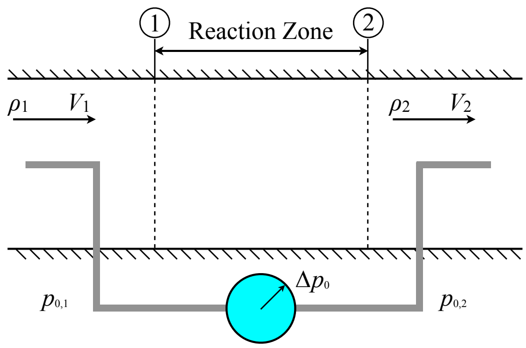
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 5.09

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin

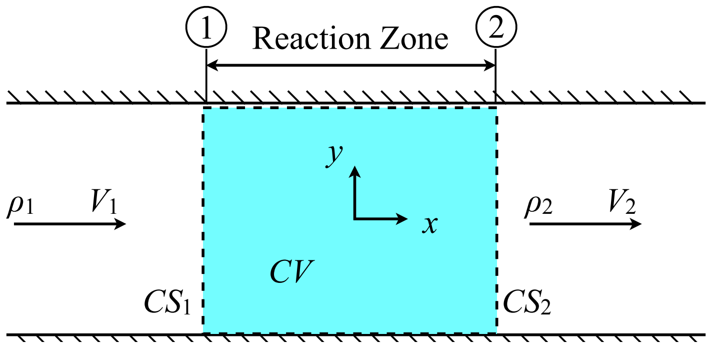


The sketch shows a liquid emulsion (a finely-divided mixture of two liquids) of mean density ρ_1 entering a reaction zone of a constant-area reactor with speed V_1. The components of the emulsion react chemically, and leave the reaction zone as a liquid at the density ρ_2. Pitot tubes are installed upstream of the reaction zone. (Pressure inside a pitot tube is stagnation pressure, $p_0 = p + \frac{1}{2}\rho V^2$).

It is agreed to assume that the flow is inviscid, steady and one-dimensional, that the original emulsion is incompressible, and that the liquid leaving the reaction zone is incompressible.

Calculate the value of $(p_{0,1} - p_{0,2})/(\frac{1}{2}\rho_1 V_1^2)$ in terms of the density ratio ρ_2/ρ_1.

Solution:



Taking the reaction zone as our control volume, and noting that the cross-sectional area of the reactor is constant, we find from our integral mass conservation relation that

$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} \tag{5.09a}$$

One might be tempted to apply the Bernoulli equation across the streamline, however, this approach would be invalid since the density changes in this region and the flow is well mixed so that it may not be possible to define a streamline here. Instead, we consider an integral momentum conservation relation to determine the pressure drop across the reaction zone. Here we consider Form A:

$$\frac{d}{dt}\int_{CV} \rho \mathbf{u} d\forall + \int_{CS} \rho \mathbf{u}(\mathbf{u} - \mathbf{u_{CS}}) \cdot \hat{n} dA = \sum \mathbf{F} \tag{5.09b}$$

Since the flow is steady in time, the first term in Eq. (5.09b) is zero and we must balance the momentum flux and forces acting on the control surfaces. Furthermore, our control volume is not moving or changing shape in time so $\mathbf{u_{CS}} = 0$. The momentum flux at CS_1 is

$$\int_{CS_1} \rho \mathbf{u}(\mathbf{u} - \mathbf{u_{CS}}) \cdot \hat{n} dA = -\rho_1 V_1^2 A \hat{e}_x \tag{5.09c}$$

Likewise, the momentum flux at CS_2 is

$$\int_{CS_2} \rho \mathbf{u}(\mathbf{u} - \mathbf{u_{CS}}) \cdot \hat{n} dA = \rho_2 V_2^2 A \hat{e}_x \tag{5.09d}$$

The forces acting on the control surfaces simply result from the pressure acting upstream and downstream of the reaction zone so that

$$\sum \mathbf{F} = (p_1 - p_2) A \hat{e}_x \tag{5.09e}$$

Substituting Eq. (5.09c), (5.09d) and (5.09e) into Eq. (5.09b), we find

$$-\rho_1 V_1^2 + \rho_2 V_2^2 = p_1 - p_2 \tag{5.09f}$$

Outside the reaction zone, the flow is well-defined such that the Bernoulli equation may be applied along a streamline, but only between two points that both lie either upstream of the reaction zone or downstream of it. The relationship between static and dynamic pressure is $p_0 = p + \frac{1}{2}\rho V^2$, so the relationships between p_1 and $p_{0,1}$ and between p_2 and $p_{0,2}$ are

$$p_{0,1} = p_1 + \frac{1}{2}\rho_1 V_1^2 \quad \& \quad p_{0,2} = p_2 + \frac{1}{2}\rho_2 V_2^2 \tag{5.09g}$$

Substituting these results into Eq. (5.09f), we obtain

$$-\rho_1 V_1^2 + \rho_2 V_2^2 = p_{0,1} - \frac{1}{2}\rho_1 V_1^2 - p_{0,2} + \frac{1}{2}\rho_2 V_2^2 \tag{5.09h}$$

or alternatively

$$p_{0,1} - p_{0,2} = \frac{1}{2}\left(\rho_2 V_2^2 - \rho_1 V_1^2\right) \tag{5.09i}$$

Dividing this expression by $\frac{1}{2}\rho_1 V_1^2$ to obtain a dimensionless pressure drop, we have

$$\frac{p_{0,1} - p_{0,2}}{\frac{1}{2}\rho_1 V_1^2} = \frac{\rho_2}{\rho_1}\left(\frac{V_2}{V_1}\right)^2 - 1 \tag{5.09j}$$

Substituting our relationship from Eq. (5.09a), we at last have

$$\frac{p_{0,1} - p_{0,2}}{\frac{1}{2}\rho_1 V_1^2} = \frac{\rho_1}{\rho_2} - 1 \tag{5.09k}$$

□

Problem Solution by TJO, Fall 2010

