SECOND-ORDER WAVE FORCES

- IN RESPONSE TO AN AMBIENT WAVE RECORD CONSISTING OF THE LINEAR SUPERPOSITION OF REGULAR WAVES, THE LINEAR WAVE FORCE ON A FLOATING STRUCTURE, OR ANY OTHER LINEAR EFFECT TAKES THE FORM:

$$f_1(t) = \mathrm{Re} \sum_k A_k F_k^{(1)}(\omega_k) e^{i\omega_k t + i\phi_k}$$

THE COMPLEX FORCE TRANSFER FUNCTION $F_k^{(1)}(\omega_k)$ IS EVALUATED ANALYTICALLY OR NUMERICALLY AS DESCRIBED EARLIER.

- THE DEFINITION OF THE CORRESPONDING SECOND-ORDER FORCE (TO BE ADDED TO THE LINEAR ONE DEFINED ABOVE) MAY BE CAST IN THE FORM:

$$f_2(t) = \mathrm{Re} \sum_k \sum_l A_k A_l F_{kl}^{(2)+} e^{i(\omega_k+\omega_l)t + i(\phi_k+\phi_l)}$$

$$+ \mathrm{Re} \sum_k \sum_l A_k A_l^* F_{kl}^{(2)-} e^{i(\omega_k-\omega_l)t + i(\phi_k-\phi_l)}$$

- THE SECOND-ORDER COMPLEX TRANSFER FUNCTIONS $F_{k\ell}^{(2)\pm}(\omega_k, \omega_\ell)$ FOR THE SUM AND DIFFERENCE FREQUENCY PROBLEMS ARE THE PRINCIPAL QUANTITIES TO BE EVALUATED FROM THE SOLUTION OF THE SECOND-ORDER PROBLEM.

- FOR VALUES OF (k, ℓ) THAT MAY BE AS LARGE AS 30-50 FOR REALISTIC SEASTATES AND FOR COMPLEX THREE-DIMENSIONAL STRUCTURES, THIS REPRESENTS A VERY SIGNIFICANT COMPUTATIONAL TASK. IT CAN BE REDUCED BY A FACTOR OF 2 BY IMPOSING THE SYMMETRY CONDITION FOR HERMITIAN FORMS:

$$F_{k\ell}^{\pm} = F_{\ell k}^{\pm *}.-$$

YET THIS LEAVES A SIGNIFICANT COMPUTATIONAL EFFORT WHICH IN ITS GENERALITY IS CARRIED OUT BY THREE-DIMENSIONAL PANEL METHODS SOLVING THE SUM- AND DIFFERENCE FREQUENCY RADIATION AND DIFFRACTION PROBLEMS AROUND FLOATING STRUCTURES._

- IN THE DIFFERENCE-FREQUENCY PROBLEM A USEFUL AND ACCURATE APPROXIMATION FOR $f_2(t)$ IS AVAILABLE KNOWN AS NEWMAN'S APPROXIMATION.

  IN UNIDIRECTIONAL WAVES IT APPROXIMATES THE TRANSFER FUNCTION AS FOLLOWS:

$$F^-_{k\ell}(\omega_k, \omega_\ell) \cong \left\{ \mathbb{D}_k(\omega_k)\, \mathbb{D}_\ell(\omega_\ell) \right\}^{1/2}$$

  WHERE $\mathbb{D}_k(\omega_k)$ IS THE MEAN DRIFT FORCE ON THE FLOATING PLATFORM AT FREQUENCY ω_k WHICH IS EVALUATED WITH THE METHODS DISCUSSED EARLIER.

- THE RESULT OF THIS APPROXIMATION IS THAT THE SOLUTION OF THE SECOND-ORDER PROBLEM IS CIRCUMVENTED ENTIRELY! THIS IS IN PRACTICE VERY DESIRABLE DUE TO THE COMPLEXITY AND EFFORT IN SOLVING SECOND-ORDER PROBLEMS.

- THE EVALUATION OF THE RESULTING SECOND-ORDER FORCE RECORD $f_2^-(t)$ MAY BE CARRIED OUT USING FFT ROUTINES.

SPECTRAL DENSITY OF SECOND-ORDER EFFECTS

- IN PRACTICE IT IS NECESSARY TO BE ABLE TO DETERMINE THE STATISTICAL PROPERTIES OF LINEAR AND SECOND-ORDER FORCES AND VESSEL RESPONSES.

- THIS MAY BE ACCOMPLISHED VIA DIRECT SIMULATION IN THE TIME DOMAIN WHEN APPROPRIATE NUMERICAL METHODS ARE AVAILABLE OR BY SPECTRAL ANALYSIS IN COMBINATION WITH APPROXIMATE LINEAR RESPONSE MODELS. —

- IN THE CASE OF SECOND-ORDER WAVE EFFECTS ON OFFSHORE PLATFORMS, FOR EXAMPLE THE SLOW-DRIFT EXCITATION AND RESPONSE OF SEMISUBMERSIBLES, SPAR'S AND TLP'S, MOORED OR TETHERED TO THE OCEAN FLOOR, LINEAR RESPONSE MODELS EXIST. THEY ARE EXCITED BY A SECOND-ORDER FORCE THE SPECTRAL DENSITY OF WHICH IS USEFUL TO KNOW.

- THE REMAINDER OF THIS SECTION DEFINES THE SPECTRAL DENSITY OF THE SUM AND DIFFERENCE FREQUENCY FORCE ON ANY FLOATING BODY.

- LET $S(\omega)$ BE THE SPECTRUM OF THE AMBIENT SEASTATE ASSUMED UNIDIRECTIONAL

THE RELATION BETWEEN $S(\omega)$ AND THE AMPLITUDES OF THE POLYCHROMATIC AMBIENT SEASTATE A_i WAS GIVEN EARLIER AND REPEATED HERE

$$S(\omega_i)\,\Delta\omega = \frac{1}{2} A_i^2$$

FOR A SMALL FREQUENCY BAND $\Delta\omega$ CENTERED AROUND ω_i.

- <u>LINEAR SYSTEM THEORY</u>

GIVEN $S(\omega)$ AND THE COMPLEX TRANSFER FUNCTION OF A LINEAR FORCE (OR OTHER EFFECT) $F(\omega)$, THE SPECRAL DENSITY OF F IS GIVEN BY

$$S_F(\omega) = S(\omega)\,|F(\omega)|^2 .$$

THIS RESULT IS EXTENDED BELOW TO SECOND ORDER FORCES (AND MOMENTS).

SUM-FREQUENCY PROBLEM

- THE SPECTRAL DENSITY OF A SUM-FREQUENCY FORCE WITH COMPLEX QUADRATIC TRANSFER FUNCTION $F^{+}(\omega_k, \omega_\ell)$ IS GIVEN BY THE EXPRESSION (STATED HERE WITHOUT PROOF):

$$S_F^{(2)+}(\Omega) = 8 \int_0^{\Omega/2} S(\omega)\, S(\Omega-\omega) \times \left| F^{+}(\omega, \Omega-\omega) \right|^2 d\omega .$$

WHERE $S(\omega)$ IS THE SPECTRUM OF THE AMBIENT SEASTATE

DIFFERENCE-FREQUENCY PROBLEM

- THE CORRESPONDING EXPRESSION FOR THE DIFFERENCE-FREQUENCY FORCE IS:

$$S_F^{(2)-}(\Omega) = 8 \int_0^{\infty} S(\omega)\, S(\Omega-\omega) \times \left| F^{-}(\omega, \omega+\Omega) \right|^2 d\omega$$

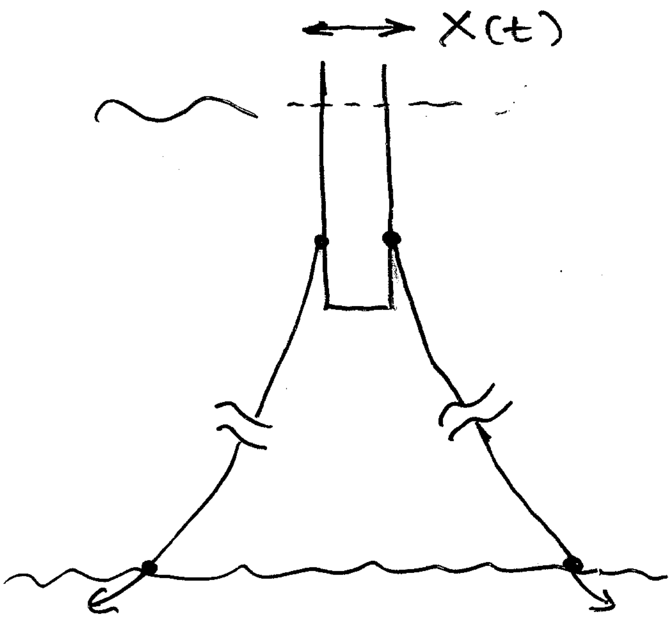
AND BY INVOKING NEWMAN'S APPROXIMATION:

$$S_F^{(2)-}(\Omega) \cong 8 \int_0^{\infty} S(\omega)\, S(\Omega-\omega) \times \mathbb{D}(\omega)\, \mathbb{D}(\omega+\Omega)\, d\omega .$$

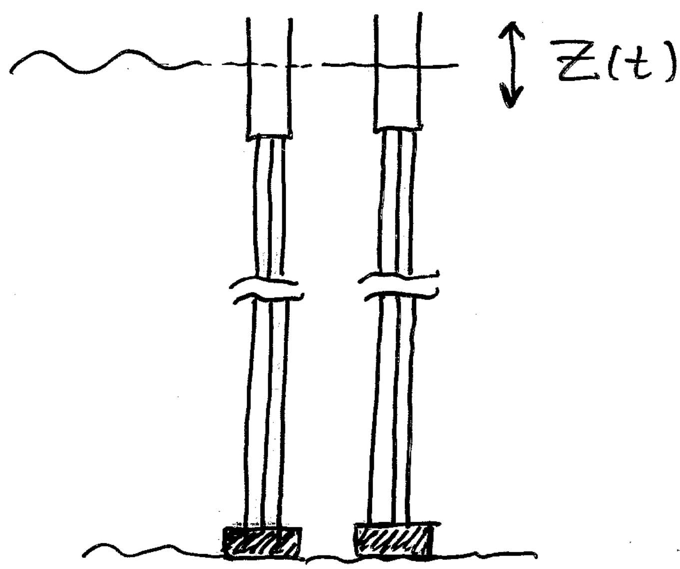
ALSO IT IS USEFUL TO RECALL THE MEAN VALUE OF THE DRIFT FORCE IN AN AMBIENT SEASTATE WITH SPECTRAL DENSITY $S(\omega)$:

$$\bar{D} = 2 \int_0^\infty S(\omega)\, \mathbb{D}(\omega)\, d\omega \,.-$$

NONLINEAR RESPONSES OF COMPLIANT OFFSHORE PLATFORMS EXCITED BY SECOND-ORDER EFFECTS



SPAR PLATFORM



TENSION LEG PLATFORM

$X(t) \equiv$ SLOW-DRIFT RESPONSE (LOW FREQUENCY)

$Z(t) \equiv$ SPRINGING OR RINGING RESPONSE (HIGH FREQUENCY)

THE ABOVE TWO FIGURES ILLUSTRATE TWO TYPICAL NONLINEAR RESPONSES OF COMPLIANT OFFSHORE PLATFORMS DESIGNED TO OPERATE IN WATERS OF LARGE DEPTH FOR THE EXTRACTION OF OIL AND GAS FROM SUB-SEA RESERVOIRS

- <u>SLOW-DRIFT RESPONSES $X(t)$</u>

  THESE ARE TYPICALLY LARGE AMPLITUDE LOW-FREQUENCY RESPONSES OF THE ORDER OF MINUTES EXCITED BY SECOND-ORDER DIFFERENCE-FREQUENCY EFFECTS, AS WELL AS TIME DEPENDENT WIND & CURRENTS. THE LATTER ARE OF VISCOUS NATURE.

  AN EFFECTIVE MODEL FOR $X(t)$ IS:

- $$(M + A)\ddot{X}(t) + B\dot{X}(t) + C\,X(t) = \bar{f}_2(t)$$

  THE FORCING IN THE RIGHT-HAND SIDE HAS BEEN STUDIED ABOVE AND ITS SPECTRAL DENSITY IS ASSUMED KNOWN. THE REMAINING COEFFICIENTS ARE SELECTED SO AS TO ALLOW THIS LINEAR SYSTEM BE AS CLOSE TO REALITY. —

- SPRINGING RESPONSES OF TLP'S $Z(t)$

THEY ARISE BECAUSE OF THE FLEXURAL RESPONSE OF THE PRETENSIONED TETHERS OF TENSION LEG PLATFORMS OPERATING IN WATERS OF LARGE DEPTH. THEY ARE EXCITED PRIMARILY BY SECOND-ORDER OR MORE EXTREME NONLINEAR EFFECTS AND CAN ALSO BE MODELED EFFECTIVELY BY A LINEAR SYSTEM:

$$(M+A)\ddot{Z}(t) + B\dot{Z}(t) + CZ(t) = f_2^{+}(t)$$

THE ABOVE DESCRIPTION DOES NOT EXPLICITLY RECOGNIZE IMACT TYPE LOADS KNOWN AS RINGING THAT MAY BE ADDED TO THE RIGHT-HAND SIDE.

- THE SPECTRAL DENSITY OF $f_2^{+}(t)$ IS DETERMINED AS DISCUSSED ABOVE

- IN BOTH TYPES OF PROBLEMS IT IS ESSENTIAL TO MODEL PROPERLY AND INCLUDE INTO A LINEAR DAMPING COEFFICIENT B (OR A NONLINEAR MORISON TYPE TERM) ALL EFFECTS THAT MAY INFLUENCE THE PLATFORM RESPONSE AT RESONANCE.—