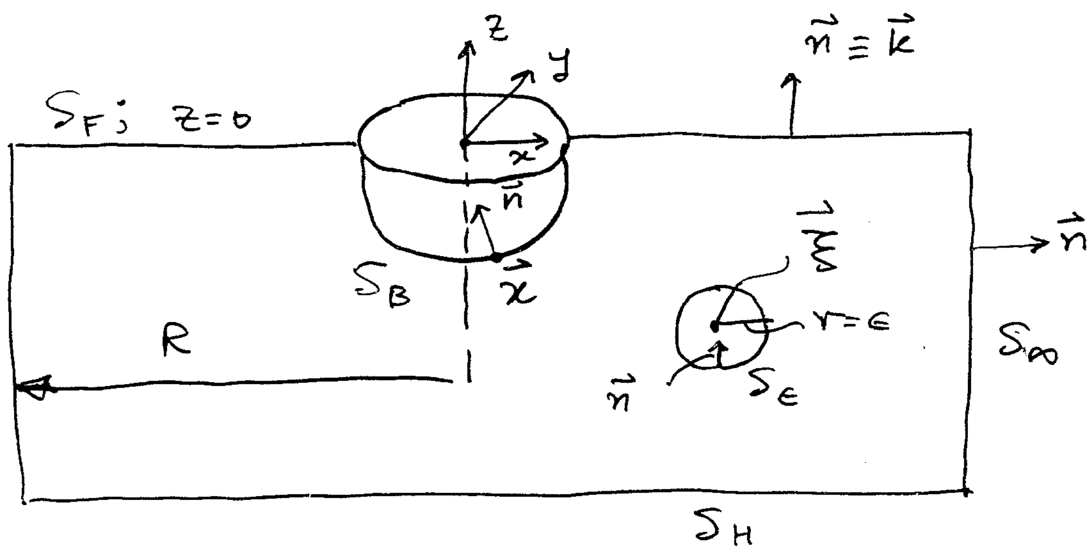
- GREEN'S THEOREM GENERATES A BOUNDARY INTEGRAL EQUATION FOR THE COMPLEX POTENTIAL φ OVER THE BODY BOUNDARY S_B FOR THE PROPER CHOICE OF THE GREEN FUNCTION:



$$\oiint_S \left(\varphi_1 \frac{\partial \varphi_2}{\partial n} - \varphi_2 \frac{\partial \varphi_1}{\partial n}\right) ds = 0$$

FOR ANY φ_1, φ_2 THAT SOVLE THE LAPLACE EQ. IN A CLOSED VOLUME $\mathcal{V}$.

DEFINE THE VOLUME $\forall$ AND S AS FOLLOWS:



THE FLUID VOLUME $\forall$ IS ENCLOSED BY THE UNION OF SEVERAL SURFACES

$$S \equiv S_B + S_F + S_\infty + S_H + S_\epsilon$$

S_B : MEAN POSITION OF BODY SURFACE

S_F : MEAN POSITION OF THE FREE SURFACE

S_∞ : BOUNDING CYLINDRICAL SURFACE WITH RADIUS $R = (x^2+y^2)^{1/2}$. WILL BE ALLOWED TO EXPAND AFTER THE STATEMENT OF GREEN'S THEOREM

S_H : SEAFLOOR (ASSUMED FLAT) OR A SURFACE WHICH WILL BE ALLOWED TO APPROACH $z=-\infty$

S_ϵ : SPHERICAL SURFACE WITH RADIUS $r=\epsilon$ CENTERED AT POINT $\vec{\xi}$ IN THE FLUID DOMAIN

$\vec{n}$: UNIT NORMAL VECTOR ON S, AT POINT $\vec{x}$ ON S

DEFINE TWO VELOCITY POTENTIALS $\varphi_i(\vec{x})$:

$\varphi_1(\vec{x}) = \varphi(\vec{x}) \equiv$ UNKOWN COMPLEX RADIATION OR DIFFRACTION POTENTIAL

$\varphi_2(\vec{x}) = G(\vec{x};\vec{\xi}) \equiv$ GREEN FUNCTION VALUE AT POINT $\vec{x}$ DUE TO A SINGULARITY CENTERED AT POINT $\vec{\xi}$.

TWO TYPES OF GREEN FUNCTIONS WILL BE USED:

- RANKINE SOURCE: $\nabla_x^2 G = 0$

$$G(\vec{x};\vec{\xi}) = -\frac{1}{4\pi}\left|\vec{x}-\vec{\xi}\right|^{-1} = -\frac{1}{4\pi r} =$$

$$= -\frac{1}{4\pi}\left\{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2\right\}^{-1/2}$$

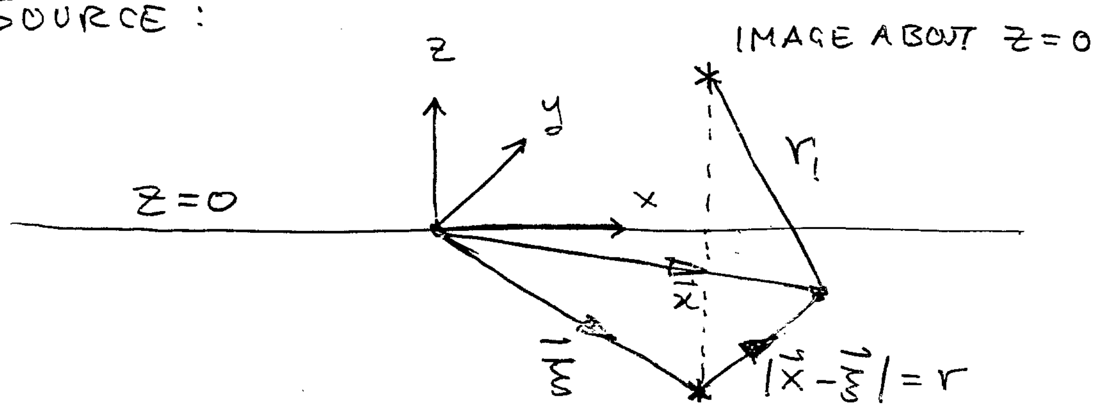
NOTE THAT THE FLUX OF FLUID EMITTED FROM $\vec{\xi}$ IS EQUAL TO 1. (VERIFY)

– THIS RANKINE SOURCE AND ITS GRADIENT WITH RESPECT TO $\vec{\xi}$ (DIPOLES) IS THE GREEN FUNCTION THAT WILL BE USED IN THE SHIP SEAKEEPING PROBLEM

HAVELOCK'S WAVE SOURCE POTENTIAL

... ALSO KNOWN AS THE $U=0$ WAVE GREEN FUNCTION IN THE FREQUENCY DOMAIN.

— SATISFIES THE FREE SURFACE CONDITION AND NEAR $\vec{\xi}=0$ BEHAVES LIKE A RANKINE SOURCE:



THE FOLLOWING CHOICE FOR $G(\vec{x};\vec{\xi})$ SATISFIES THE LAPLACE EQUATION AND THE FREE-SURFACE CONDITION:

$$G(\vec{x},\vec{\xi}) = -\frac{1}{4\pi}\left(\frac{1}{r}+\frac{1}{r_1}\right)$$

$$-\frac{k}{2\pi}\int_0^\infty \frac{du}{u-k}\, e^{u(z+\zeta)}\, J_0(uR)$$

WHERE: $k=\omega^2/g$

$R^2=(x-\xi)^2+(y-\eta)^2$

$J_0(uR)=$ BESSEL FUNCTION OF ORDER ZERO

$\int$: CONTOUR INDENTED ABOVE POLE $u=k$

VERIFY THAT WITH RESPECT TO THE ARGUMENT $\vec{x}$, THE VELOCITY POTENTIAL $\varphi_2(\vec{x}) \equiv G(\vec{x};\vec{\xi})$ SATISFIES THE FREE SURFACE CONDITION:

$$\frac{\partial \varphi_2}{\partial z} - k\varphi_2 = 0, \quad z = 0$$

$$\varphi_2 \sim -\frac{1}{4\pi} r^{-1}, \quad \vec{x} \to \vec{\xi}$$

AS $kR \to \infty$:

$$G \sim -\frac{i}{2} k e^{k(z+\zeta)} H_0^{(2)}(kR)$$

WHERE $H_0^{(2)}(kR)$ IS THE HANKEL FUNCTION OF THE SECOND KIND AND ORDER ZERO.

As $kR \to \infty$:

$$H_0^{(2)}(kR) \sim \sqrt{\frac{2}{\pi kR}}\, e^{-i\left(kR - \frac{\pi}{4}\right)} + O(1/R)$$

- THEREFORE THE REAL VELOCITY POTENTIAL

$$\mathbb{G} = \mathrm{Re}\{G e^{i\omega t}\}$$

REPRESENTS OUTGOING RING WAVES OF THE FORM $\propto e^{i(\omega t - kR)}$ HENCE SATISFYING THE RADIATION CONDITION.

- A SIMILAR FAR-FIELD RADIATION CONDITION IS SATISFIED BY THE VELOCITY POTENTIAL $\varphi_1(\vec{x}) \equiv \varphi(\vec{x})$

$$\varphi_1 \sim \frac{A(\theta)}{(kR)^{1/2}} e^{kz - ikR} + O(1/R)$$

IT FOLLOWS THAT ON S_∞:

- $\frac{\partial \varphi_1}{\partial n} = \frac{\partial \varphi_1}{\partial R} = -ik\varphi_1$.—
- $\frac{\partial \varphi_2}{\partial n} = \frac{\partial \varphi_2}{\partial R} = -ik\varphi_2$.—

THEREFORE:

$$\varphi_1 \frac{\partial \varphi_2}{\partial n} - \varphi_2 \frac{\partial \varphi_1}{\partial n} = -ik(\varphi_1\varphi_2 - \varphi_2\varphi_1) = 0$$

WITH ERRORS THAT DECAY LIKE $R^{-3/2}$, HENCE FASTER THAN R, WHICH IS THE RATE AT WHICH THE SURFACE S_∞ GROWS AS $R \to \infty$.—

ON S_F $(z=0)$: $\frac{\partial \varphi_1}{\partial n} = \frac{\partial \varphi_1}{\partial z}$, $\frac{\partial \varphi_2}{\partial n} = \frac{\partial \varphi_2}{\partial z}$.—

$$\varphi_1 \frac{\partial \varphi_2}{\partial n} - \varphi_2 \frac{\partial \varphi_1}{\partial n} = \varphi_1 \frac{\partial \varphi_2}{\partial z} - \varphi_2 \frac{\partial \varphi_1}{\partial z} = \nu(\varphi_1\varphi_2 - \varphi_2\varphi_1)$$
$$= 0 .—$$

- IT FOLLOWS THAT UPON APPLICATION OF GREEN'S THEOREM ON THE UNKNOWN POTENTIAL $\varphi_1 \equiv \varphi$ AND THE WAVE GREEN FUNCTION $\varphi_2 \equiv G$ ONLY THE INTEGRALS OVER S_B AND S_ϵ SURVIVE.

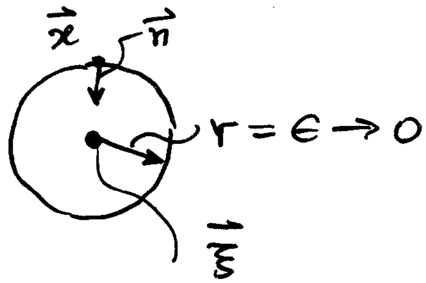
- OVER S_H, EITHER $\frac{\partial \varphi_1}{\partial n} = \frac{\partial \varphi_2}{\partial n} = 0$ BY VIRTUE OF THE BOUNDARY CONDITION IF THE WATER DEPTH IS FINITE OR $\frac{\partial \varphi_1}{\partial z} \to 0$, $\frac{\partial \varphi_2}{\partial z} \to 0$ AS $Z \to -\infty$ BY VIRTUE OF THE VANISHING OF THE RESPECTIVE FLOW VELOCITIES AT LARGE DEPTHS.

- THERE REMAINS TO INTERPRET AND EVALUATE THE INTEGRAL OVER S_ϵ AND S_B. START WITH S_ϵ:

$$I_\epsilon = \iint_{S_\epsilon} \left(\varphi_1 \frac{\partial \varphi_2}{\partial n} - \varphi_2 \frac{\partial \varphi_1}{\partial n} \right) ds$$

OR

$$I_\epsilon = \iint_{S_\epsilon} \left(\varphi \frac{\partial G}{\partial n} - G \frac{\partial \varphi}{\partial n} \right) ds_x ,$$



NOTE THAT THE INTEGRAL OVER S_ϵ IS OVER THE $\vec{x}$ VARIABLE WITH $\vec{\xi}$ BEING THE FIXED POINT WHERE THE SOURCE IS CENTERED.

NEAR $\vec{\xi}$: $\quad G \sim -\dfrac{1}{4\pi r}$, $\quad \dfrac{\partial G}{\partial n} = -\dfrac{\partial G}{\partial r} \sim \dfrac{1}{4\pi r^2}$

$$\varphi \to \varphi(\vec{\xi}) = \varphi(\vec{x}) \quad \text{AS } \epsilon \to 0, \ \vec{x} \leftrightarrow \vec{\xi}$$

IN THE LIMIT AS $r \to 0$, THE INTEGRAND OVER THE SPHERE S_ϵ BECOMES SPHERICALLY SYMMETRIC AND WITH VANISHING ERRORS

$$I_\epsilon \xrightarrow[r \to 0]{} 4\pi r^2 \left[\varphi(\vec{\xi}) \frac{1}{4\pi r^2} + G \frac{\partial \varphi}{\partial r} \right]$$

$$= \varphi(\vec{\xi})$$

IN SUMMARY:

$$\varphi(\vec{\xi}) + \iint_{S_B} \left[\varphi(\vec{x}) \frac{\partial G(\vec{x};\vec{\xi})}{\partial n_x} - G(\vec{x};\vec{\xi}) \frac{\partial \varphi}{\partial n_x} \right] ds_x = 0$$

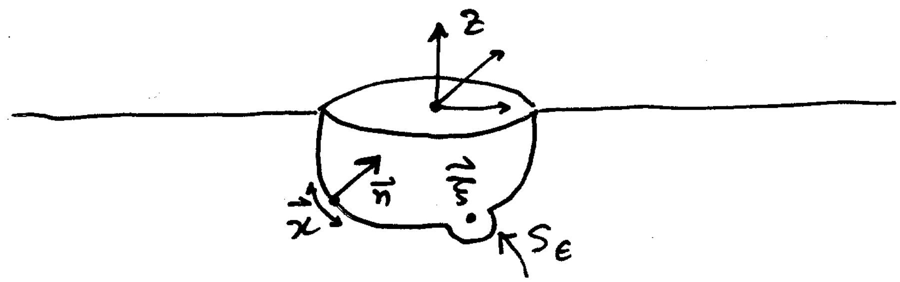
ON S_B: $\dfrac{\partial \varphi}{\partial n_x} = V(x)$ = KNOWN FROM THE BOUNDARY CONDITION OF THE RADIATION AND DIFFRACTION PROBLEMS

- IT FOLLOWS THAT A RELATIONSHIP IS OBTAINED BETWEEN THE VALUE OF $\varphi(\vec{\xi})$ AT SOME POINT IN THE FLUID DOMAIN AND ITS VALUES $\varphi(\vec{x})$ AND NORMAL DERIVATIVES OVER THE BODY BOUNDARY:

$$\varphi(\vec{\xi}) + \iint_{S_B} \varphi(\vec{x}) \frac{\partial G(\vec{x};\vec{\xi})}{\partial n_x} ds_x = \iint_{S_B} G(\vec{x};\vec{\xi}) V(\vec{x}) ds_x$$

STATED DIFFERENTLY, KNOWLEDGE OF φ AND $\dfrac{\partial \varphi}{\partial n}$ OVER THE BODY BOUNDARY ALLOWS THE DETERMINATION OF φ AND UPON DIFFERENTIATION OF $\nabla \varphi$ IN THE FLUID DOMAIN.

IN ORDER TO DETERMINE $\varphi(\vec{x})$ ON THE BODY BOUNDARY S_B, SIMPLY ALLOW $\vec{\xi} \rightarrow S_B$ IN WHICH CASE THE SPHERE S_ϵ BECOMES A ½ SPHERE AS $\epsilon \rightarrow 0$:



- NOTE THAT $\vec{\xi}$ IS A FIXED POINT WHERE THE POINT SOURCE IS CENTERED AND $\vec{x}$ IS A DUMMY INTEGRATION VARIABLE MOVING OVER THE BODY BOUNDARY S_B.

- THE REDUCTION OF GREEN'S THEOREM DERIVED ABOVE SURVIVES ALMOST IDENTICALLY WITH A FACTOR OF ½ NOW MULTIPLYING THE I_ϵ INTEGRAL SINCE ONLY ½ OF THE S_ϵ SURFACE LIES IN THE FLUID DOMAIN IN THE LIMIT AS $\epsilon \rightarrow 0$ AND FOR A BODY SURFACE WHICH IS SMOOTH. IT FOLLOWS THAT:

- $$\frac{1}{2}\varphi(\vec{\xi}) + \iint_{S_B} \varphi(\vec{x}) \frac{\partial G(\vec{x};\vec{\xi})}{\partial n_x}\, dS_x$$

$$= \iint_{S_B} G(\vec{x};\vec{\xi})\, V(\vec{x})\, dS_x$$

WHERE NOW BOTH $\vec{x}$ AND $\vec{\xi}$ LIE ON THE BODY SURFACE. THIS BECOMES AN INTEGRAL EQUATION FOR $\varphi(\vec{x})$ OVER A SURFACE S_B OF BOUNDED EXTENT. ITS SOLUTION IS CARRIED OUT WITH PANEL METHODS DESCRIBED BELOW

THE INTERPRETATION OF THE DERIVATIVE UNDER THE INTEGRAL SIGN IS AS FOLLOWS:

$$\frac{\partial G}{\partial n_x} \equiv \vec{n}_x \cdot \nabla_x G(\vec{x};\vec{\xi})$$

$$\equiv \left(n_1 \frac{\partial}{\partial x} + n_2 \frac{\partial}{\partial y} + n_3 \frac{\partial}{\partial z}\right) G(\vec{x};\vec{\xi})$$

WHERE DERIVATIVES ARE TAKEN WRT THE FIRST ARGUMENT FOR A POINT SOURCE CENTERED AT POINT $\vec{\xi}$.

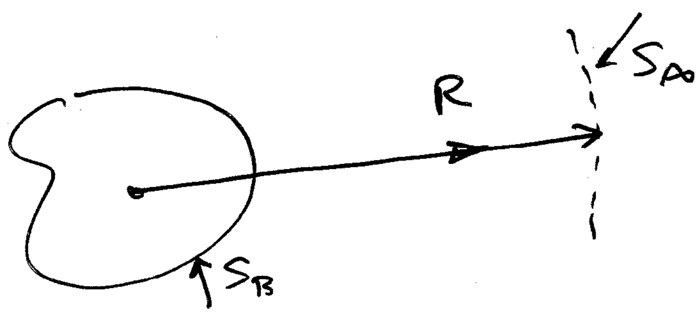
INFINITE DOMAIN POTENTIAL FLOW SOLUTIONS

IN THE ABSENCE OF THE FREE SURFACE, THE DERIVATION OF THE GREEN INTEGRAL EQUATION REMAINS ALMOST UNCHANGED USING:

$$\varphi_2(\vec{x}) = -\frac{1}{4\pi} |\vec{x} - \vec{\xi}|^{-1}$$

$$\equiv G(\vec{x}; \vec{\xi})$$

THE RANKINE SOURCE AS THE GREEN FUNCTION AND USING THE PROPERTY THAT AS $R \to \infty$



$$|\nabla \varphi| \sim 1/R^2$$

$$|\nabla G| \sim 1/R^2$$

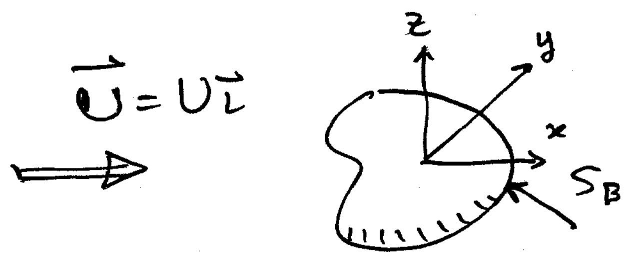
FOR CLOSED BOUNDARIES S_B WITH NO SHED WAKES RESPONSIBLE FOR LIFTING EFFECTS

THE RESULTING INTEGRAL EQUATION FOR $\varphi(\vec{x})$ OVER THE BODY BOUNDARY BECOMES:

- $$\frac{1}{2}\varphi(\vec{\xi}) + \iint_{S_B} \varphi(\vec{x}) \frac{\partial G(\vec{x};\vec{\xi})}{\partial n_x}\, dS_x = \iint_{S_B} G(\vec{x};\vec{\xi})\, V(\vec{x})\, dS_x$$

WITH $V(\vec{x}) = \dfrac{\partial \varphi}{\partial n}$, on S_B

EXAMPLE: UNIFORM FLOW PAST S_B



$$\Phi = Ux + \varphi, \qquad \frac{\partial \Phi}{\partial n} = 0, \text{ on } S_B$$

$$\Rightarrow \frac{\partial \varphi}{\partial n} = -\frac{\partial}{\partial n}(Ux) = -n_1 U \equiv V(\vec{x})$$

SO THE RHS OF THE GREEN EQUATION BECOMES:

$$RHS = \iint_{S_B} G(\vec{x};\vec{\xi})\,(-U n_{1x})\, dS_x$$