

Name: _____

2.57/2.570 Midterm Exam No. 1
March 31, 2010
11:00 am -12:30 pm

Instructions:

- (1) 2.57 students: try all problems
- (2) 2.570 students: Problem 1 plus one of two long problems. You can also do both long problems, and one will be considered for bonus points.
- (3) Clearly state your assumptions.
- (4) Closed book, one sheet of notes allowed.

Unless otherwise noted, assume electron mass equals that of free electron in vacuum.

1. Answer the following short questions. Briefly explain your answer **(60 Points)**

(1) Infrared light at 10 micron is emitted between the first two energy levels of a one-dimensional quantum well with infinite potential barrier height. What is the width of the quantum well?

(2) Air molecules at 1 atm have a mean free path approximately 70 nm. Estimate the mean free path at 1 torr, i.e., 1/760 of 1 atm.

(3) Light is incident from a medium of refractive index $n=2$ towards an interface with vacuum. Calculate the reflectivity at the interface if

(3a) The angle of incidence is 0° .

(3b) If the angle of incidence is 65° .

(4) Consider blackbody radiation at 400 K in vacuum. On average, how many photons are there in a quantum state that has a photon wavelength of $10 \mu\text{m}$?

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(5) An electron with an energy of 1 eV encounters a potential barrier of height 0.5 eV. The electron effective mass equals that of free electrons. What is the probability that the electron can go over the potential barrier?

(6) The thermal conductivity of air at 300 K and 1 atmosphere is 0.026 W/m-K. Estimate its thermal conductivity at 600 K and 1 atm.

(7) Copper has an fcc lattice with a lattice constant of 3.61 Å (one Cu atom per lattice point). Each Cu atom contributes one valence electron. Calculate its Fermi level.

(8) A material has an fcc structure with two atoms as the basis. The lattice constant of the conventional cubic unit cell is 5.0 Å. The average phonon velocity is 2000 m/s and phonon mean free path is 50 nm. Answer the following two questions:

(8a) What is the phonon specific heat per unit volume at high temperatures?

(8b) What is the high temperature thermal conductivity of the material?

Long Problems:

2. Linear Dispersion. Sometimes, the electron energy-wavevector dispersion can be linear (an example is in graphene at special points in k-space). Consider a simplified linear dispersion model,

$$E - E_c = vk = v[k_x^2 + k_y^2 + k_z^2]$$

Where $k_x, k_y, k_z = \pm \frac{2\pi}{L}, \pm \frac{4\pi}{L}, \dots$. We assume that the number of electrons in the material is fixed at n per unit volume. Answer the following questions

- (a) Derive an expression for the electron density of states per unit volume and per energy interval.
- (b) Derive an integral expression that determines the chemical potential, using Fermi-Dirac statistics.
- (c) Assuming that Fermi-Dirac statistics can be well approximated by Boltzmann statistics, derive an explicit expression for the electron chemical potential.

In doing the energy integration, you can assume that upper limit is energy approaches infinity, despite you know that the dispersion is limited to the first Brillouin zone.

3. Phonon Reflection at a One-Dimensional Atomic Chain Interface. Consider an infinite one-dimensional atomic chain as shown in the following figure. On the left side, the atoms have a mass m . On the right hand side, the atoms have a different mass M . The spring constant and interatomic spacing are identical on the two sides, respectively. The phonon dispersion on the two sides are given by

$$\omega = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \left| \sin \frac{k_\ell a}{2} \right| \quad \omega = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \left| \sin \frac{k_r a}{2} \right|$$

where k_r or $k_\ell = -\frac{\pi}{a}, \dots, \frac{\pi}{a}$ are wavevectors in the first Brillouin zone for phonons on the left and the right hand side.

We now consider a lattice wave comes in from the left hand side, and the atomic displacement from equilibrium position is expressed as

$$u_i(x_i) = A \exp[-i(\omega t - k_\ell x_i)]$$

where x_i is the atomic position at equilibrium. The reflected atomic wave is

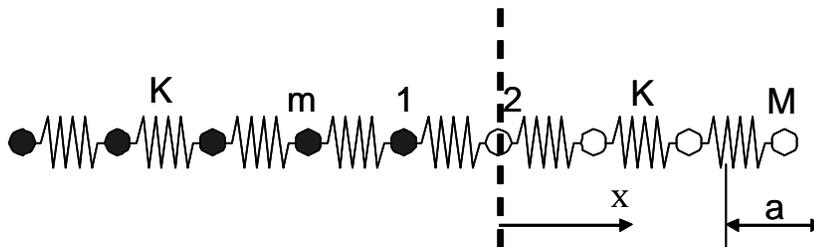
$$u_r(x_i) = B \exp[-i(\omega t + k_\ell x_i)]$$

and transmitted wave is

$$u_t(x_i) = C \exp[-i(\omega t - k_r x_i)]$$

We know that when wave transmit across the interface, the frequency remains constant. Answer the following questions,

- Find out a relation between k_r and k_ℓ .
- Write down the equations of motion for atoms 1 and 2 at the interface.
- Determine the reflection coefficient B/A , and transmission coefficient C/A .
- Consider that power equals force times velocity, determine power reflectance and transmittance at the interface.



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