

- Zernicke phase mask.** You are given an imaging system which consists of two thin transparencies T1, T2 and two thin lenses L1, L2 arranged as shown in Figure A. The shapes and dimensions of T1, T2 are shown immediately below in Figure B.

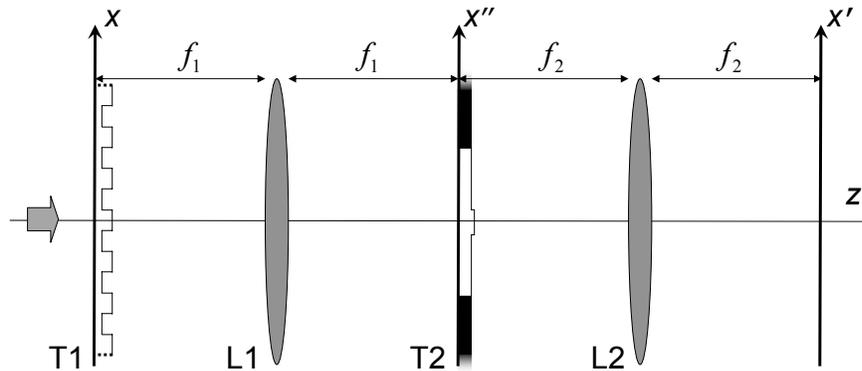


Figure A (not to scale)

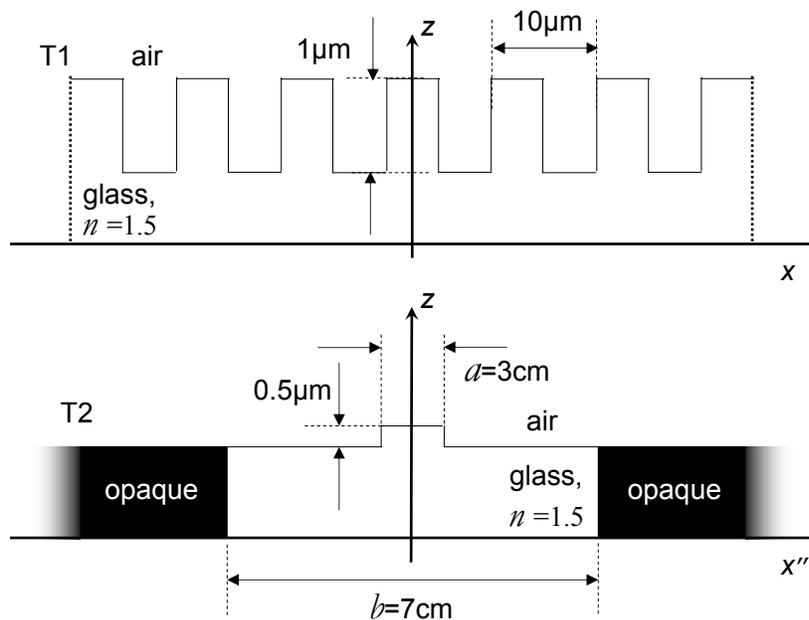


Figure B (not to scale)

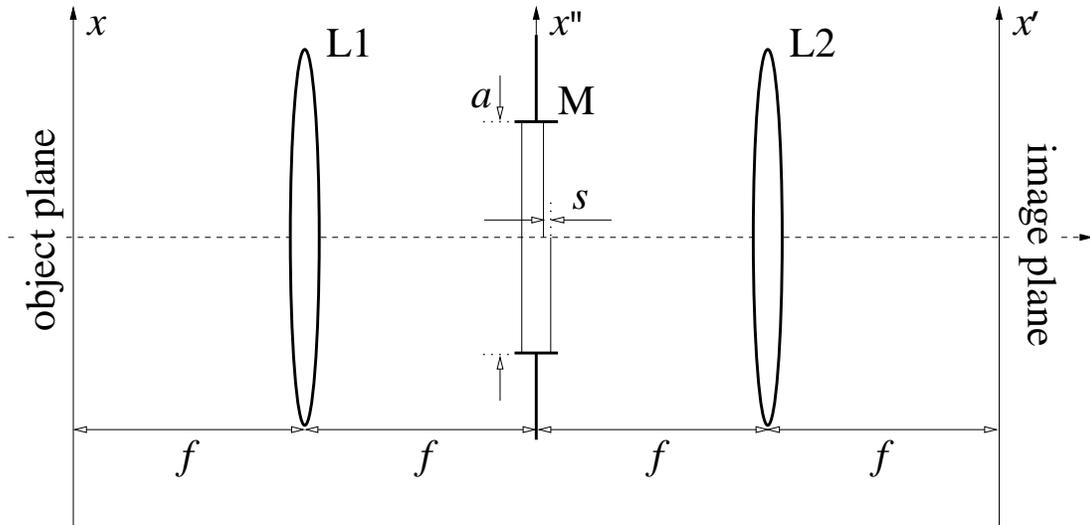
Transparency T1 is infinitely large in the  $x$  dimension. Lenses L1, L2 are identical, with infinitely large apertures and focal lengths  $f_1 = f_2 = 10\text{cm}$ . Numerical

values for the symbols  $a$ ,  $b$  denoting lateral feature sizes of T2 are defined in Figure B.

The illumination is an on-axis plane wave at wavelength  $\lambda = 1\mu\text{m}$ . The observation plane is located one focal distance behind L2.

- 1.a) What is the intensity immediately after T1?
- 1.b) What is the optical field immediately before T2?
- 1.c) What is the intensity measured at the observation plane?
- 1.d) Comparing your answers (a) and (c), how is T2 helpful in imaging the phase object T1?
- 1.e) Consider the limit  $b \rightarrow \infty$ . How does then answer (c) change? Is the larger aperture helpful in this case?
- 1.f) If  $a = 0.5\text{cm}$  and  $b \rightarrow \infty$ , is your answer (d) still valid? If yes, why? If not, what has gone wrong?

2. **Signum phase mask.** The imaging system shown below is illuminated by an on-axis plane wave at wavelength  $\lambda = 1\mu\text{m}$ . Lenses L1, L2 are identical with sufficiently large aperture and focal length  $f = 10\text{cm}$ .



The pupil mask M has aperture  $a = 1\text{cm}$ . Inside the aperture there is a piece of glass of refractive index  $n = 1.5$ . The glass has been partially etched to form a step of height  $s = 1\mu\text{m}$ , and the step is precisely aligned with the optical axis, as shown.

- 2.a) Sketch the amplitude transfer function (ATF) of this optical system.

**2.b)** A thin transparency with amplitude transmission function

$$g_t(x) = \cos\left(2\pi\frac{x}{20\ \mu\text{m}}\right)$$

is placed at the object plane. Sketch the magnitude and phase of the transparency  $g_t(x)$ .

**2.c)** With the transparency  $g_t(x)$  in place at the object plane, the system is illuminated with a plane wave on-axis. What is the optical field at the image plane?

**2.d)** With the same transparency  $g_t(x)$  in place at the object plane, the system is illuminated with a plane wave off-axis, propagating at an angle of  $+0.1\text{rad}$  with respect to the optical axis. What is the optical field at the image plane?

**2.e)** Suggest an intuitive description of this system's operation under spatially coherent, on-axis plane wave illumination (as in question c).

**3. Image processing.** Download an image from your favorite image-intensive website (e.g., imdb.com), convert it to grayscale by adding the R, G, B color values at each pixel, and crop its central portion  $g(x, y)$  so that it have square shape (e.g.,  $128 \times 128$ .) Please do not use any images that might be considered offensive.

**3.a)** Plot your image next to the amplitude  $|G(u, v)|$  of its Fourier transform  $G(u, v)$  (more details may be visible if instead you plot  $\log_{10}|G(u, v)|$ .)

**3.b)** Select a  $5 \times 5$  square region  $\mathcal{S}$  around the origin of the Fourier transform domain and define the new function  $G_1(u, v)$  such that

$$G_1(u, v) = \begin{cases} G(u, v) & \text{outside } \mathcal{S} \\ 0 & \text{inside } \mathcal{S} \end{cases}$$

Plot the inverse Fourier transform  $g_1(x, y)$  of  $G_1(u, v)$ .

**3.c)** Define the new function  $G_2(u, v)$  such that

$$G_2(u, v) = \begin{cases} G(u, v) & \text{inside } \mathcal{S} \\ 0 & \text{outside } \mathcal{S} \end{cases}$$

Plot the inverse Fourier transform  $g_2(x, y)$  of  $G_2(u, v)$ .

**3.d)** Comment on the appearances of  $g_1(x, y)$ ,  $g_2(x, y)$  and how these appearances are affected by the size of the region  $\mathcal{S}$ .

If you use MATLAB to solve this problem, you will find the following functions useful: (i) `fft2` computes the 2D Fourier transform of an image and returns it with some quadrants swapped, (ii) `fftshift` rearranges the quadrants of the

Fourier transform in their proper order, (iii) `ifft2` computes the inverse 2D Fourier transform (iv) `imagesc; colormap gray` displays a real grayscale image, (v) `print -dps [filename]` prints a figure into a postscript file which you can then print at any printer using the `lpr` command.

4. The Fourier transform may be regarded as a mapping of functions into their transforms and therefore satisfies the definition of a system.

4.a) Is this system linear?

4.b) Can you specify a transfer function for this system? If yes, what is it? If not, why not?

5. Compute the following convolution analytically in the *simplest* possible way:

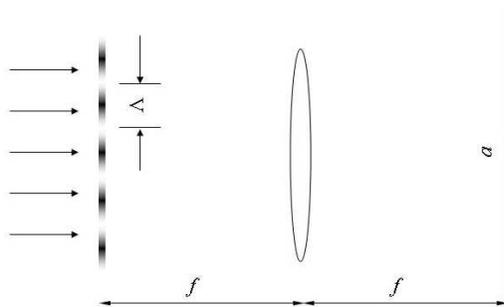
$$g(x', y') = f(x, y) \star h(x, y),$$

where

$$f(x, y) = \frac{1}{2} \left\{ 1 + \sin \left( 2\pi \frac{x}{\Lambda} \right) \right\},$$

$$h(x, y) = \text{sinc} \left( \frac{x}{x_0} \right).$$

6. Consider the optical system drawn below, where a sinusoidal amplitude grating of perfect contrast and period  $\Lambda$  is placed at the front focal plane of a lens of focal length  $f$ . A rectangular aperture (slit) of size  $a$  is placed at the back focal plane of the lens.



6.a) What is the field immediately past the aperture?

6.b) What is the relationship between the answer to this problem and the previous problem? Quantify.

MIT OpenCourseWare  
<http://ocw.mit.edu>

2.71 / 2.710 Optics  
Spring 2009

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.