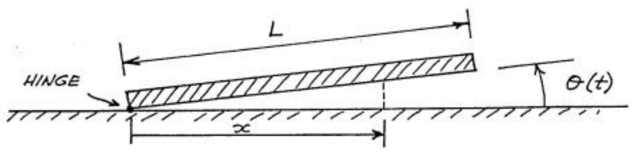
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 3.05

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin

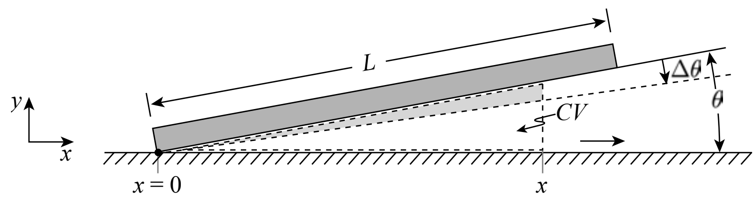


A long, flat plate of breadth L (L being small compared with the length perpendicular to the sketch) is hinged at the left side to a flat wall, and the gap betweeen the plate and wall is filled with an incompressible liquid of density ρ. If the plate is at a *small* angle $\theta(t)$ and is depressed at an angular rate

$$\omega(t) = -\frac{d\theta}{dt} \ ,$$

obtain an expression for the average liquid speed $u(x,t)$ in the x-direction at station x and time t.

Solution:



$\mathbf{u} = (u, v)$

Conservation of mass:

$$\begin{pmatrix}\text{time rate of change}\\ \text{of mass in the volume}\end{pmatrix} = \begin{pmatrix}\text{rate of mass}\\ \text{flow in}\end{pmatrix} - \begin{pmatrix}\text{rate of mass}\\ \text{flow out}\end{pmatrix} \tag{3.05a}$$

If the plate moves $\Delta\theta$ in time, Δt, fluid in the shaded area, *must flow out* through the control surface (unit depth into the page).

$$\begin{pmatrix}\text{time rate of change}\\ \text{of mass in the volume}\end{pmatrix} = \frac{\rho}{2}x(x\underbrace{\tan\Delta\theta}_{\approx\Delta\theta})\frac{1}{\Delta t} = -\frac{1}{2}x^2\omega\rho\ ;$$

$$(\text{rate of mass flow in}) = 0\ ;$$

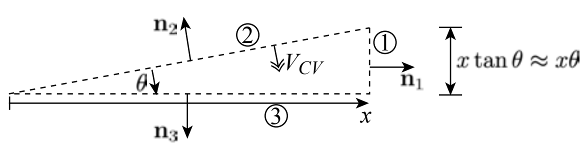
$$(\text{rate of mass flow out}) = u_{\text{av}}(x\underbrace{\tan\theta}_{\approx\theta})\rho$$

From (3.05a)

$$-\frac{1}{2}x^2\omega = 0 - u_{\text{av}}\theta x \Rightarrow \boxed{u_{\text{av}} = \frac{x\omega}{2\theta}}\ .$$

More formally by conservation of mass:

$$\frac{\partial}{\partial t}\int_{\forall}\rho\, dV = -\int_s \rho(\mathbf{u} - \mathbf{V}_{CV})\cdot\mathbf{n}\, ds\ . \tag{3.05b}$$



$$\int_{\forall}\rho\, dV = \rho\int_0^x dx'\int_0^{x'\theta} dy' = \frac{1}{2}x(x\theta) \Rightarrow \frac{d}{dt}\int_{\forall}\rho\, dV = -\frac{1}{2}\rho x^2\omega$$

$$\int_{③} = 0, \qquad \int_{②} = 0, \qquad \text{b.c. } \mathbf{u} = \mathbf{V}_{CV} \text{ (no flux through a solid boundary)}$$

$$\int_{①} = -\int_{\theta}^{x\theta}\rho \underset{=\mathbf{u}\cdot\mathbf{n}}{\underset{\uparrow}{u}}\, dy = \rho x\theta\underbrace{\frac{1}{x\theta}\int_0^{x\theta} u\, dy}_{u_{\text{av}}}$$

Putting this together in (3.05b)

$$-\frac{1}{2}\rho x^2\omega = -\rho x\theta u_{\text{av}} \Rightarrow \boxed{u_{\text{av}} = \frac{x\omega}{2\theta}}\ .$$

□

Problem Solution by Sungyon Lee, Fall 2005

