NONLINEAR EFFECTS

SOME OF THE MOST IMPORTANT NONLINEAR EFFECTS ARISING IN CONNECTION WITH WAVE-BODY INTERACTIONS ARE

- DRIFT FORCES. THEY ARE THE MEAN FORCES EXERTED ON FLOATING OR SUBMERGED BODIES BY AMBIENT WAVES MAY BE TREATED VERY WELL BY PERTURBATION THEORY

- SLAMMING. THESE ARE HIGHLY NONLINEAR EFFECTS ARISING WHEN A SHIP SECTION IMACTS UPON THE WATER SURFACE OR WHEN A STEEP OR BREAKING WAVE IMPINGES UPON A FLOATING STRUCTURE. MAY BE MODELED BY FULLY OR PARTIALLY NONLINEAR POTENTIAL FLOW MODELS OF ANALYTICAL OR NUMERICAL NATURE

- FORCES DUE TO VISCOUS FLOW SEPARATION AROUND FLOATING

STRUCTURES AND THEIR SUBSYSTEMS, E.G. RISERS, MOORING LINES ETC. VORTEX INDUCED VIBRATIONS (VIV) IS AN IMPORTANT EXAMPLE. SUCH EFFECTS CAN BE TREATED EXPERIMENTALLY AND COMPUTATIONALLY BY SOLVING THE NAVIER-STOKES EQUATIONS

- NONLINEAR SHIP MOTIONS IN STEEP WAVES. THESE EFFECTS ARE MOSTLY OF POTENTIAL-FLOW NATURE AND ARE BEING TREATED BY NONLINEAR RANKINE PANEL METHODS. THE PRIMARY NONLINEARITY IS THE VARIABLE WETNESS OF THE SHIP HULL, THE NONLINEARITY OF THE KINEMATICS OF AMBIENT WAVES AND THE NUMERICAL SOLUTION OF THE EQUATIONS OF MOTION IN THE TIME DOMAIN

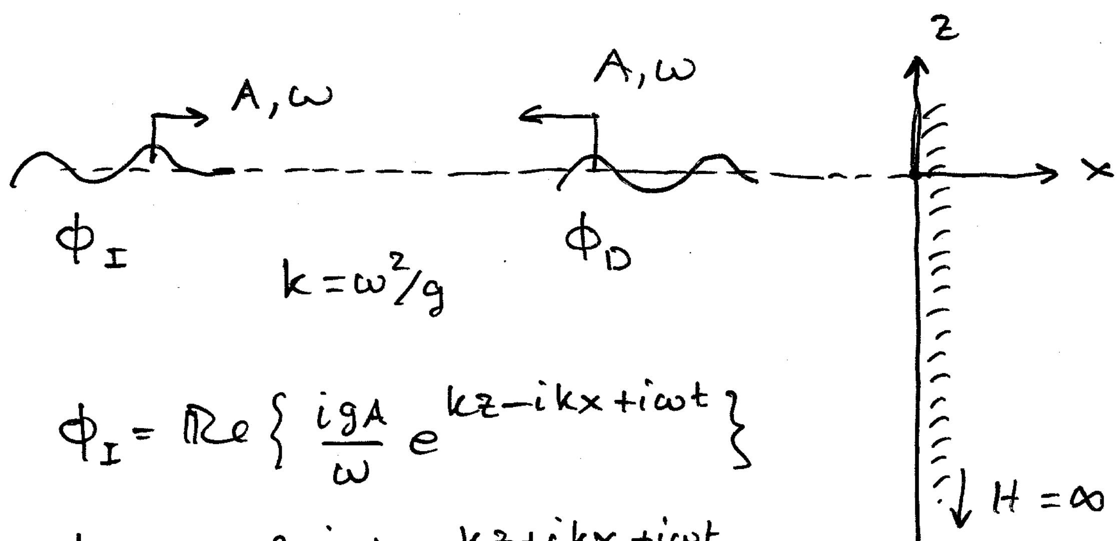
- NONLINEAR RESPONSES OF DEEP WATER OFFSHORE PLATFORMS IN CERTAIN FLEXURAL MODES OF THEIR TETHERS. THESE EFFECTS ARE KNOWN AS SPRINGING & RINGING AND ARE TREATED BY A COMBINATION OF PERTURBATION AND NONLINEAR METHODS AND EXPERIMENTS. —

DRIFT FORCES

- DRIFT FORCES WILL FIRST BE CONSIDERED IN REGULAR WAVES. THE MAIN RESULTS WILL THEN BE EXTENDED IN RANDOM WAVES

- ONE VERY IMPORTANT PROPERTY OF DRIFT FORCES OTHER THAN THEIR PRACTICAL SIGNIFICANCE IS THAT THEY DEPEND ONLY ON THE LINEAR SOLUTION

→ MEAN DRIFT FORCE ON A VERTICAL WALL



$$\phi_I = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} \right\}$$

$$\phi_D = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz + ikx + i\omega t} \right\}$$

$$\phi = \phi_I + \phi_D \; ; \quad \phi_x = 0, \quad x = 0 \quad (\text{VERIFY})$$

- NONLINEAR HYDRODYNAMIC PRESSURE:

$$p = -\rho\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\cdot\nabla\phi + gz\right)$$

- NONLINEAR HORIZONTAL FORCE ON THE WALL:

$$F_x = \int_{-\infty}^{\zeta} p\,dz, \qquad \zeta = -\frac{1}{g}\frac{\partial\phi}{\partial t}\bigg|_{z=0} + O(A^2)$$

- WE NEED TO EVALUATE F_x CORRECT TO $O(A^2)$ NOTING THAT THE MEAN TIME VALUE OF EFFECTS OF $O(A)$ WHICH ARE LINEAR, IS ZERO.

ASSUME A PERTURBATION EXPANSION FOR ϕ:

$$\phi = \underbrace{\phi_1}_{O(A)} + \underbrace{\phi_2}_{A^2} + \underbrace{\phi_3}_{A^3} + \cdots$$

WHERE THE POTENTIAL DERIVED EARLIER IS THE LINEAR TERM DENOTED ABOVE BY ϕ_1.

- IN EVALUATING THE LEADING ORDER EFFECT IN THE MEAN HORIZONTAL FORCE WE DROP TERMS WITH ZERO MEAN VALUES OR OF ORDER A^3 AND HIGHER. WE NOTE WITHOUT PROOF THAT

$$\overline{\phi_2(t)}^{\,t} = 0$$

THE TOTAL HORIZONTAL FORCE F_x ALSO ACCEPTS THE EXPANSION:

$$F_x = \underbrace{F_1}_{O(A)} + \underbrace{F_2}_{O(A^2)} + \underbrace{F_3}_{O(A^3)} + \cdots$$

- $$F_1 = \int_{-\infty}^{0} p_1 \, dz = -\rho \int_{-\infty}^{0} \frac{\partial \phi}{\partial t} dz = O(A)$$

$$\overline{F_1(t)}^{\,t} = 0 \quad \text{(VERIFY)}$$

- $$F_2 = \int_{-\infty}^{0} p_2 \, dz + \int_{0}^{\zeta} p_1 \, dz = O(A^2)$$

$$= -\rho \int_{-\infty}^{0} \left(\frac{\partial \phi_2}{\partial t} + \frac{1}{2} \nabla \phi_1 \cdot \nabla \phi_1 \right) dz$$

$$-\rho \int_{0}^{\zeta} \left(\frac{\partial \phi_1}{\partial t} + gz \right) dz + O(A^3)$$

WITH ERRORS OF $O(A^3)$, THE LAST INTEGRAL MAY BE APPROXIMATED BY TAYLOR EXPANDING ABOUT $z=0$ THE FIRST TERM AND BY DIRECT INTEGRATION OF THE SECOND. IT FOLLOWS THAT:

$$\int_{0}^{\zeta} \left(\frac{\partial \phi_1}{\partial t} + gz \right) dz = \zeta \left. \frac{\partial \phi_1}{\partial t} \right|_{z=0} + \frac{1}{2} g \zeta^2 + O(A^3)$$

$$= -g\zeta^2 + \frac{1}{2} g \zeta^2 = -\frac{1}{2} g \zeta^2 \cdot$$

COLLECTING TERMS:

- $$F_2(t) = -\rho \int_{-\infty}^{0} \frac{\partial \phi_2}{\partial t} dz - \frac{1}{2}\rho \int_{-\infty}^{0} \nabla\phi_1 \cdot \nabla\phi_1 \, dz + \frac{1}{2}\rho g \zeta^2(t) + O(A^3)$$

THE MEAN TIME VALUE OF $\overline{\partial \phi_2 / \partial t}^{\,t} = 0$ FOR ANY STATIONARY SIGNAL $\phi_2(t)$. SO THE SECOND-ORDER POTENTIAL DOES NOT CONTRIBUTE TO THE MEAN DRIFT FORCE AS STATED ABOVE!

- OF THE REMAINING TWO TERMS THE QUADRATIC BERNOULLI TERM CONTRIBUTES A SUCTION FORCE WHICH IS "PULLING" THE WALL INTO THE WAVE (COUNTERINTUITIVE BUT TRUE!) WHILE THE LAST TERM IS ALWAYS POSITIVE PUSHING THE WALL IN THE DIRECTION OF THE WAVE AS EXPECTED

- IT FOLLOWS THAT ALL OF THE MEAN DRIFT FORCE ARISES FROM THE PRESSURE INTEGRATION OVER THE SURF-ZONE WHICH IS MORE THAN ENOUGH TO OVERCOME THE SUCTION FORCE.

UPON SUBSTITUTION OF THE LINEAR VELOCITY POTENTIAL DERIVED ABOVE AND USE OF THE FAMILIAR IDENTITY :

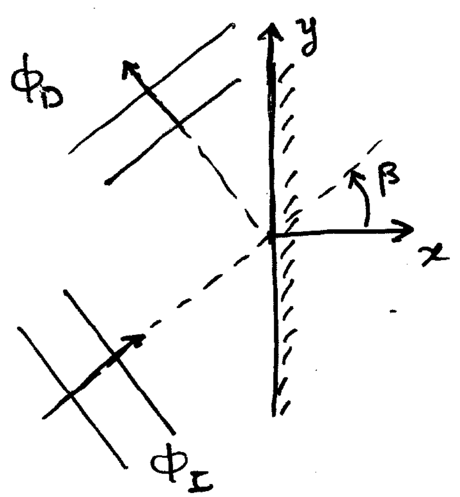
$$\overline{\mathrm{Re}(A_1 e^{i\omega t})\,\mathrm{Re}(A_2 e^{i\omega t})} = \frac{1}{2}\,\mathrm{Re}\{A_1 A_2^*\}$$

IT IS EASY TO VERIFY :

$$\overline{F_x}^{\,t} = \frac{1}{2}\rho g A^2 + O(A^3)$$

SO THE MEAN HORIZONTAL FORCE ON A VERTICAL WALL OF INFINITE DRAFT BY A PLANE PROGRESSIVE WAVE HAS A FINITE MEAN VALUE $\sim A^2$.

IF THE PLANE REGULAR WAVE IS INCIDENT AT AN ANGLE THE MEAN HORIZONTAL FORCE MAY BE SHOWN TO TAKE THE VALUE :

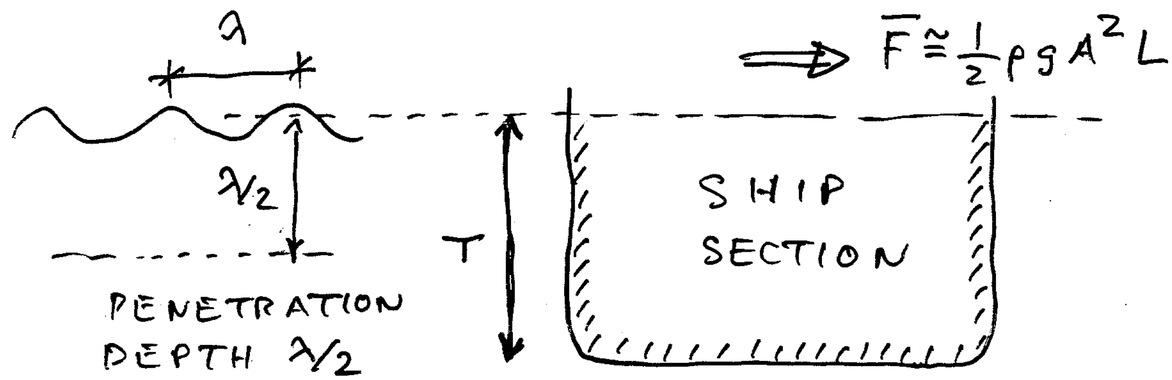


$$\overline{F_x} = \frac{1}{2}\rho g A^2 \cos^2\beta .$$

(VERIFY).

IN SPITE OF THEIR SIMPLICITY THE ABOVE RESULTS HAVE A NUMBER OF USEFUL APPLICATIONS IN PRACTICE WHEN WAVES THAT ARE SUFFICIENTLY SHORT INTERACT WITH FLOATING STRUCTURES.

SOME EXAMPLES FOLLOW:

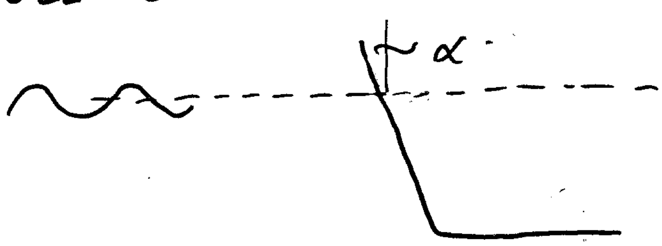
1. 

$\lambda/2 < T \Rightarrow \lambda < T/2$: SHIP WITH WALL SIDED GEOMETRY ACTS LIKE A WALL

- FOR A BARGE LIKE SHIP WITH LENGTH L THE MEAN SWAY DRIFT FORCE IN REASONABLY SHORT WAVES IS APPROXIMATELY GIVEN BY:

$$\bar{F}_y \simeq \frac{1}{2}\rho g A^2 L$$

2. EXTEND THE ABOVE RESULT WHEN THE SHIP HULL SECTION HAS FLARE WITH SLOPE α.



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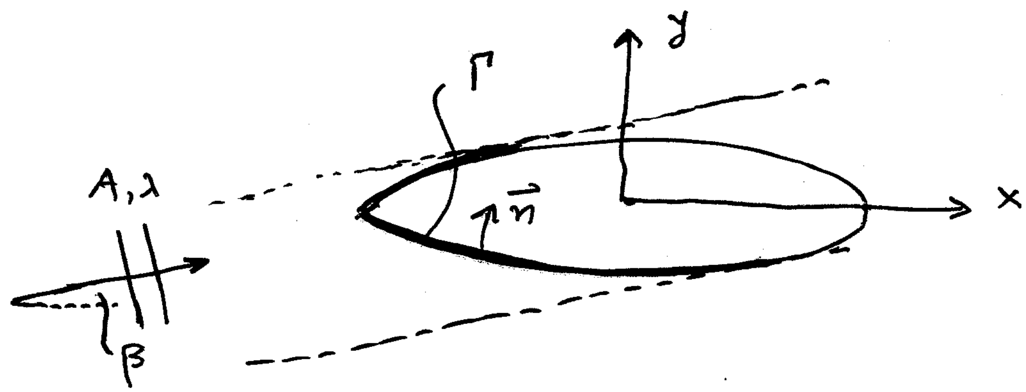


- SHORT WAVES INCIDENT UPON A SHIP AT AN ANGLE ARE LOCALLY REFLECTED AS IF THEY ENCOUNTER A CONTINUUM OF VERTICAL WALLS INCLINED AT VARYING ANGLES. WE CAN THUS APPLY THE RESULT DERIVED ABOVE.

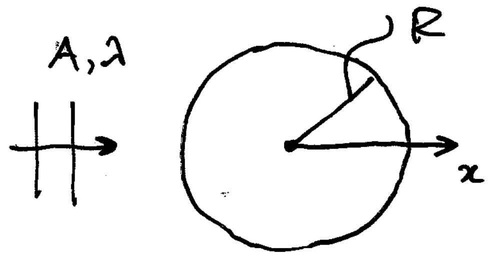
- ONLY PART OF THE SHIP WATERLINE WILL ENCOUNTER WAVES. THIS REGION IS BOLDFACED ABOVE AND CAN BE DETERMINED BY A SIMPLE GEOMETRICAL ARGUMENT. DENOTE THIS PORTION OF THE SHIP WATERLINE BY Γ. IT IS EASY TO SHOW THAT THE MEAN DRIFT FORCE IN THE (X,Y) DIRECTIONS IS GIVEN BY

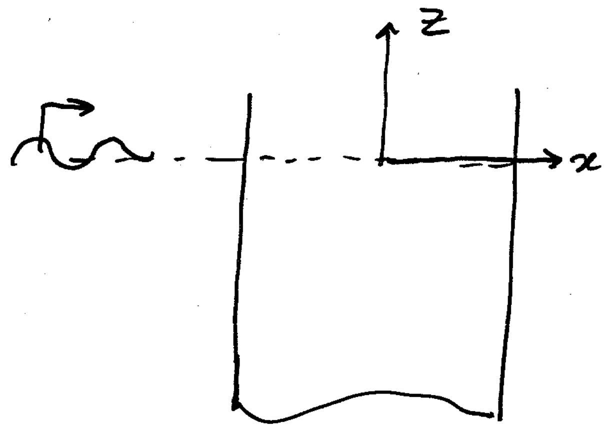
$$\vec{F} = \begin{pmatrix} \bar{F}_x \\ \bar{F}_y \end{pmatrix} = \frac{1}{2} \rho g A^2 \int_{\Gamma} \left| \vec{n} \cdot \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \right|^2 \vec{n} \, dl$$

(VERIFY)

THIS RESULT IS KNOWN AS RAY THEORY.—

4. CONSIDER A VERTICAL CIRCULAR CYLINDER PIERCING THE FREE SURFACE AND SHORT WAVES INCIDENT UPON IT:

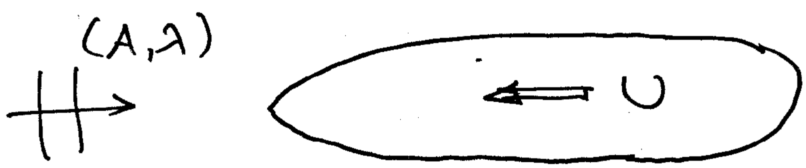




USE THE RESULT FROM RAY THEORY STATED ABOVE TO SHOW THAT THE MEAN DRIFT FORCE IN THE X-DIRECTION IS GIVEN BY:

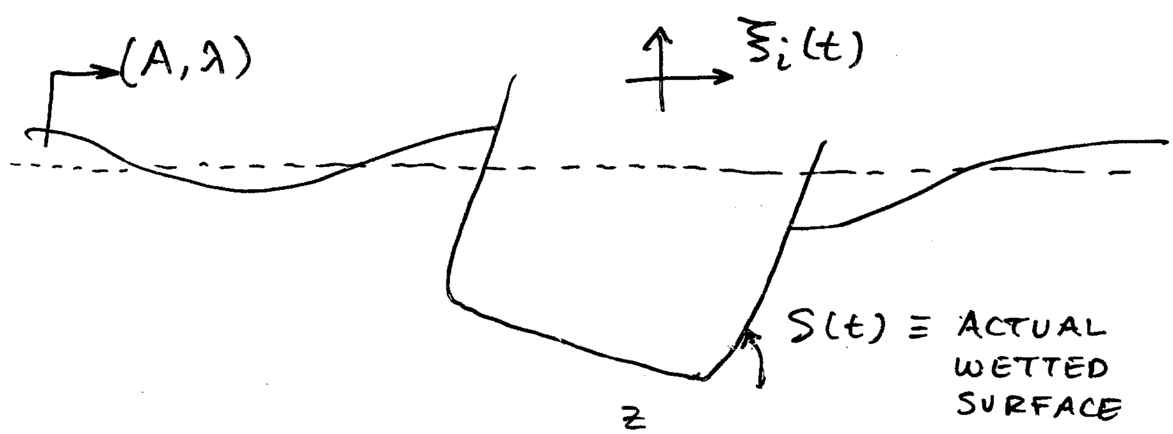
$$\bar{F}_x = \frac{2}{3} \rho g A^2 R.$$

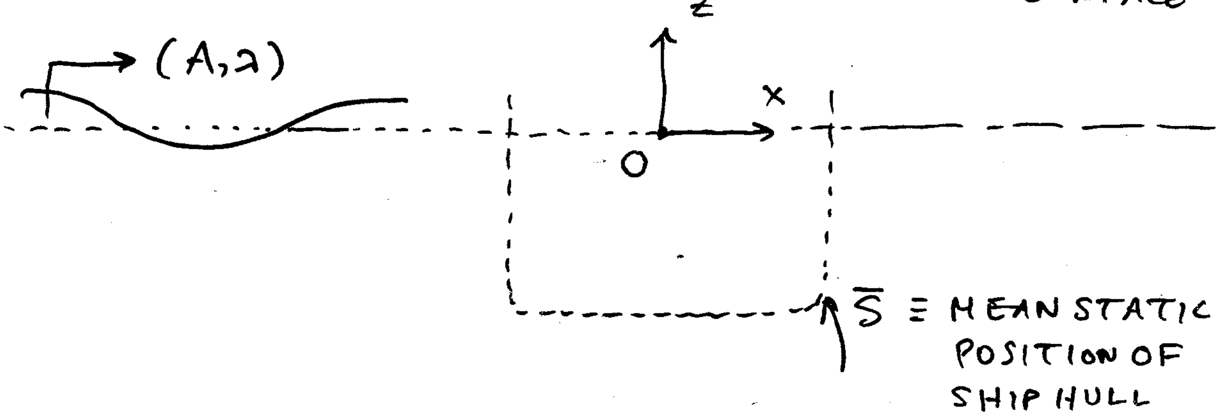
5. IT IS POSSIBLE TO EXTEND RAY THEORY WITH LITTE ADDITIONAL COMPLEXITY TO THE CASE WHERE THE VESSEL ADVANCES WITH A MODERATE FORWARD SPEED. (SEE OMF)



DRIFT FORCES BY PRESSURE INTEGRATION

- IN THE MORE GENERAL CASE OF SURFACE WAVES INTERACTING WITH FLOATING BODIES OSCILLATING DUE TO THE AMBIENT WAVES, A MORE GENERAL EXPRESSION MAY BE DERIVED BY DIRECTLY INTEGRATING THE PRESSURE OVER THE <u>INSTANTANEOUS</u> POSITION OF THE VESSEL WETTED SURFACE AND <u>LINEARIZING</u> ABOUT ITS MEAN POSITION KEEPING CONSISTENTLY TERMS OF $O(A^2)$. THE EXPRESSION IS NOT GIVEN HERE FOR SIMLICITY.





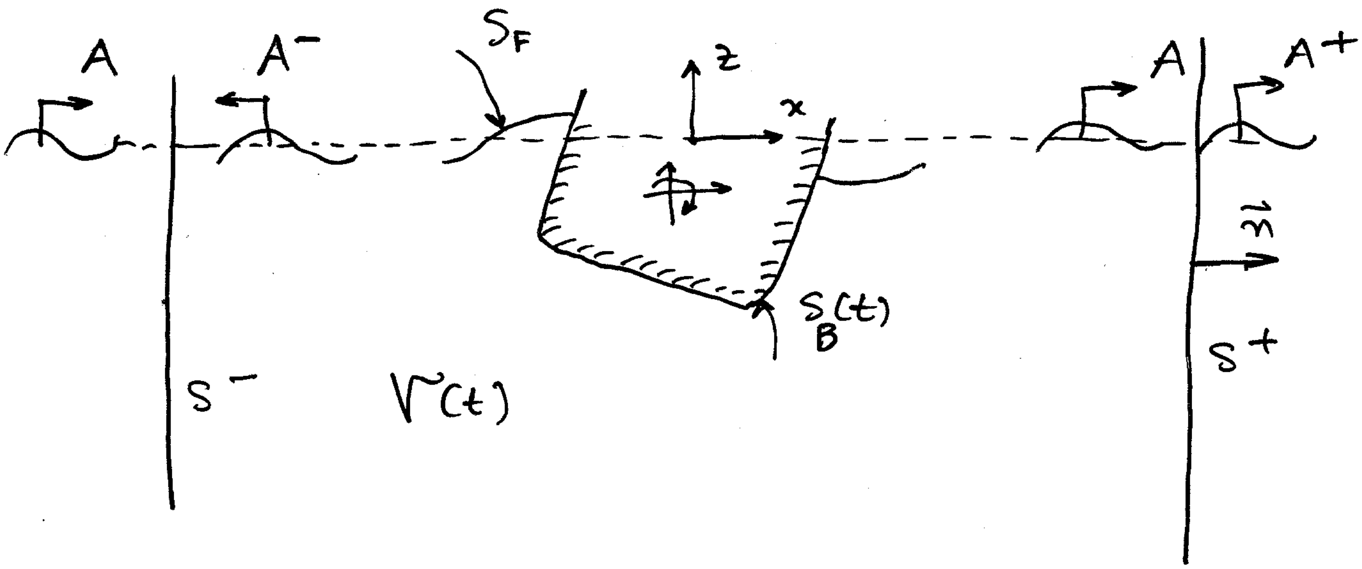
DRIFT FORCES BY MOMENTUM CONSERVATION

- IT IS POSSIBLE TO DERIVE AN EXPRESSION FOR THE MEAN DRIFT FORCE BY APPLYING THE MOMENTUM CONSERVATION PRINCIPLE DERIVED EARLIER. THIS APPROACH PROVIDES AN EXPRESSION FOR THE DRIFT FORCE IN TERMS OF THE WAVE SYSTEMS IN THE FAR FIELD AND CAN BE SHOWN TO PRODUCE THEORETICALLY AN IDENTICAL RESULT TO THAT OBTAINED FROM THE CONSIDERABLY MORE ELABORATE TASK OF USING NEAR-FIELD PRESSURE INTEGRATION

- THE FULL THREE-DIMENSIONAL RESULT WILL NOT BE PRESENTED HERE, AS FOR THE PRESSURE INTEGRATION, AND CAN BE FOUND IN THE REFERENCES CITED IN <u>OMF</u> AND <u>W&L</u>. IT IS NOTED THAT IT IS NUMERICALLY SUPERIOR TO PRESSURE INTEGRATION AND IS OFTEN PREFERED IN PRACTICE

- BELOW WE PROVE FROM FIRST PRINCIPLES THE RESULT IN TWO DIMENSIONS WHICH IS ITSELF QUITE USEFUL IN PRACTICE AND EDUCATIONAL.—

MEAN DRIFT FORCE ON A FLOATING 2D BODY



- AMBIENT REGULAR WAVES WITH AMPLITUDE A ARE INCIDENT ON A FLOATING BODY IN 2D WHICH IS ALLOWED TO OSCILLATE FREELY IN HEAVE, SWAY AND ROLL

- AS A RESULT OF THE DIFFRACTION AND RADIATION WAVE DISTURBANCES, A REFLECTED PLANE PROGRESSIVE WAVE WITH COMPLEX AMPLITUDE A^- APPEARS AT $X = -\infty$ OVER A CONTROL SURFACE S^- AND A RADIATED/DIFFRACTED WAVE WITH AMPLITUDE A^+ APPEARS AT $X = +\infty$ OVER S^+

- APPLYING THE MOMENTUM CONSERVATION PRINCIPLE IN THE X-DIRECTION WITHIN THE VOLUME $\forall(t)$ BOUNDED BY $S^+ + S^- + S_F(t) + S_B(t)$ WE OBTAIN:

$$D \equiv \overline{F_x}^{\,t} = -\overline{\iint_{S^+ + S^-} \left[p\vec{n} + \rho \vec{V}(V_n - U_n) \right] ds}^{\,t}$$

WHERE THE MOMENTUM CONSERVATION PRINCIPLE DERIVED EARLIER HAS BEEN APPLIED. MEAN VALUES IN TIME ARE TAKEN LEADING TO A VANISHING MEAN VALUE FOR THE MOMENTUM RATE OF CHANGE WITHIN THE VOLUME $\mathcal{V}(t)$, FOR FIXED $S^{\pm}$.

- THE MEAN MOMENTUM FLUX ACROSS $S_F(t)$ IS ZERO
- THE MEAN HORIZONTAL MOMENTUM FLUX ACROSS THE BODY SECTION IS THE DRIFT FORCE $D \equiv \overline{F_x}^{\,t}$
- THERE REMAINS TO EVALUATE THE INTEGRALS OVER $S^{\pm}$ WHICH ARE SURFACES FIXED IN SPACE AND THEREFORE, $U_n = 0$ on $S^{\pm}$. RECALL THAT $\vec{V} = \nabla\phi$ IS THE TOTAL FLUID VELOCITY AND p IS GIVEN BY THE BERNOULLI EQUATION
- THE DEFINITION OF THE PLANE PROGRESSIVE WAVE FORMS DEFINED ABOVE WILL BE INTRODUCED AND AN EXPRESSION WILL BE DERIVED FOR D IN TERMS OF A, A^+, A^-. THE RESULT IS "MILDLY" SURPRISING!

DEFINE THE VELOCITY POTENTIALS FOR THE PLANE PROGRESSIVE WAVES DESCRIBED ABOVE:

$$\phi_I = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz-ikx+i\omega t} \right\}$$

$$\phi^+ = \mathrm{Re}\left\{ \frac{igA^+}{\omega} e^{kz-ikx+i\omega t} \right\}$$

$$\phi^- = \mathrm{Re}\left\{ \frac{igA^-}{\omega} e^{kz+ikx+i\omega t} \right\}$$

UPON SUBSTITUTION INTO THE MOMENTUM FLUX EXPRESSION DEFINED ABOVE AND AFTER SOME SIMPLE ALGEBRA THAT HAS BEEN ILLUSTRATED EARLIER, IT FOLLOWS THAT:

$$\mathbb{D} = \frac{\rho g A}{4}\left(A^+ + A^{+*}\right) - \frac{\rho g}{4}\left\{ |A^+|^2 - |A^-|^2 \right\}$$

WHERE (*) DENOTES THE COMPLEX CONJUGATE. NOTE THAT A IS REAL AND $A^{\pm}$ ARE COMPLEX QUANTITIES.

- ALSO NOTE THAT THE DRIFT FORCE AS STATED ABOVE DOES NOT APPEAR TO DEPEND ON A SINCE THE AMBIENT WAVE APPEARS AT BOTH INFINITIES. THE ABOVE EXPRESSION SIMPLIFIES A LOT BY INVOKING ENERGY CONSERVATION.

- THE FLOW AROUND THE FLOATING BODY IS CONSERVATIVE IF VISCOUS EFFECTS OR BREAKING WAVE EFFECTS ARE IGNORED. THE FORMER ARE OFTEN PRESENT WHEN VORTICES ARE SHED AROUND CORNERS OF THE BODY, BILOGE KEELS ETC. - IN SUCH CASES THE EXPESSIONS BELOW MAY BE APPROXIMATELY VALID OR INVALID.

- IN THE ABSENCE OF ENERGY LOSSES THE MEAN ENERGY FLUX ACROSS S^- MUST BE IDENTICAL TO THAT ACROSS S^+ WITHIN THE LIMITATIONS OF LINEAR THEORY:

→ <u>ENERGY FLUX ACROSS S^+:</u>

$$P^+ = \frac{1}{4} \rho g V_p |A + A^+|^2 \quad \left(\begin{array}{c} \text{VERIFY;} \\ \text{EASY} \end{array} \right)$$

→ <u>ENERGY FLUX ACROSS S^-:</u>

$$P^- = \frac{1}{4} \rho g V_p \left(|A|^2 - |A^-|^2 \right) \quad \left(\begin{array}{c} \text{VERIFY;} \\ \text{FAIRLY} \\ \text{EASY} \end{array} \right)$$

$$P^+ \equiv P^- \quad \Rightarrow$$

$$|A|^2 = |A^-|^2 + |A + A^+|^2$$

$$= |A^-|^2 + |A|^2 + |A^+|^2 + A(A^+ + A^{+*})$$

$$\Rightarrow \quad A(A^+ + A^{+*}) = -\left(|A^+|^2 + |A^-|^2\right).$$

- THIS RELATION (RESTRICTION) UPON THE WAVE AMPLITUDES IS THE RESULT OF ENERGY CONSERVATION.

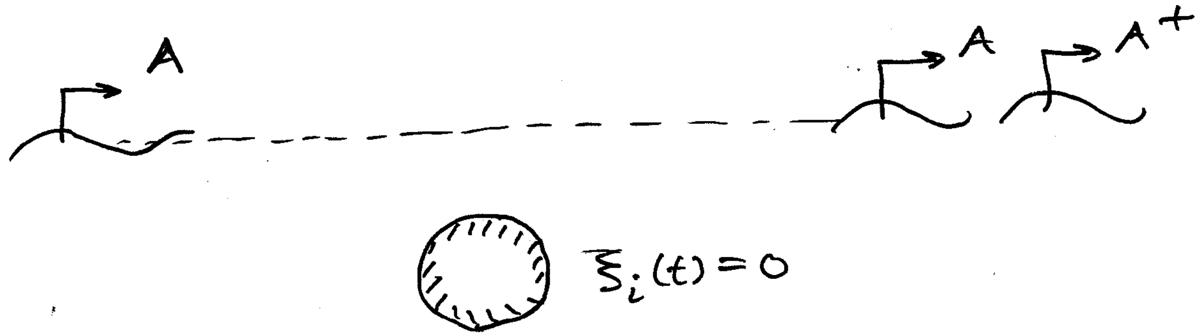
UPON SUBSTITUTION IN $\mathbb{D}$:

$$\mathbb{D} = \frac{1}{2} \rho g |A^-|^2 .$$

THIS INTERESTING RESULT STATES THAT THE MEAN DRIFT FORCE ON A TWO-DIMENSIONAL BODY FREELY FLOATING IN AMBIENT REGULAR WAVES (THAT SHEDS NO VORTICES) IS IDENTICALLY EQUAL, ACCORDING TO LINEAR THEORY, TO THE ABOVE EXPRESSION THAT DEPENDS ONLY ON THE AMPLITUDE OF THE RELECTED WAVE.

- IF THE AMPLITUDE $|A^-| = 0$ THEN THE BODY WILL EXPERIENCE ZERO MEAN DRIFT FORCE!

SUBMERGED FIXED CIRCLE



IT CAN BE SHOWN THAT A SUBMERGED FIXED CIRCLE DOES NOT RELECT ANY OF THE AMBIENT WAVE DISTURBANCE. HENCE

$$|A^-| = 0 \implies \mathbb{D} = 0 \; !$$

- THIS IS THE ONLY CASE WE KNOW IN 2D OR 3D WHERE THE MEAN DRIFT FORCE IS ZERO. IF THE CIRCLE IS ALLOWED TO MOVE THEN $\mathbb{D} > 0$

- WE KNOW OF NO OTHER SHAPE IN 2D WITH THE SAME PROPERTY. IT IS NOTEWORTHY THAT $\mathbb{D} > 0$ IN THE CASE OF A CIRCLE FIXED IN FINITE DEPTH.

- IN PRACTICE THE FLOW OVER THE CIRCLE WILL SEPARATE AND THE ABOVE RESULT IS NOT VALID. SEE ONF FOR A DISCUSSION OF VISCOUS EFFECTS AROUND BLUFF BODIES. —