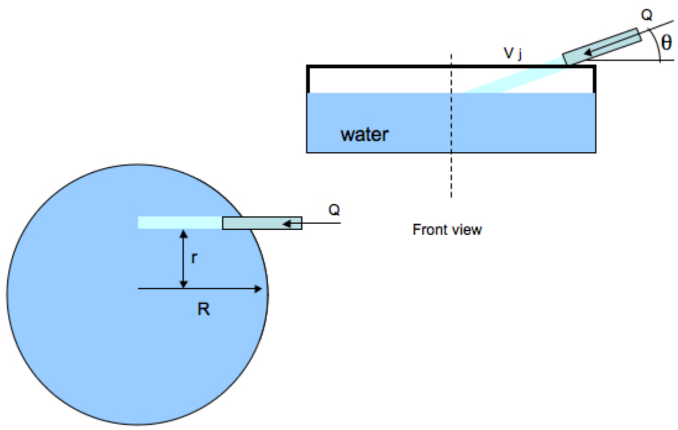
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 5.33

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin

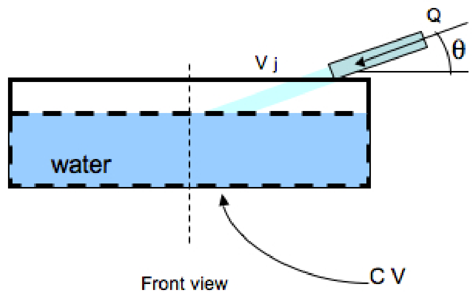


At $t = 0$, a circular tank of radius R contains water at rest, with a depth h. Between $0 < t < \tau$, a water hose is sprayed onto the surface of the water in the tank at a volume flow rate Q and an exit velocity V_j. The jet impacts tangentially on the water at a radius R_j, with an angle θ relative t the horizontal.

After the time τ, the hose is turned off. Eventually, because of friction within the water, all the water in tank will end up rotating like a solid body.

Derive an expression for the final angular rate of rotation Ω of the water, assuming shear forces between the water and the walls of the tank are negligible.

Solution:



Applying mass conservation to this CV,

$$\frac{\partial}{\partial t}\left(\int_{CV}\rho dVol\right)+\int_{CS}\rho(\underline{V}-\underline{V}_{CS})\cdot\hat{\underline{n}}dA=0, \tag{5.33a}$$

then

$$\rho\pi R^2\frac{dh}{dt}-\rho Q=0, \tag{5.33b}$$

for which, when $t=\tau$, then

$$\Delta h=h_{new}=\frac{Q\tau}{\pi R^2}. \tag{5.33c}$$

Applying angular momentum to the CV, we have

$$\frac{d}{dt}\left(\int_{CV}\rho\underline{r}\times\underline{V}dVol\right)+\int_{CS}\rho\underline{r}\times\underline{V}(\underline{V}-\underline{V}_{CS})\cdot\hat{\underline{n}}dA=0,\Rightarrow \tag{5.33d}$$

$$d\left(\int_{CV}\rho\underline{r}\times\underline{V}dVol\right)+\left(\int_{CS}\rho\underline{r}\times\underline{V}(\underline{V}-\underline{V}_{CS})\cdot\hat{\underline{n}}dA\right)dt=0,\Rightarrow \tag{5.33e}$$

$$d\left(\int_0^R(\rho r^2\omega)2\pi r(h+h_{new})dr\right)=(\rho R_jV_j\cos\theta Q)dt, \tag{5.33f}$$

where h_{new} is the increment in the tank water level due to the added hose water.

From (5.33c), we have, $h_{new}=\frac{Q\tau}{\pi R^2}$ for any $t<\tau$. Also, note that writing $dV=2\pi rhdr$ is only an approximation, since the free surface is not straight but rather curved! (recall the calculation of this surface equation during the fluid statics lecture).

As the water hose is sprayed, its radial component adds angular momentum in the CV (finally stabilizing as rigid body rotation). If τ_{rigid} is the time, after the hose stops, for the fluid to stabilize into rigid body rotation, we can integrate the right hand side of (5.33e) from 0 to $(\tau+\tau_{rigid})$, and the left hand side form 0 to R [1], thus

$$\left[\frac{\pi\omega R^4}{2}\left(h+\underbrace{h_{new}}_{=\frac{Q\tau}{\pi R^2},\ t>\tau}\right)\right]_{0,0}^{\tau+\tau_{rigid},\Omega}=(\rho R_jV_j\cos\theta Qt)_0^{\tau+\tau_{rigid}}. \tag{5.33g}$$

Now, noting that $Q=0$ for $\tau<t<\tau+\tau_{rigid}$, and $\omega=\Omega$ at $t=\tau+\tau_{rigid}$, we have

[1]Note that since the fluid has stabilized, ω is nor a function of the radius anymore.

$$\frac{\pi\Omega R^4}{2}\left(h + \frac{Q\tau}{\pi R^2}\right) = R_j V_j \cos\theta Q\tau, \Rightarrow \tag{5.33h}$$

$$\Omega = \frac{2R_j V_j \cos\theta Q\tau}{\pi R^4\left(h + \frac{Q\tau}{\pi R^2}\right)}. \tag{5.33i}$$

□

Problem Solution by ATP/MC(Updated), Fall 2008

