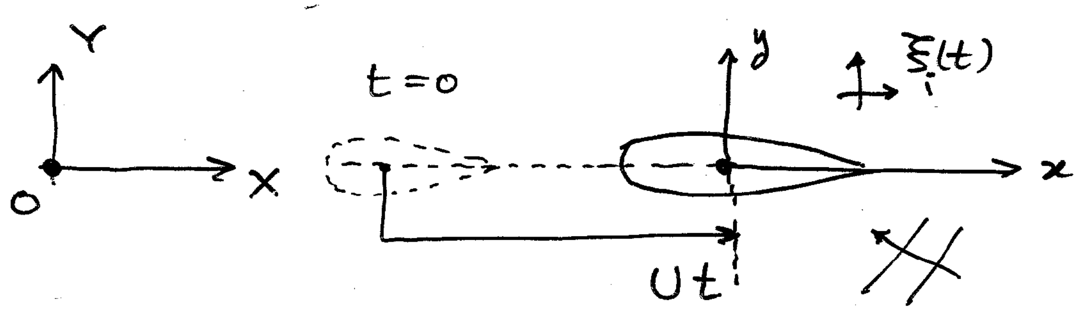
RANKINE INTEGRAL EQUATIONS FOR SHIP FLOW PROBLEMS WITH FORWARD SPEED

- THE GREEN INTEGRAL EQUATION EXTENDS EASILY TO FLOWS PAST SHIPS IN CALM WATER AND IN WAVES WHEN THE FREE SURFACE CONDITION IS MORE COMPLEX THAN THAT OF THE $U=0$ FREQUENCY DOMAIN PROBLEM

- NEUMANN-KELVIN PROBLEM IN TIME DOMAIN



CONSIDER A VESSEL WHICH STARTS FROM REST AT $t=0$ AND TRANSLATES FORWARD WITH CONSTANT VELOCITY U AND ALSO POSSIBLY OSCILLATING WITH AMPLITUDES $\xi_i(t)$ IF AMBIENT WAVES ARE PRESENT.

IT WAS SHOWN EARLIER THAT THE SIMPLEST FORWARD SPEED FREE SURFACE CONDITION FOR THE FORWARD SPEED PROBLEM TAKES THE FORM:

$$\begin{cases} \left(\dfrac{\partial}{\partial t} - U\dfrac{\partial}{\partial x}\right)^2 \varphi + g\dfrac{\partial \varphi}{\partial z} = 0, & z = 0 \\ \dfrac{\partial \varphi}{\partial n} = V, & \text{on } \bar{S}_B \end{cases}$$

RELATIVE TO THE SHIP FRAME. THE NORMAL VELOCITY V ON $\bar{S}_B$ CAN BE OF THREE FORMS:

$$V(\vec{x}) = \begin{cases} U n_1, & t > 0: \text{ FORWARD TRANSLATION} \\ n_i \xi_i(t), & t > 0: \text{ RADIATION} \\ -\dfrac{\partial \varphi_I}{\partial n} & t > 0: \text{ DIFFRACTION} \end{cases}$$

- MORE GENERAL FREE-SURFACE CONDITIONS WITH SPACE DEPENDENT COEFFICIENTS ARISING FROM GRADIENTS OF THE DOUBLE-BODY FLOW EXIST AND ARE DESCRIBED IN THE LITERATURE THE STEPS IN DERIVING THE RELEVANT INTEGRAL EQUATIONS ARE VERY SIMILAR TO THE ONES THAT FOLLOW:

- WAVE GREEN FUNCTIONS THAT SATISFY ANALYTICALLY THE TIME-DOMAIN FREE SURFACE CONDITION STATED ABOVE EXIST AND ARE DERIVED IN W & L. THEIR EVALUATION IS HOWEVER TIME-CONSUMING AND THEY APPLY ONLY TO THE NEUMANN-KELVIN FORMULATION.—

- PROCEEDING WITH THE DERIVATION OF THE GREEN INTEGRAL EQUATION AS ABOVE AND USING THE RANKINE SOURCE AS THE GREEN FUNCTION :

$$\varphi_2(\vec{x}) = -\frac{1}{4\pi}|\vec{x}-\vec{\xi}|^{-1}$$
$$\equiv G(\vec{x};\vec{\xi})$$

WE OBTAIN:

$$\frac{1}{2}\varphi(\vec{\xi}) + \iint_{S_B} \varphi(\vec{x})\frac{\partial G(\vec{x};\vec{\xi})}{\partial n_x}\,dS_x$$
$$+ \iint_{S_F(z=0)} \left[\varphi(\vec{x})\frac{\partial G}{\partial z}(\vec{x};\vec{\xi}) - G(\vec{x};\vec{\xi})\frac{\partial \varphi}{\partial z}\right]dx\,dy$$
$$= \iint_{S_B} G(\vec{x};\vec{\xi})\,V(\vec{x})\,dS_x .—$$

- NOTE THAT THE INTEGRAL OVER THE FREE SURFACE ($z=0$) DOES NOT VANISH SINCE WE HAVE NOT USED THE RELEVANT WAVE GREEN FUNCTION.

- OTHERWISE THE REMAINING INTEGRAL OVER S_B RETAINS ITS FORM. THE INTEGRAL OVER S_∞ CAN BE SHOWN TO VANISH. THE PROOF IS NON-TRIVIAL AND MAY BE FOUND IN REFERENCES.

OVER $z=0$, IT FOLLOWS FROM THE FREE-SURFACE CONDITION:

$$\frac{\partial \varphi}{\partial z} = -\frac{1}{g}\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)^2 \varphi \quad , \quad z=0$$

UPON SUBSTITUTION, THE SECOND INTEGRAL OVER S_F BECOMES:

$$I_F = \iint_{z=0} \left[\varphi(\vec{x}) \frac{\partial G(\vec{x};\vec{\xi})}{\partial z} + \frac{1}{g} G(\vec{x};\vec{\xi}) \left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)^2 \varphi(\vec{x}) \right] ds$$

IT FOLLOWS THAT OVER $z=0$, ONLY VALUES AND TANGENTIAL GRADIENTS OF $\varphi(\vec{x})$ ARE NOW PRESENT LEADING TO AN INTEGRO-DIFFERENTIAL EQUATION:

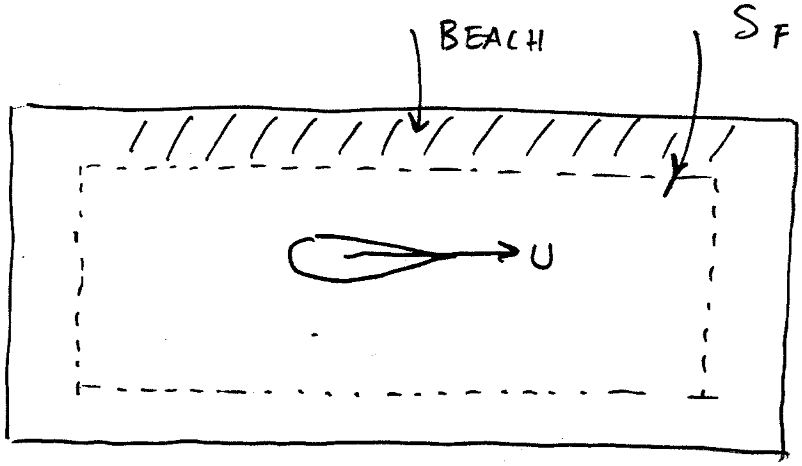
$$\frac{1}{2}\varphi(\vec{\xi}) + \iint_{S_B} \varphi(\vec{x}) \frac{\partial G(\vec{x};\vec{\xi})}{\partial n_x} \, dS_x$$

$$+ \iint_{S_F(z=0)} \left[\varphi(\vec{x}) \frac{\partial G(\vec{x};\vec{\xi})}{\partial z} + \frac{1}{g} G(\vec{x};\vec{\xi}) \left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x} \right)^2 \varphi(\vec{x}) \right] dx\,dy$$

$$= \iint_{S_B} V(\vec{x}) \, G(\vec{x};\vec{\xi}) \, dS_x$$

- UNKNOWN IS $\varphi(\vec{x})$ OVER S_B & S_F. ITS X-DERIVATIVES MAY BE APPROXIMATED BY CAREFULLY SELECTED NUMERICAL DIFFERENTIATION SCHEMES FORMING A CORE PART OF RANKINE PANEL METHODS, DISCUSSED BELOW

- THE INTEGRAL OVER THE INFINITE FREE SURFACE $S_F(z=0)$ IS TRUNCATED AT SOME FINITE DISTANCE FROM THE SHIP AS DRAWN BELOW



- A DOMAIN DENOTED BY THE SHADED AREA IS ALSO INTRODUCED DEFINED AS THE "BEACH". THIS IS LOCATED AS THE OUTER BOUNDARY OF S_F AND SELECTED SO THAT OVER ITS SURFACE THE FOLLOWING FREE SURFACE CONDITION IS ENFORCED:

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)^2 \varphi + g\frac{\partial \varphi}{\partial z} + 2\nu\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\varphi + \nu^2 \varphi = 0, \qquad z = 0$$

- THIS CONDITION DIFFERS FROM THE NEUMANN-KELVIN CONDITION BY THE ADDITION OF THE TERMS THAT ARE MULTIPLIED BY THE DISSIPATIVE PARAMETER $\nu(\vec{x})$ WHICH VARIES FROM $\nu = 0$ AT THE INNER BOUNDARY TO A FINITE VALUE AT THE OUTER BOUNDARY OF THE BEACH.

- IT CAN BE SHOWN THAT CONVERTING FROM THE TIME TO THE FREQUENCY DOMAIN VIA $\frac{\partial}{\partial t} \rightarrow i\omega$, ν IS THE FAMILIAR RAYLEIGH VISCOSITY THAT PLAYS A KEY ROLE IN THE ENFORCEMENT OF THE RADIATION CONDITIONS.—

(SEE W & L)