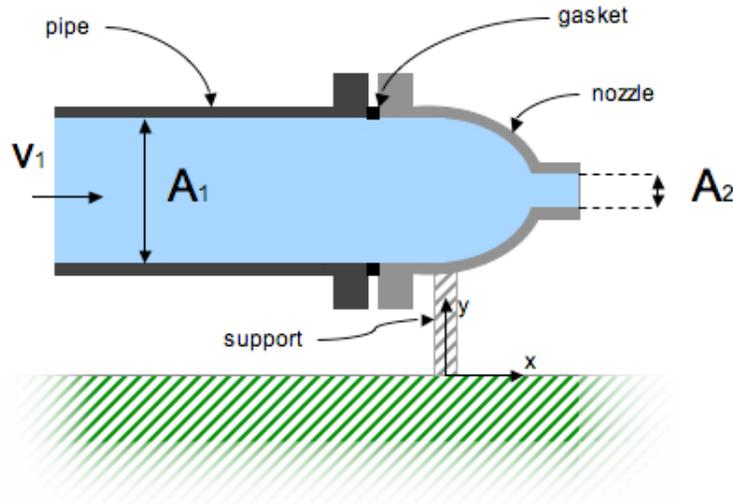


MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 5.02

This problem is from “Advanced Fluid Mechanics Problems” by A.H. Shapiro and A.A. Sonin



A pipe of area A_1 carries a gas at density ρ and velocity V_1 . A converging nozzle is mounted at the end of the pipe, as shown, to increase the gas velocity as it emerges into the atmosphere. The flow in the nozzle is incompressible.

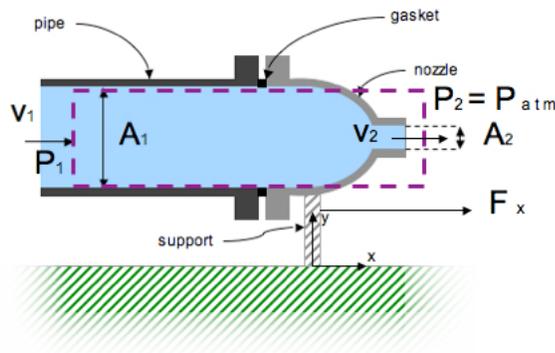
- (a) Use the momentum theorem to derive the x and y components of force, in excess of those required to support weight, exerted by the nozzle on its support.
- (b) What gage pressure will the presence of the nozzle induce at the pipe (where the area is A_1)? You may model the velocity at station (1) as being uniform and assume that the velocity is also uniform at (2). As regards part (a), there is clearly ambiguity in the problem as being stated, since the x component of the force on the support will depend on the compression force applied to the gasket, as well as on the fluid flow. In answering (a), consider just the flow-induced force which will be exerted when the compression force on the gasket is zero.
- (c) Apart from the assumption that conditions at (2) have attained uniformity, does the result in (a) depend in any way on the contour of the nozzle between (1) and (2)?
- (d) What is the direction of the force (a) if $A_2 < A_1$? If $A_2 > A_1$? Explain.

Note: See also problem 4.4.

Solution:

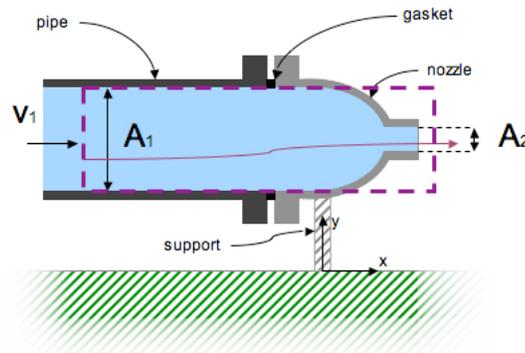
- (a) Let's use mass conservation in the CV shown to get more information about the entrance and exit velocities, V_1 and V_2 respectively.

$$\underbrace{\frac{\partial}{\partial t}(\text{Mass}_{CV})}_{\text{Steady} \Rightarrow = 0} - V_1 A_1 + V_2 A_2 = 0, \Rightarrow V_2 = V_1 \frac{A_1}{A_2}. \tag{5.02a}$$



Now, in order to perform a momentum conservation analysis, we still need to know the pressures at the control volume boundaries. Let's use Bernoulli to figure out the pressure inside the pipe ¹, then

$$P_1 + \frac{1}{2}\rho V_1^2 = \underbrace{P_2}_{=P_{atm}} + \frac{1}{2}\rho \underbrace{V_2^2}_{=V_1^2 \frac{A_1^2}{A_2^2}}, \Rightarrow P_1 = P_{atm} + \frac{1}{2}\rho V_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right). \tag{5.02b}$$



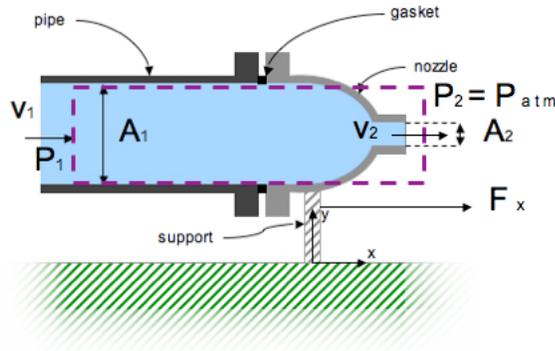
Now, let's use a CV to find out the extra force needed to hold the nozzle, considering that the system is steady,

$$(\rho V_1)A_1(-V_1) + (\rho V_2)A_2(V_2) = F_x + A_1 \left(\underbrace{P_1}_{=P_{atm} + \frac{1}{2}\rho V_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right)} \right) - A_1 \left(\underbrace{P_2}_{=P_{atm}} \right), \tag{5.02c}$$

¹Notice that we are assuming that the flow is inviscid to use Bernoulli, and that is possible to draw a streamline between (1) and (2), thus the flow is assumed not turbulent too.

then,

$$-\rho V_1^2 A_1 + \rho \underbrace{V_2^2}_{=V_1^2 \frac{A_1^2}{A_2^2}} A_2 = F_x + A_1 \frac{1}{2} \rho V_1^2 \left(\frac{A_1^2}{A_2^2} - 1 \right), \Rightarrow \rho V_1^2 \frac{A_1^2}{A_2^2} A_2 = F_x + A_1 \frac{1}{2} \rho V_1^2 \left(\frac{A_1^2}{A_2^2} + 1 \right), \quad (5.02d)$$



finally,

$$F_x = -A_1 \frac{1}{2} \rho V_1^2 \left(\frac{A_1^2}{A_2^2} - 2 \frac{A_1}{A_2} + 1 \right) = -A_1 \frac{1}{2} \rho V_1^2 \left(\frac{A_1}{A_2} - 1 \right)^2. \quad (5.02e)$$

- (b) From Bernoulli,

$$P_1 + \frac{1}{2} \rho V_1^2 = \underbrace{P_2}_{=P_{atm}} + \frac{1}{2} \rho \underbrace{V_2^2}_{=V_1^2 \frac{A_1^2}{A_2^2}}, \Rightarrow P_1 = P_{atm} + \frac{1}{2} \rho V_1^2 \left(\frac{A_1^2}{A_2^2} - 1 \right). \quad (5.02f)$$

- (c) Yes, if the flow exhibits separation, the use of Bernoulli is not valid and the solution is not valid too.
- (d) From the form of the solution, in both cases the force applied goes in the negative direction. The force can be divided in two parts, one part due to the pressure differences, and another from the difference in momentum fluxes. When there is a reduction in the area, the momentum flux difference dominates. On the other hand when there is a expansion, the pressure at the inlet is smaller than atmospheric and this force dominates. To show this, let's take the terms from the momentum conservation analysis before simplification

$$-\rho V_1^2 A_1 + \rho V_1^2 \frac{A_1^2}{A_2^2} A_2 = F_x + A_1 \frac{1}{2} \rho V_1^2 \left(\frac{A_1^2}{A_2^2} - 1 \right), \Rightarrow \quad (5.02g)$$

$$\underbrace{A_1 \frac{1}{2} \rho V_1^2 \left(1 - \frac{A_1^2}{A_2^2} \right)}_{Pressure} + \underbrace{\rho V_1^2 A_1 \left(\frac{A_1}{A_2} - 1 \right)}_{Momentum Flux} = F_x. \quad (5.02h)$$

□

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