

Lecture #9 Instructor Notes

A. Simple finite difference method for solving the mode eigenvalue problem

Rather than using the sound speed, let us define an equally useful quantity, the “slowness” by

$$S(z) = 1/c(z)$$

In terms of slowness, our DE for the acoustic normal modes is:

$$\frac{d^2\phi}{dz^2} + (\omega^2 S^2(z) - k_H^2)\phi = 0$$

where the first term in the round brackets is the total wavenumber and the full term in the round brackets is the vertical wavenumber.

We can use simple BC's to begin with, which we'll take as pressure release at top and bottom, i.e. $\phi(0) = 0$ and $\phi(H) = 0$. These can be modified with relative ease.

Now we write our modal equation in finite difference form, i.e.

$$\frac{\phi_{j+1} - 2\phi_j - \phi_{j-1}}{(\Delta z)^2} + \omega^2 S^2(z)\phi_j = k_H^2\phi_j$$

Define the shorthand:

$$A_j \equiv (\omega^2 S^2(z_j))$$

Also, with our pressure release BC's, we have $\phi_0 = \phi_{N+1} = 0$ for an N layer representation of the soundspeed profile.

Now one can write a set of N+2 equations from z_0 to z_{N+1} . But by taking $\phi_0 = \phi_{N+1} = 0$, we have deleted the first and last row and column, leaving N equations. There is a slight error in the central difference scheme at the endpoints, but our pressure release condition makes this negligible. Thus we write:

$$\frac{\varphi_2 - 2\varphi_1 + \overset{\varphi_0}{(0)}}{\Delta z^2} + A_1 \varphi_1 = R_H^2 \varphi_1$$

$$\frac{\varphi_3 - 2\varphi_2 + \varphi_1}{\Delta z^2} + A_2 \varphi_2 = R_H^2 \varphi_2$$

⋮

$$\frac{\varphi_N - 2\varphi_{N-1} + \varphi_{N-2}}{\Delta z^2} + A_{N-1} \varphi_{N-1} = R_H^2 \varphi_{N-1}$$

$$\overset{\varphi_{N+1}}{(0)} - 2\varphi_N + \varphi_{N-1} + A_N \varphi_N = R_H^2 \varphi_N$$

If we make two more definitions for notational convenience, as below, we can write

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