Today

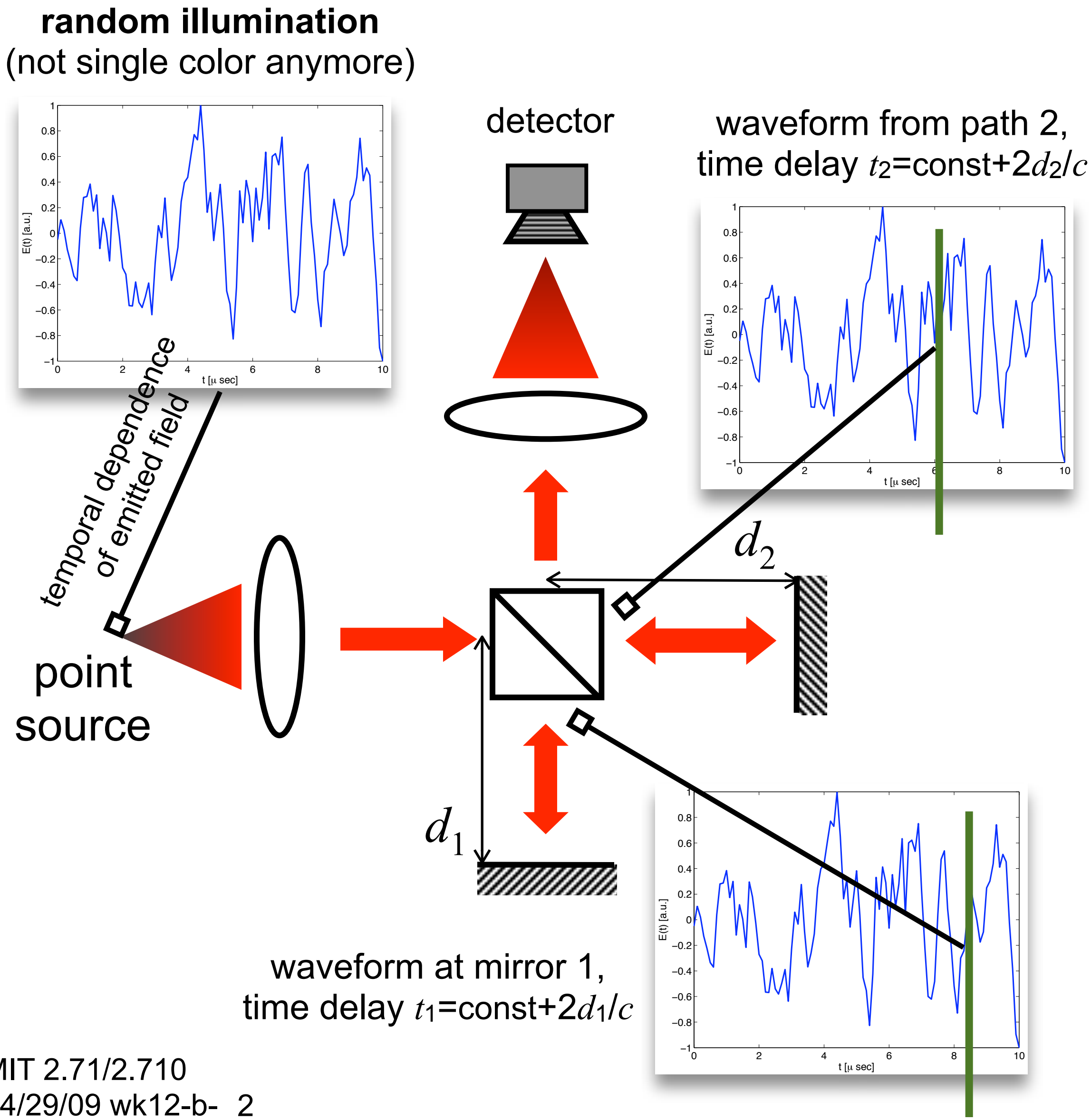
- Temporal and spatial coherence
- Spatially incoherent imaging
  - The incoherent PSF
  - The Optical Transfer Function (OTF) and Modulation Transfer Function (MTF)
  - MTF and contrast
  - comparison of spatially coherent and incoherent imaging

next two weeks

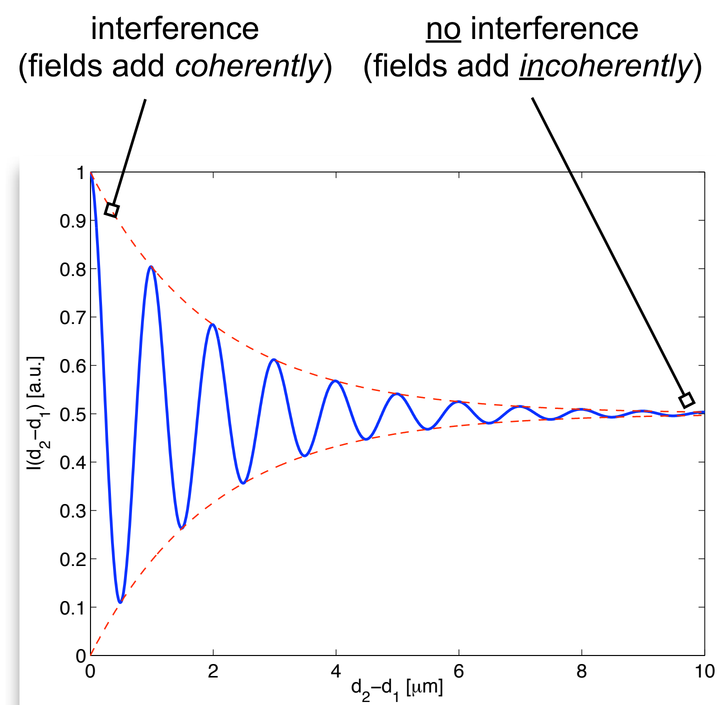
- Applications of the MTF
- Diffractive optics and holography

Temporal coherence

Michelson interferometer

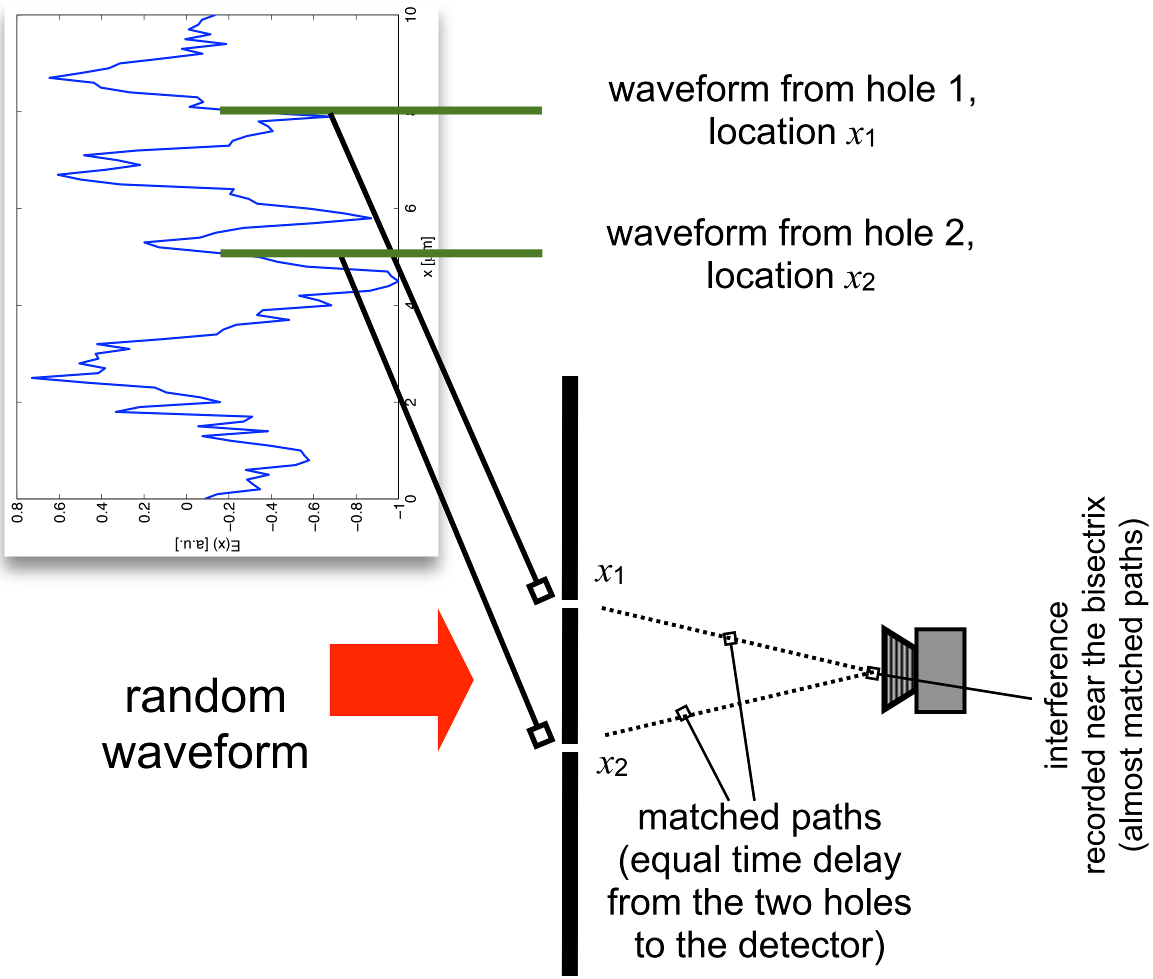


If paths 1 & 2 are matched, then the recombined waveforms at the detector are *correlated* so they produce interference fringes. However, as the difference $d_2 - d_1$ increases, the degree of correlation decreases and so does the contrast in the interference pattern.

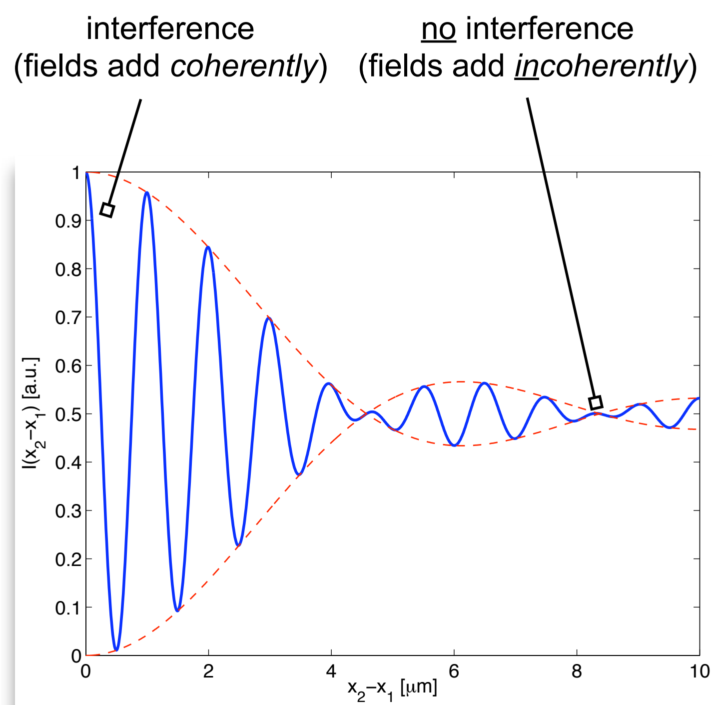


Spatial coherence

Young interferometer



If holes 1 & 2 are coincident, or very closely spaced, then the recombined waveforms at the detector are *correlated* so they produce interference fringes. However, as the difference $x_2 - x_1$ increases, the degree of correlation decreases and so does the contrast in the interference pattern.



Field intensity in the coherent and incoherent cases

The fringe pattern in the Michelson interferometer (notes `wk8-a-7`) is

$$I_M = \left(a_1^2 + a_2^2\right)\left[1 + \frac{2a_1a_2}{a_1^2+a_2^2}\cos\left(\frac{2\pi}{\lambda}2\left(d_2-d_1\right)\right)\right],$$

where a_1, a_2 are the amplitudes recombining from paths 1 and 2, respectively; and λ is the mean wavelength.

The fringe pattern in the Young interferometer (notes `wk8-b-7`) is

$$I_Y = \left(a_1^2 + a_2^2\right)\left[1 + \frac{2a_1a_2}{a_1^2+a_2^2}\cos\left(\frac{2\pi x'}{\lambda l}2\left(x_2-x_1\right)\right)\right],$$

where a_1, a_2 are the amplitudes recombining from holes 1 and 2, respectively, x' is the observation screen coordinate, and l the distance between the screen with the holes and the observation plane.

In both cases, the interference pattern can be written as

$$I = \left|a_1 e^{i\phi_1} + a_2 e^{i\phi_2}\right|^2 = I_0\left[1 + m\cos\left(\phi_2 - \phi_1\right)\right],$$

where $I_0 = a_1^2 + a_2^2 \equiv I_1 + I_2$ is the average intensity,
$m = 2\sqrt{I_1 I_2}/\left(I_1+I_2\right)$ is the contrast,
and ϕ_1, ϕ_2 the phase delays incurred in paths 1, 2, respectively.
When the fields are *mutually coherent* (temporally coherent in the Michelson and spatially coherent in the Young interferometer),
I may be as high as $I_0(1+m)$ (when $\phi_2-\phi_1 = 0, 2\pi, 4\pi, \ldots$)
or as low as $I_0(1-m)$ (when $\phi_2-\phi_1 = \pi, 3\pi, 5\pi, \ldots$).
In the perfect contrast case ($a_1 = a_2$, $m=1$) the high and low values of the interference intensity are $2I_0$ and 0, respectively.

When the fields are *incoherent*, the interference fringes are moving rapidly because the phase delay $\Delta\phi \equiv \phi_1 - \phi_2$ is itself changing rapidly. Therefore, a slow detector observes instead the time averaged

$$\langle I\rangle = \left(a_1^2+a_2^2\right)\left[1 + \frac{2a_1a_2}{a_1^2+a_2^2}\langle\cos\left(\Delta\phi\right)\rangle\right]$$

In the *perfectly incoherent* case, the $\cos\left(\Delta\phi\right)$ fluctuations average out to zero. Therefore, the average intensity in the perfectly incoherent case is

$$\langle I\rangle_{\text{incoh}} = a_1^2 + a_2^2 = \left|a_1 e^{i\phi_1}\right|^2 + \left|a_2 e^{i\phi_2}\right|^2.$$

We conclude that:

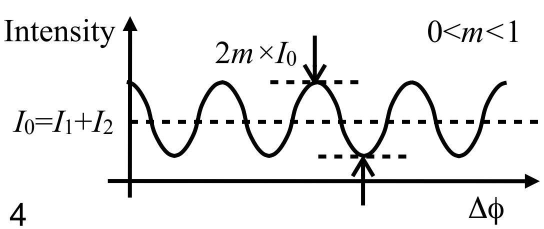
- In the perfectly coherent case, the intensity is computed as the *modulus–squared of the sum of the phasors* of the interfering fields; whereas
- in the perfectly incoherent case, the intensity is computed as the *sum of the moduli–squared of the phasors* of the interfering fields.

Of course the two extreme cases are idealized and do not occur in practice. More realistic is the *partially coherent* description of an optical field; however, that requires a more sophisticated mathematical treatment and is beyond the scope of this class (it is covered in 2.717).
The perfectly coherent–incoherent assumptions are sufficient for many cases of practical interest.

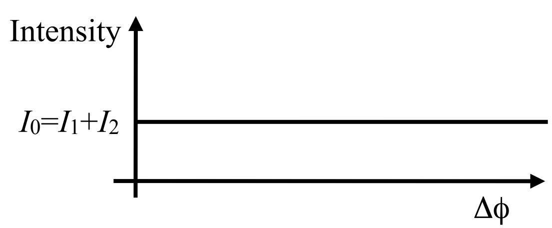
Summary

Coherent	$I = \left\|a_1 e^{i\phi_1} + a_2 e^{i\phi_2}\right\|^2$	$= \left(I_1+I_2\right)\left[1 + m\cos\left(\Delta\phi\right)\right]$;
Incoherent	$I = \left\|a_1 e^{i\phi_1}\right\|^2 + \left\|a_2 e^{i\phi_2}\right\|^2$	$= I_1 + I_2$.

Perfectly coherent



Perfectly incoherent



Coherent and incoherent sources and measurements

Temporally incoherent; spatially coherent

- ➡ White light lamp (broadband; e.g., thermal) spatially limited by a pinhole
- ➡ White light source located very far away (i.e. with extremely small NA) e.g. sun, stars, lighthouse at long distance
- ➡ Pulsed laser sources with extremely short (<nsec) pulse duration; supercontinuum sources

Temporally & spatially coherent

- ➡ Monochromatic laser sources e.g. doubled Nd:YAG (best), HeNe, Ar^+ (poorer)
- ➡ Atomic transition (quasi-monochromatic) lamps (e.g. Xe) spatially limited by a pinhole

Temporally & spatially incoherent

- ➡ White light source at a nearby distance or without spatial limitation

Temporally coherent; spatially incoherent

also referred to as **quasi-monochromatic spatially incoherent**

- ➡ Monochromatic laser sources (e.g. HeNe, doubled Nd:YAG) with a rotating diffuser (plate of ground glass) in the beam path
- ➡ Atomic transition (quasi-monochromatic) lamps (e.g. Xe) without spatial limitation

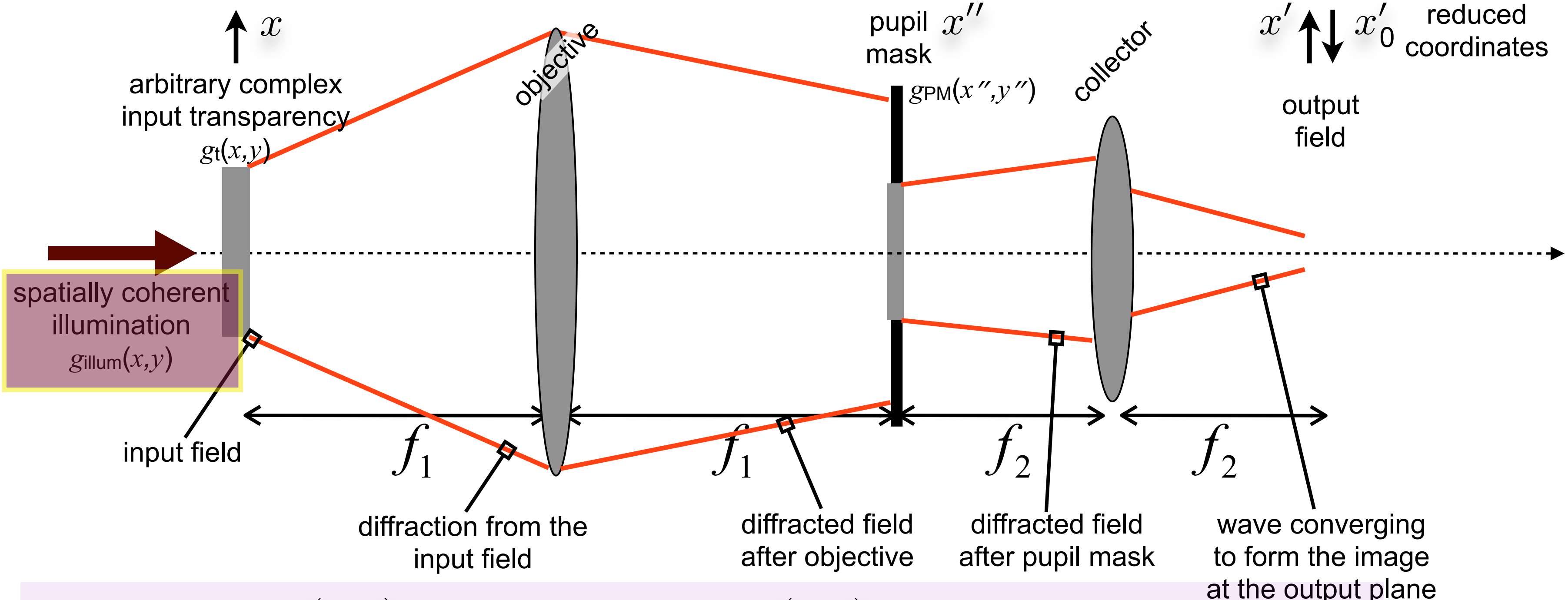
Optical instruments utilizing the degree of coherence for imaging

- ➡ Michelson interferometer [spatial; high resolution astronomical imaging at optical frequencies]
- ➡ Radio telescopes, e.g. the Very Large Array (VLA) [spatial; astronomical imaging at RF frequencies]
- ➡ Optical Coherence Tomography (OCT) [temporal; bioimaging with optical sectioning]
- ➡ Multipole illumination in optical lithography [spatial; sub-μm feature patterning]

Implications of coherence on imaging

- An optical system behaves differently if illuminated by temporally or spatially coherent or incoherent light
- Temporally incoherent illumination is typically associated with white light (or, generally, broadband) operation [Goodman 6.1.3]
  - for example, chromatic aberration is typical evidence of temporal incoherence
- The degree of spatial coherence alters the description of an optical system as a linear system
  - if the illumination is *spatially coherent*, the output *field* (phasor) is described as a convolution of the input *field* (phasor) with the "coherent" PSF $h(x,y)$
    (as we already saw)
  - if the illumination is *spatially incoherent*, the output *intensity* is described as a convolution of the input *intensity* with the "incoherent" PSF $h_I(x,y)$
    (as we are about to see)

Spatially coherent imaging with the 4F system



Illumination: $g_{\text{illum}}(x,y)$ Input transparency: $g_{\text{t}}(x,y)$

Field to the right of the input transparency (input field): $g_{\text{in}}(x,y) = g_{\text{illum}}(x,y) \times g_{\text{t}}(x,y)$

Pupil mask: $g_{\text{PM}}(x'',y'')$ Amplitude transfer function (ATF): $H(u,v) = a\, g_{\text{PM}}\left(\lambda f_1 u, \lambda f_1 v\right)$

Point Spread Function (PSF):

$$h(x',y') = a\, G_{\text{PM}}\left(\frac{x'}{\lambda f_2}, \frac{y'}{\lambda f_2}\right)$$ in actual coordinates

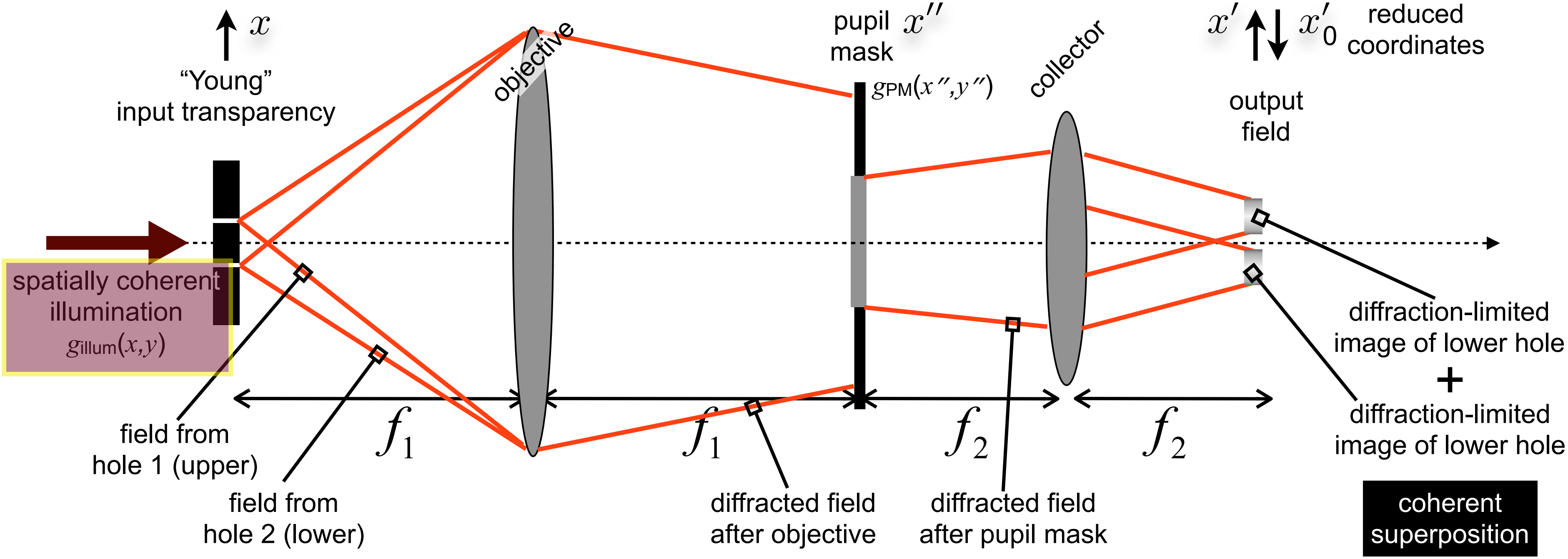
$$h(x'_0,y'_0) = a\, G_{\text{PM}}\left(-\frac{x'_0}{\lambda f_1}, -\frac{y'_0}{\lambda f_1}\right)$$ in reduced coordinates

Output field:

$$g_{\text{out}}\left(x',y'\right) = \iint g_{\text{in}}(x,y) h\left(x' + \frac{f_2}{f_1}x, y' + \frac{f_2}{f_1}y\right) \mathrm{d}x\, \mathrm{d}y$$ in actual coordinates

$$g_{\text{out}}\left(x'_0,y'_0\right) = \iint g_{\text{in}}(x,y) h\left(x'_0 - x, y'_0 - y\right) \mathrm{d}x\, \mathrm{d}y$$ in reduced coordinates

Spatially coherent imaging with the 4F system



Since the incoming illumination is spatially coherent, the diffracted images add up as phasors, i.e.

$$g_{\text{out}}(x',y') = a_1 e^{i\phi_1} h\left(x' + \frac{f_2}{f_1}x_1\right) + a_2 e^{i\phi_2} h\left(x' + \frac{f_2}{f_1}x_2\right);$$

$$I_{\text{out}}(x',y') = \left| a_1 e^{i\phi_1} h\left(x' + \frac{f_2}{f_1}x_1\right) + a_2 e^{i\phi_2} h\left(x' + \frac{f_2}{f_1}x_2\right)\right|^2$$

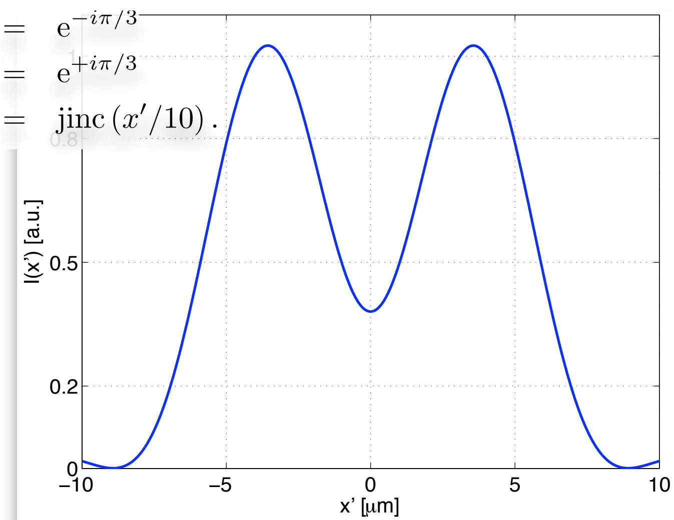
$$= a_1^2 \left| h\left(x' + \frac{f_2}{f_1}x_1\right)\right|^2 + a_2^2 \left| h\left(x' + \frac{f_2}{f_1}x_2\right)\right|^2 +$$

$$a_1 a_2 \operatorname{Re}\left\{ e^{i(\phi_2-\phi_1)} h^*\left(x' + \frac{f_2}{f_1}x_1\right) h\left(x' + \frac{f_2}{f_1}x_2\right)\right\},$$

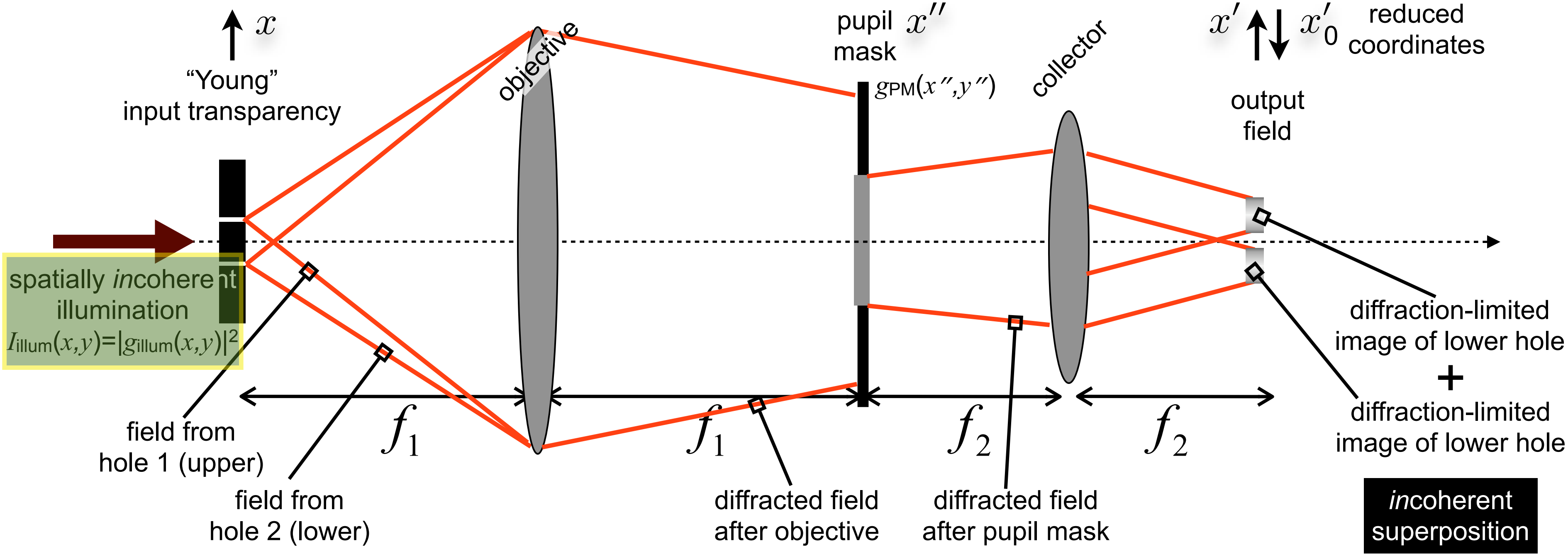
where x_j, $a_j e^{i\phi_j}$ are the location of and field emanating from hole j, respectively.

interference term, or "cross-term"

e.g.
$$a_1 e^{i\phi_1} = e^{-i\pi/3}$$
$$a_2 e^{i\phi_2} = e^{+i\pi/3}$$
$$h(x') = \operatorname{jinc}(x'/10).$$



Spatially *in*coherent imaging with the 4F system

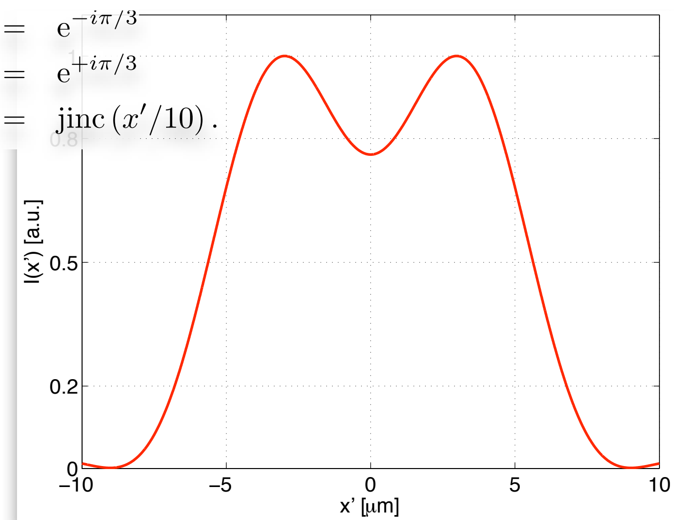


Since the incoming illumination is spatially *in*coherent, the diffracted images add up *in intensity*, i.e.

$$
\begin{aligned}
I_{\text{out}}(x',y') &= \left|a_1 e^{i\phi_1} h\left(x' + \frac{f_2}{f_1}x_1\right)\right|^2 + \left|a_2 e^{i\phi_2} h\left(x' + \frac{f_2}{f_1}x_2\right)\right|^2 \\
&= a_1^2 \left|h\left(x' + \frac{f_2}{f_1}x_1\right)\right|^2 + a_2^2 \left|h\left(x' + \frac{f_2}{f_1}x_2\right)\right|^2 .
\end{aligned}
$$

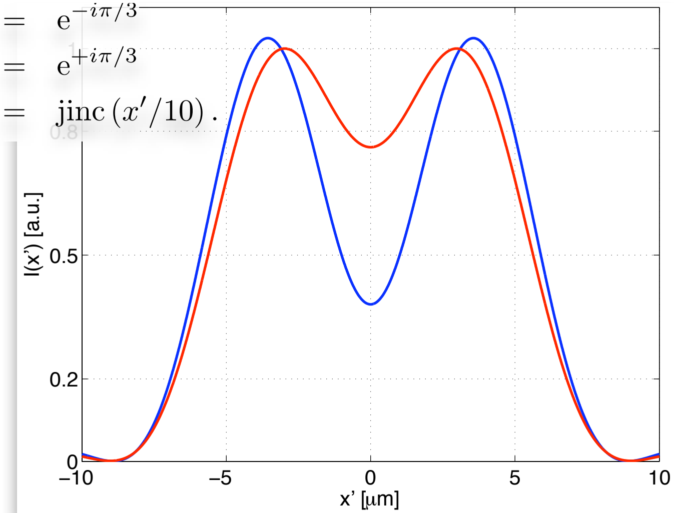
there is no interference term, or "cross-term"

$$
\begin{aligned}
\text{e.g. } a_1 e^{i\phi_1} &= e^{-i\pi/3} \\
a_2 e^{i\phi_2} &= e^{+i\pi/3} \\
h(x') &= \text{jinc}\,(x'/10)\,.
\end{aligned}
$$

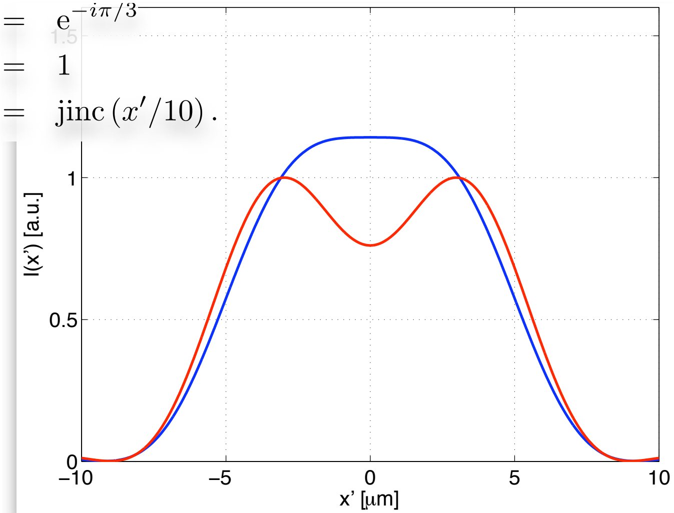


Spatially coherent vs *in*coherent imaging: two point sources

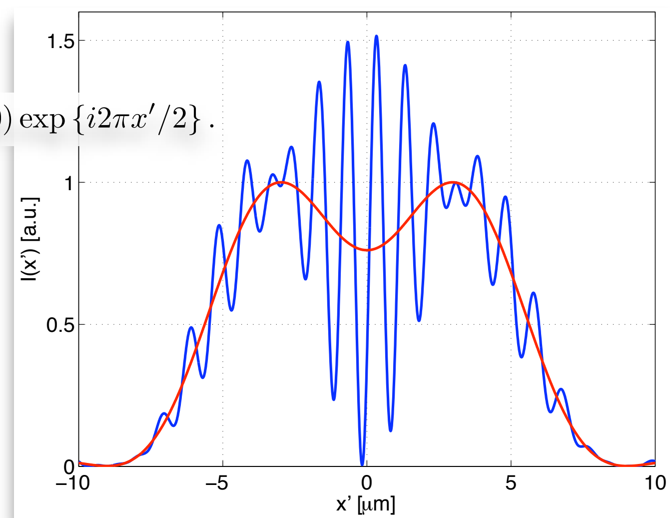
$$\begin{aligned} \text{e.g. } a_1 e^{i\phi_1} &= e^{-i\pi/3} \\ a_2 e^{i\phi_2} &= e^{+i\pi/3} \\ h(x') &= \mathrm{jinc}\,(x'/10)\,. \end{aligned}$$



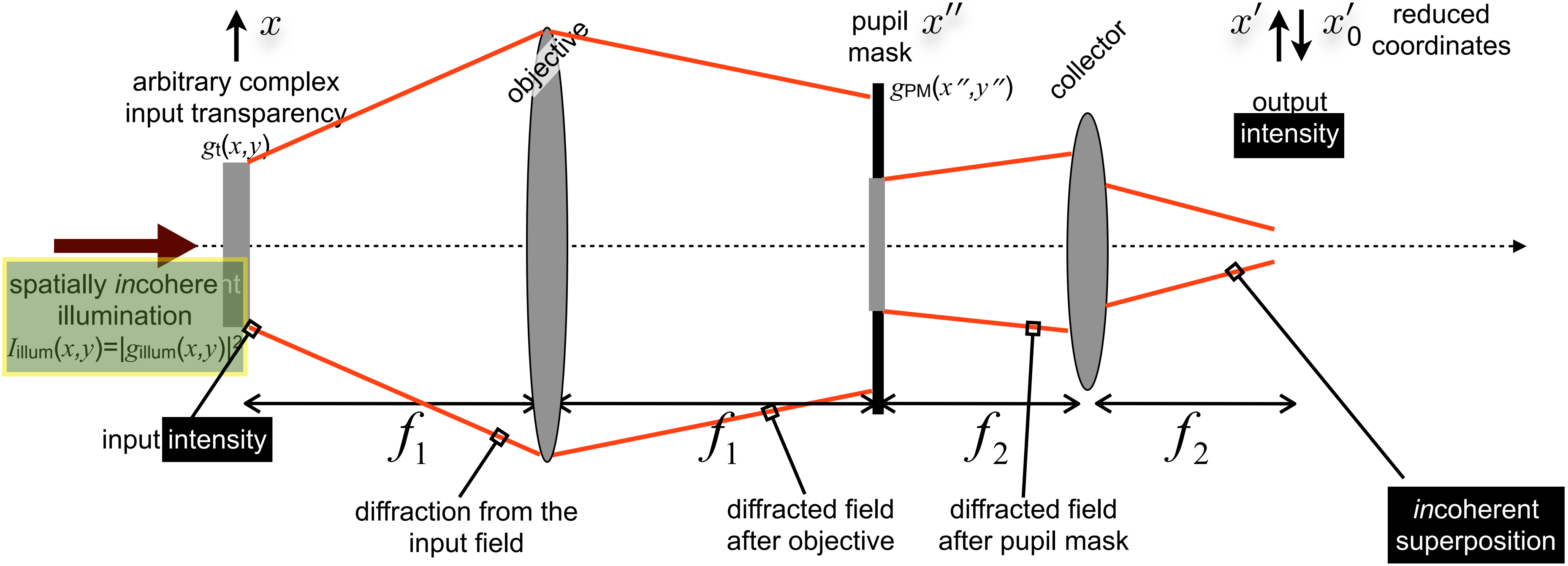
$$\begin{aligned} \text{e.g. } a_1 e^{i\phi_1} &= e^{-i\pi/3} \\ a_2 e^{i\phi_2} &= 1 \\ h(x') &= \mathrm{jinc}\,(x'/10)\,. \end{aligned}$$



$$\begin{aligned} \text{e.g. } a_1 e^{i\phi_1} &= e^{-i\pi/3} \\ a_2 e^{i\phi_2} &= e^{+i\pi/3} \\ h(x') &= \mathrm{jinc}\,(x'/10)\exp\{i2\pi x'/2\}\,. \end{aligned}$$



Spatially *in*coherent PSF of the 4F system



Generalizing the principle of coherent superposition in the case of an arbitrary complex input transparency, we find that the intensity at the output plane is

$$
\begin{aligned}
I_{\text{out}}(x',y') &= \iint I_{\text{illum}}(x,y)\,|g_t(x,y)|^2 \left| h\left(x'+\frac{f_2}{f_1}x_1, y'+\frac{f_2}{f_1}y_1\right)\right|^2 dx dy \\
&\equiv \iint I_{\text{illum}}(x,y)\,|g_t(x,y)|^2\, h_I\left(x'+\frac{f_2}{f_1}x_1, y'+\frac{f_2}{f_1}y_1\right) dx dy
\end{aligned}
$$

or, assuming uniform input illumination $I_{\text{illum}}(x,y)=1$,

$$
\begin{aligned}
I_{\text{out}}(x',y') &= \iint |g_t(x,y)|^2 \left| h\left(x'+\frac{f_2}{f_1}x_1, y'+\frac{f_2}{f_1}y_1\right)\right|^2 dx dy \\
&\equiv \iint |g_t(x,y)|^2\, h_I\left(x'+\frac{f_2}{f_1}x_1, y'+\frac{f_2}{f_1}y_1\right) dx dy.
\end{aligned}
$$

where $h_I(x,y) \equiv |h(x,y)|^2$.

is the **incoherent Point Spread Function (iPSF)**

Derivation of the Optical Transfer Function (OTF)

Since the spatially coherent PSF $h(x,y)$ and ATF $H(u,v)$ are a Fourier transform pair, we can write the PSF as the Fourier integral of the ATF

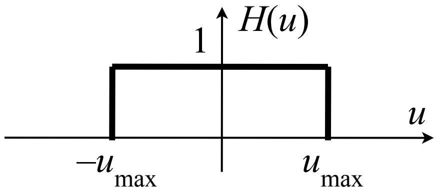
$$h(x,y) = \iint H(u,v) \exp\{i2\pi(ux+vy)\}\, \mathrm{d}u\mathrm{d}v.$$

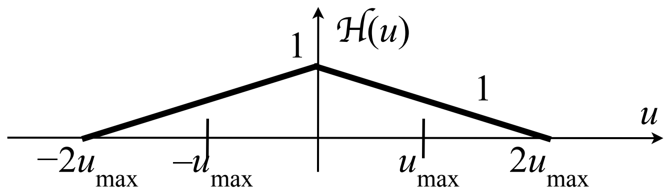
The spatially incoherent PSF is

$$\begin{aligned}
h_{\mathrm{I}}(x,y) &= |h(x,y)|^2 \\
&= \iint H(u_1,v_1)\exp\{i2\pi(u_1x+v_1y)\}\,\mathrm{d}u_1\mathrm{d}v_1 \times \\
&\quad \iint H^*(u_2,v_2)\exp\{-i2\pi(u_2x+v_2y)\}\,\mathrm{d}u_2\mathrm{d}v_2 \\
&= \iint \mathrm{d}u_1\mathrm{d}v_1 \iint \mathrm{d}u_2\mathrm{d}v_2 H(u_1,v_1)H^*(u_2,v_2) \times \\
&\quad \exp\left\{i2\pi\left[(u_1-u_2)x+(v_1-v_2)y\right]\right\} \\
&= \iint \mathrm{d}u_1\mathrm{d}v_1 \iint \mathrm{d}u\mathrm{d}v H(u_1,v_1)H^*(u_1-u,v_1-v) \times \\
&\quad \exp\left\{i2\pi(ux+vy)\right\} \\
&= \iint \mathrm{d}u\mathrm{d}v \left[\iint \mathrm{d}u_1\mathrm{d}v_1 H(u_1,v_1)H^*(u_1-u,v_1-v)\right] \times \\
&\quad \exp\left\{i2\pi(ux+vy)\right\} \\
&\equiv \iint \mathrm{d}u\mathrm{d}v \mathcal{H}(u,v)\exp\left\{i2\pi(ux+vy)\right\}, \qquad \text{where} \\
\mathcal{H}(u,v) &\equiv \iint \mathrm{d}u'\mathrm{d}v' H(u',v')H^*(u'-u,v'-v).
\end{aligned}$$

- The iPSF is the modulus squared of the cPSF
- $|h(x,y)|^2 = h(x,y) \times h^*(x,y)$ and then we write each term as a Fourier integral
- combine the integrals and rearrange the order of integration
- define new integration variables $u \equiv u_1 - u_2, \quad v \equiv v_1 - v_2.$
- rearrange the order of integration
- observe that this expression is again a Fourier integral
- correlation integral

Example: 1D OTF from ATF





It is customary to normalize such that the OTF peak is 1:

$$\mathcal{H}(0,0) = 1.$$

Consider a clear–aperture pupil mask, which therefore has a boxcar ATF

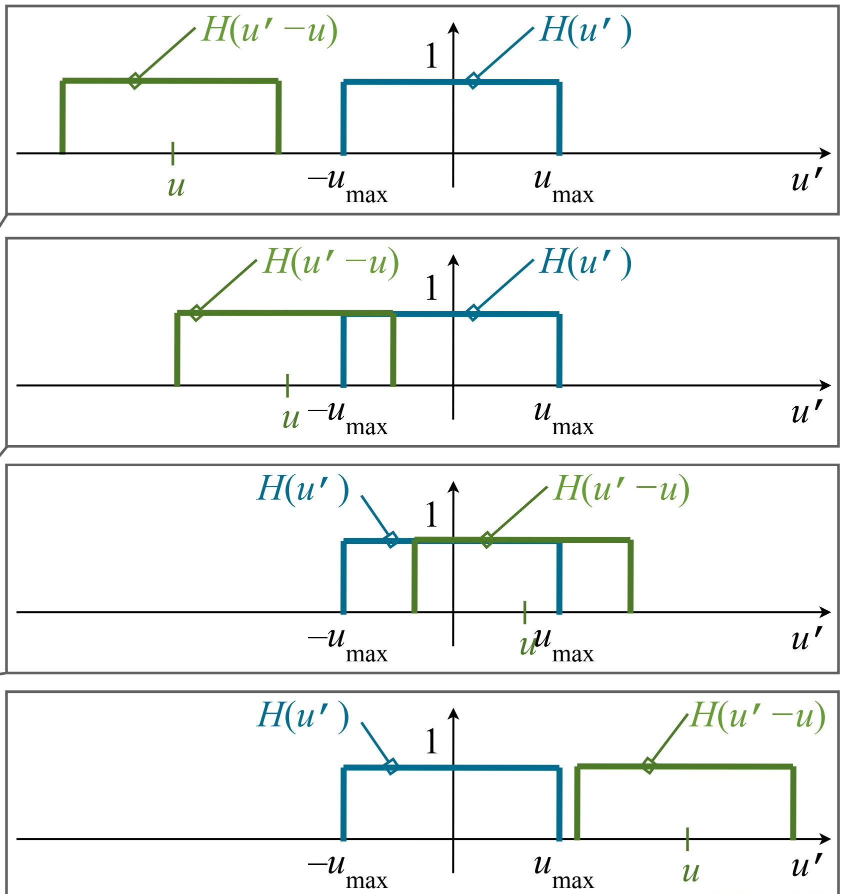
$$H(u) = \mathrm{rect}\left(\frac{u}{u_{\max}}\right),$$

where the cut–of spatial frequency $u_{\max}$ is related to the pupil aperture size $x''_{\max}$ as $u_{\max} = x''_{\max}/\left(\lambda f_1\right)$.

The same optical system, when incoherently illuminated has OTF

$$\begin{aligned}
\mathcal{H}(u) &\propto H(u) \star H(u) \\
&= \int H(u')H^*(u'-u)\mathrm{d}u' \\
&= \int \mathrm{rect}\left(\frac{u'}{u_{\max}}\right)\mathrm{rect}\left(\frac{u'-u}{u_{\max}}\right)\mathrm{d}u' \\
&= \begin{cases} 0, & \text{if } u < -2u_{\max} \\ \int_{-u_{\max}}^{u+u_{\max}} \mathrm{d}u' = u + 2u_{\max}, & \text{if } -2u_{\max} < u < 0 \\ \int_{u-u_{\max}}^{u_{\max}} \mathrm{d}u' = u - 2u_{\max}, & \text{if } 0 < u < 2u_{\max} \\ 0, & \text{if } u > 2u_{\max} \end{cases} \\
&\propto \mathrm{tri}\left(\frac{u}{2u_{\max}}\right),
\end{aligned}$$

where the triangle function is denoted as $\Lambda(.)$ in Goodman pp. 12–13.



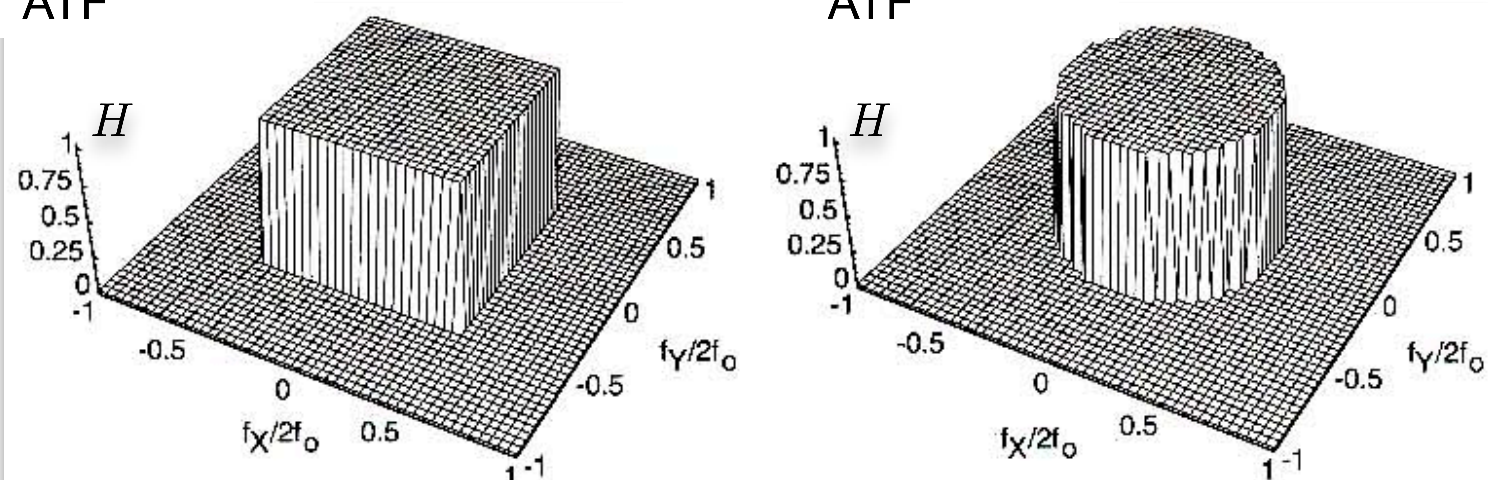
Examples: ATF *vs* OTF in 2D

rectangular aperture

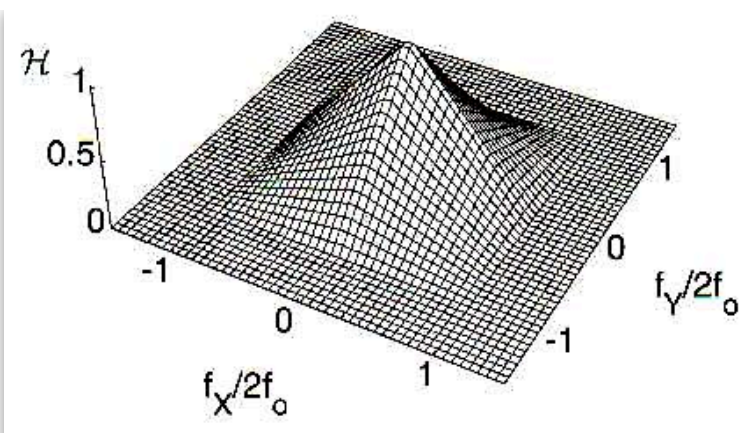
ATF

circular aperture

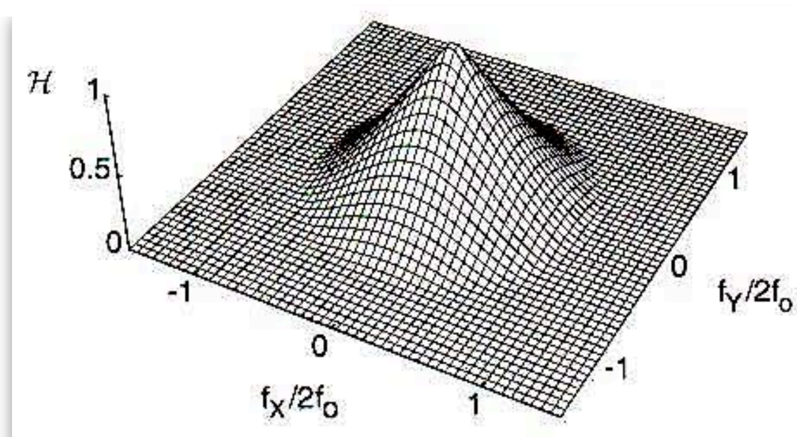
ATF



OTF



OTF



Goodman pp. 138, 144, 146

Terminology and basic relationships

coherent
Point Spread Function **PSF**

$$h(x,y)$$

physical meaning: *optical field* produced when the illumination is a point source

incoherent
Point Spread Function **iPSF**

$$h_{\mathrm{I}}(x,y)=|h(x,y)|^2$$

physical meaning: *intensity* produced when the illumination is a point source

Amplitude Transfer Function **ATF**
(coherent illumination)

in the 4F system: mathematically identical to the pupil mask $g_{\mathrm{PM}}(x'',y'')$, within a coordinate scaling operation
$H(u,v)=a g_{\mathrm{PM}}(\lambda f_1 u, \lambda f_1 v)$

$$\begin{aligned} H(u,v) &= \mathcal{F}\{h(x,y)\} \\ &= \iint h(x,y)\mathrm{e}^{-i2\pi(ux+vy)}\mathrm{d}x\mathrm{d}y \end{aligned}$$

Optical Transfer Function **OTF**
(incoherent illumination)

autocorrelation of the ATF

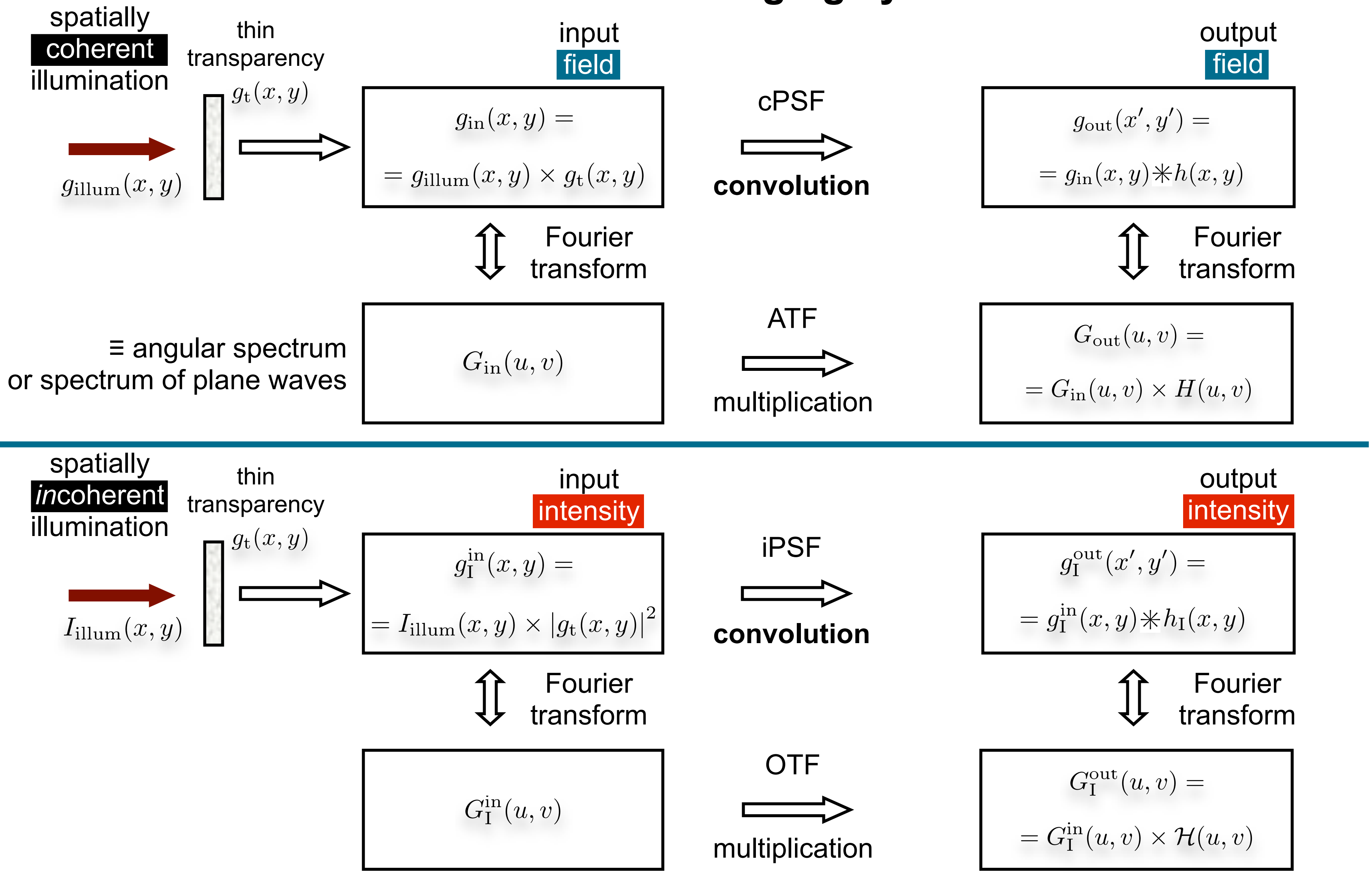
$$\begin{aligned} \mathcal{H}(u,v) &= H(u,v)\star H(u,v) \\ &= \iint H(u',v')H^*(u'-u,v'-v)\mathrm{d}u'\mathrm{d}v' \end{aligned}$$

Modulation Transfer Function **MTF**
(incoherent illumination)

modulus of the MTF

$$\hat{H}(u,v)=|\mathcal{H}(u,v)|$$

Block diagrams for coherent and incoherent linear shift invariant imaging systems



Interpretation of the MTF /1

Consider a grating whose *amplitude* transmission function is:

$$g_{\mathrm{t}}(x) = \mathrm{e}^{i\phi(x)}\sqrt{\frac{1}{2}\left\{1 + m\cos\left(2\pi\frac{x}{\Lambda}\right)\right\}}$$

where $\varphi(x)$ is an arbitrary phase function.

If illuminated uniformly, the *intensity* past the grating is:

$$g_{\mathrm{I}}(x) \equiv |g_{\mathrm{t}}(x)|^2 = \frac{1}{2}\left\{1 + m\cos\left(2\pi\frac{x}{\Lambda}\right)\right\}$$

The Fourier transform of this intensity transmission function is:

$$\begin{aligned}\mathcal{F}\left\{g_{\mathrm{I}}(x)\right\} &\equiv G_{\mathrm{I}}(u) = \\ &= \frac{1}{2}\left\{\delta(u) + \frac{m}{2}\left[\delta\left(u - \frac{1}{\Lambda}\right) + \delta\left(u + \frac{1}{\Lambda}\right)\right]\right\}\end{aligned}$$

The output of an optical system with 1:1 magnification resulting from this sinusoidal input signal is:

$$\begin{aligned}\mathcal{F}\left\{g_{\mathrm{I}}^{\mathrm{out}}(x')\right\} &\equiv G_{\mathrm{I}}^{\mathrm{out}}(u) = G_{\mathrm{I}}(u) \times \mathcal{H}(u) \\ &= \frac{1}{2}\left\{\mathcal{H}(0)\delta(u) + \frac{m}{2}\left[\mathcal{H}\left(+\frac{1}{\Lambda}\right)\delta\left(u - \frac{1}{\Lambda}\right) + \mathcal{H}\left(-\frac{1}{\Lambda}\right)\delta\left(u + \frac{1}{\Lambda}\right)\right]\right\}\end{aligned}$$

Interpretation of the MTF /2

Since the incoherent point-spread function $h_{\mathrm{I}}(x)$ is positive, its Fourier transform $\mathcal{H}(u)$ must be Hermitian, *i.e.* it must satisfy the relationship $\mathcal{H}(-u)=\mathcal{H}^*(u)$ (show this!)

A complex function with this property is called *Hermitian*; therefore, **the OTF of a physically realizable optical system must be Hermitian**.

Therefore, after some algebraic manipulation, and using the normalization $\mathcal{H}(0)\equiv 1$ we find that the intensity image at the output of the optical system is:

$$g_{\mathrm{I}}^{\mathrm{out}}(x')=\frac{1}{2}\left\{1+m\left|\mathcal{H}\left(\frac{1}{\Lambda}\right)\right|\cos\left(2\pi\frac{x}{\Lambda}\right)\right\}$$

Recall the definition of contrast

$$\mathcal{V}\equiv\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}$$

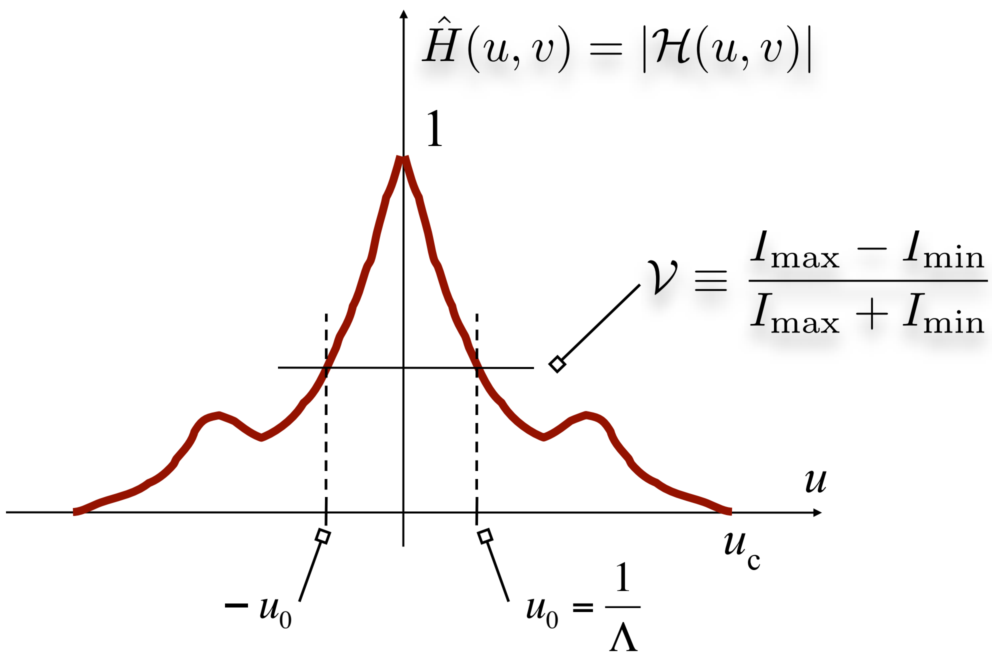
which for our case is applied as

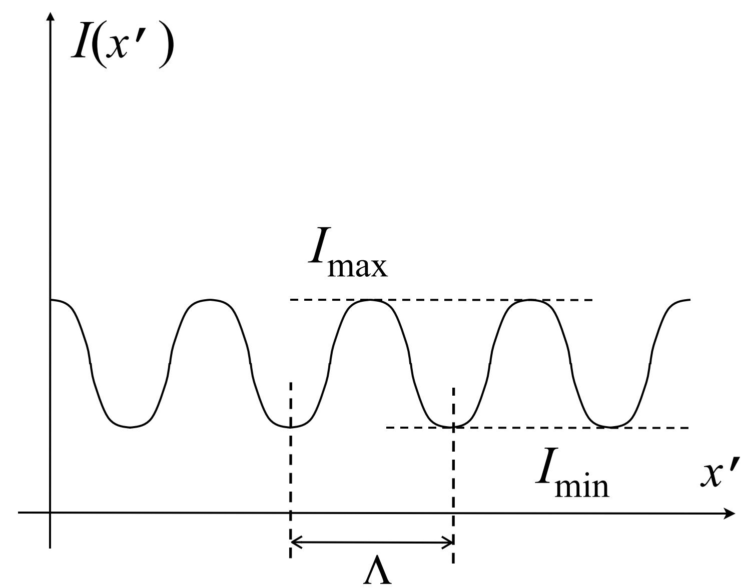
$$\mathcal{V}=\frac{g_{I,\max}^{\mathrm{out}}-g_{I,\min}^{\mathrm{out}}}{g_{I,\max}^{\mathrm{out}}-g_{I,\min}^{\mathrm{out}}}=m\left|\tilde{H}\left(\frac{1}{\Lambda}\right)\right|$$

We conclude that the value of the MTF at a given spatial frequency expresses the **contrast at that spatial frequency *relative* to the contrast of the same spatial frequency in the input intensity pattern.** The contrast change is the result of propagation through the optical system, including free space diffraction and the effect of the pupil mask.

Interpretation of the MTF /3

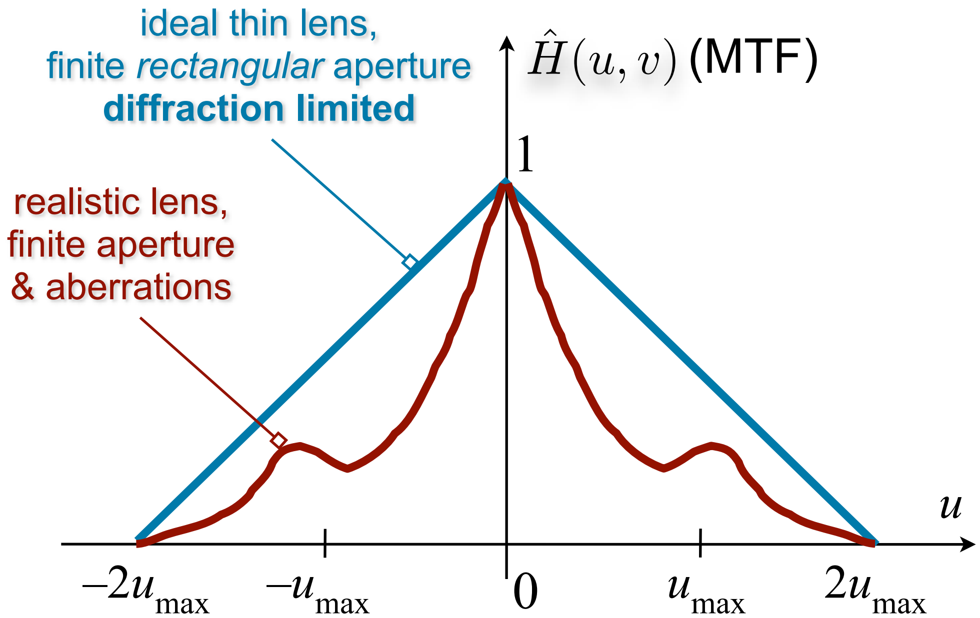
Graphical interpretation: (assuming *m*=1)



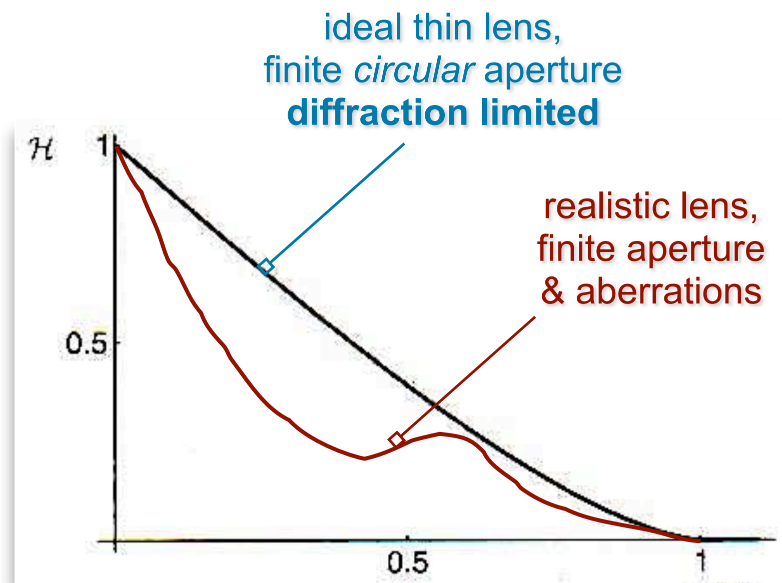


$$g_I(x) \equiv |g_t(x)|^2 = \frac{1}{2}\left\{1 + m\cos\left(2\pi\frac{x}{\Lambda}\right)\right\} \Rightarrow g_I^{\text{out}}(x') = \frac{1}{2}\left\{1 + m\left|\mathcal{H}\left(\frac{1}{\Lambda}\right)\right|\cos\left(2\pi\frac{x}{\Lambda}\right)\right\}$$

Diffraction limited vs. aberrated OTF



u_{max} is the cutoff frequency of the corresponding coherent imaging system.



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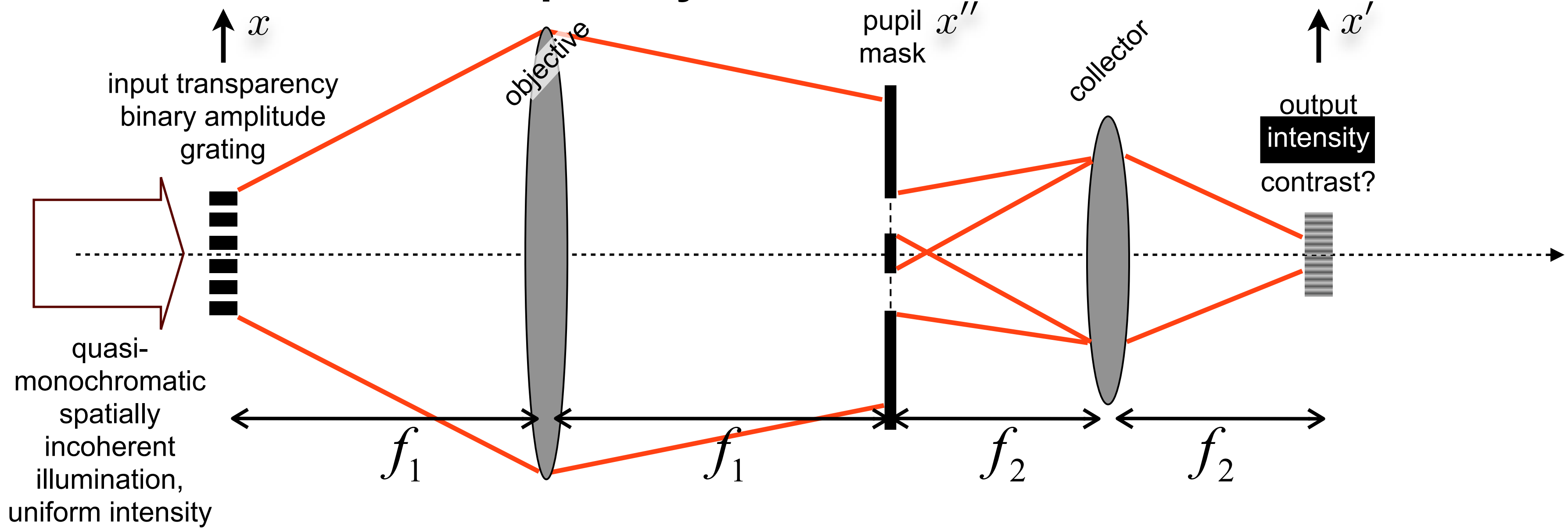
normalized polar spatial frequency

➡ In a diffraction limited optical system with clear rectangular aperture and no aberrations, using spatially incoherent illumination, the contrast (fringe visibility) at the image of a sinusoidal thin transparency of spatial frequency u_0 decreass linearly with u_0, according to the triangle function

➡ In a diffraction limited optical system with circular aperture, the contrast decreases approximately linearly with u_0, according to the autocorrelation function of the circ function

➡ In an aberrated optical system, the contrast is generally less than in the diffraction limited system of the same cut-off frequency.

Example: band pass filtering a binary amplitude grating with spatially incoherent illumination



Consider the optical system from lecture 19, slide 16, with a pupil mask consisting of two holes, each of diameter (aperture) 1cm and centered at ±1cm from the optical axis, respectively. Recall that the wavelength is λ=0.5μm and the focal lengths $f_1=f_2=f$=20cm. However, now the illumination is spatially incoherent.

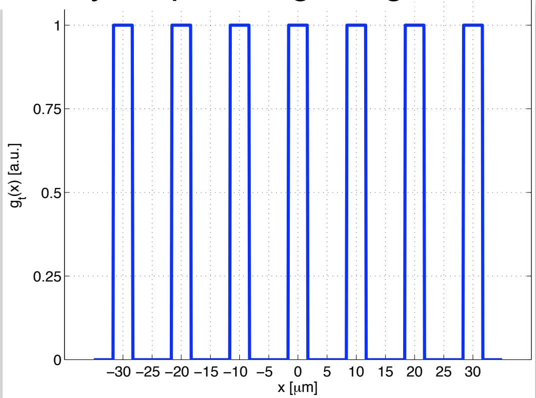
What is the intensity observed at the output (image) plane?

The sequence to solve this kind of problem is:

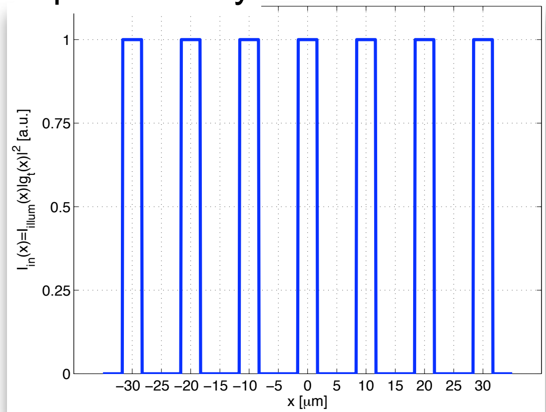
- ➡ calculate the input intensity as $I_{\text{in}}(x) = I_{\text{illum}}(x) \times |g_{\text{t}}(x)|^2$ and calculate its Fourier transform $G_{\text{I}}(u)$
- ➡ obtain the ATF as $H(u) = g_{\text{PM}}(x''/\lambda f_1)$
- ➡ obtain the OTF $\mathcal{H}(u)$ as the autocorrelation of $H(u)$ and multiply the OTF by $G_{\text{I}}(u)$
- ➡ Fourier transform the product and scale to the output plane coordinates $x' = u\lambda f_2$

Example: band pass filtering a binary amplitude grating with spatially incoherent illumination

binary amplitude grating



input intensity



The illuminating intensity is uniform, i.e.

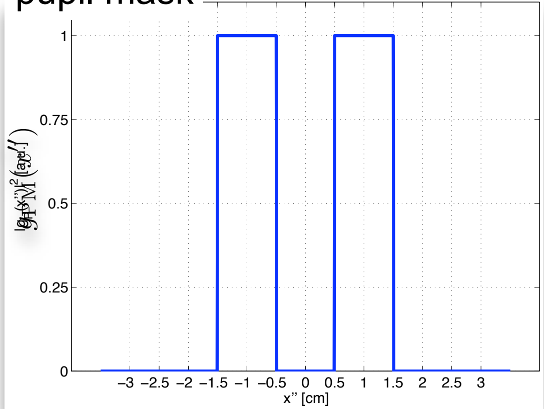
$$I_{\text{illum}}(x) = 1$$

The input intensity after the transparency is

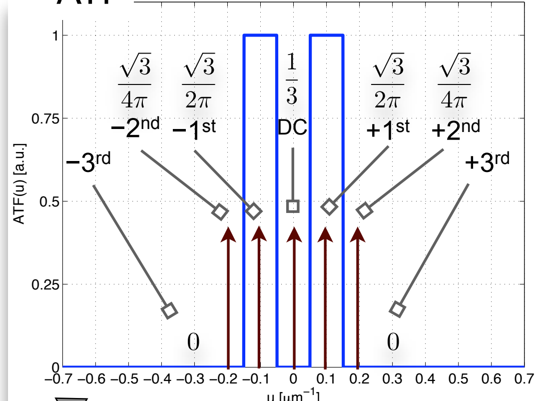
$$I_{\text{in}}(x) = I_{\text{illum}}(x) \times |g_{\text{t}}(x)|^2$$

(since the transparency is binary, i.e. either ON (bright) or OFF (dark) and the illumination is also uniform, the input intensity is either 0 or 1^2=1, i.e. it has the same binary dependence on x as the input transparency.)

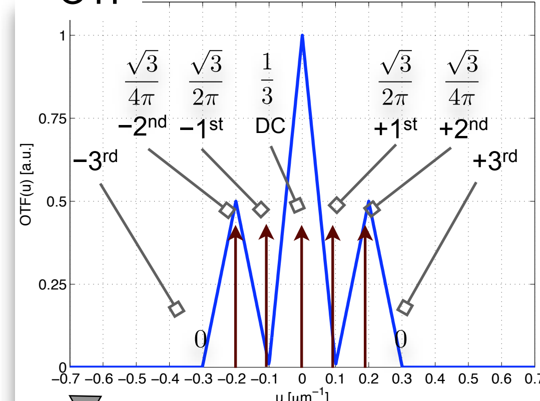
pupil mask



ATF



OTF



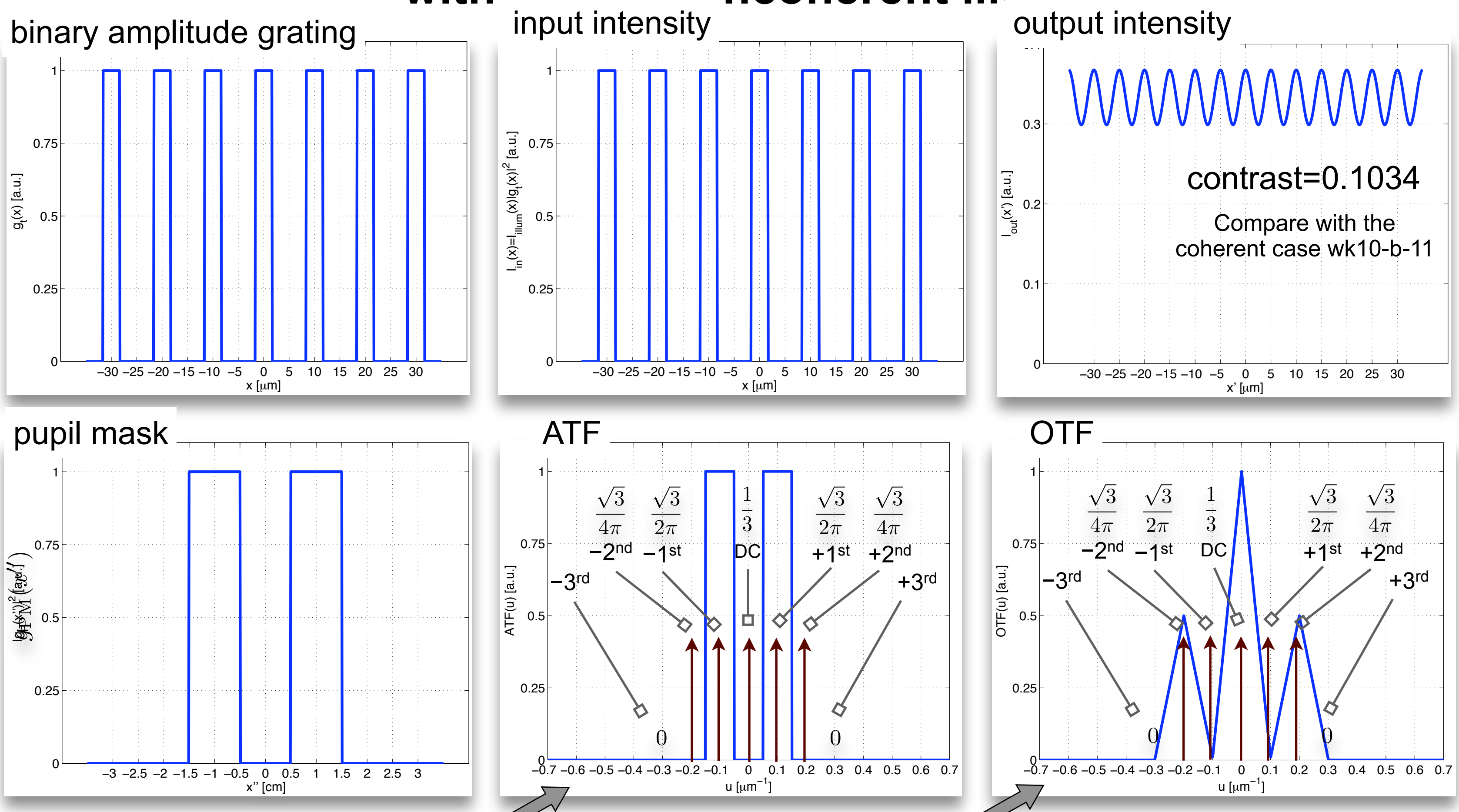
$$H(u) = g_{\text{PM}}(x''/\lambda f_1)$$

$$\mathcal{H}(u) = H(u) \star H(u)$$

$$\begin{aligned} G_{\text{I}}^{\text{out}}(u) &= G_{\text{I}}^{\text{in}}(u) \times \mathcal{H}(u) \\ &= \frac{1}{3} + 0 \times \left(\frac{\sqrt{3}}{2\pi}\delta\left(u - 0.1\mu m^{-1}\right) + \frac{\sqrt{3}}{2\pi}\delta\left(u + 0.1\mu m^{-1}\right) \right) \\ &\quad + \frac{1}{2} \times \left(\frac{\sqrt{3}}{4\pi}\delta\left(u - 2\times 0.1\mu m^{-1}\right) + \frac{\sqrt{3}}{4\pi}\delta\left(u + 2\times 0.1\mu m^{-1}\right) \right). \end{aligned}$$

$$\begin{aligned} I_{\text{out}}(x') &= \frac{1}{3} + \frac{1}{2} \times \left(\frac{\sqrt{3}}{4\pi}\exp\left\{ i2\pi\frac{2x'}{10\mu m} \right\} + \frac{\sqrt{3}}{4\pi}\exp\left\{ -i2\pi\frac{2x'}{10\mu m} \right\} \right) \\ &= \frac{1}{3} + \frac{\sqrt{3}}{4\pi}\cos\left(i2\pi\frac{2x'}{10\mu m} \right). \end{aligned}$$

Example: band pass filtering a binary amplitude grating with spatially incoherent illumination



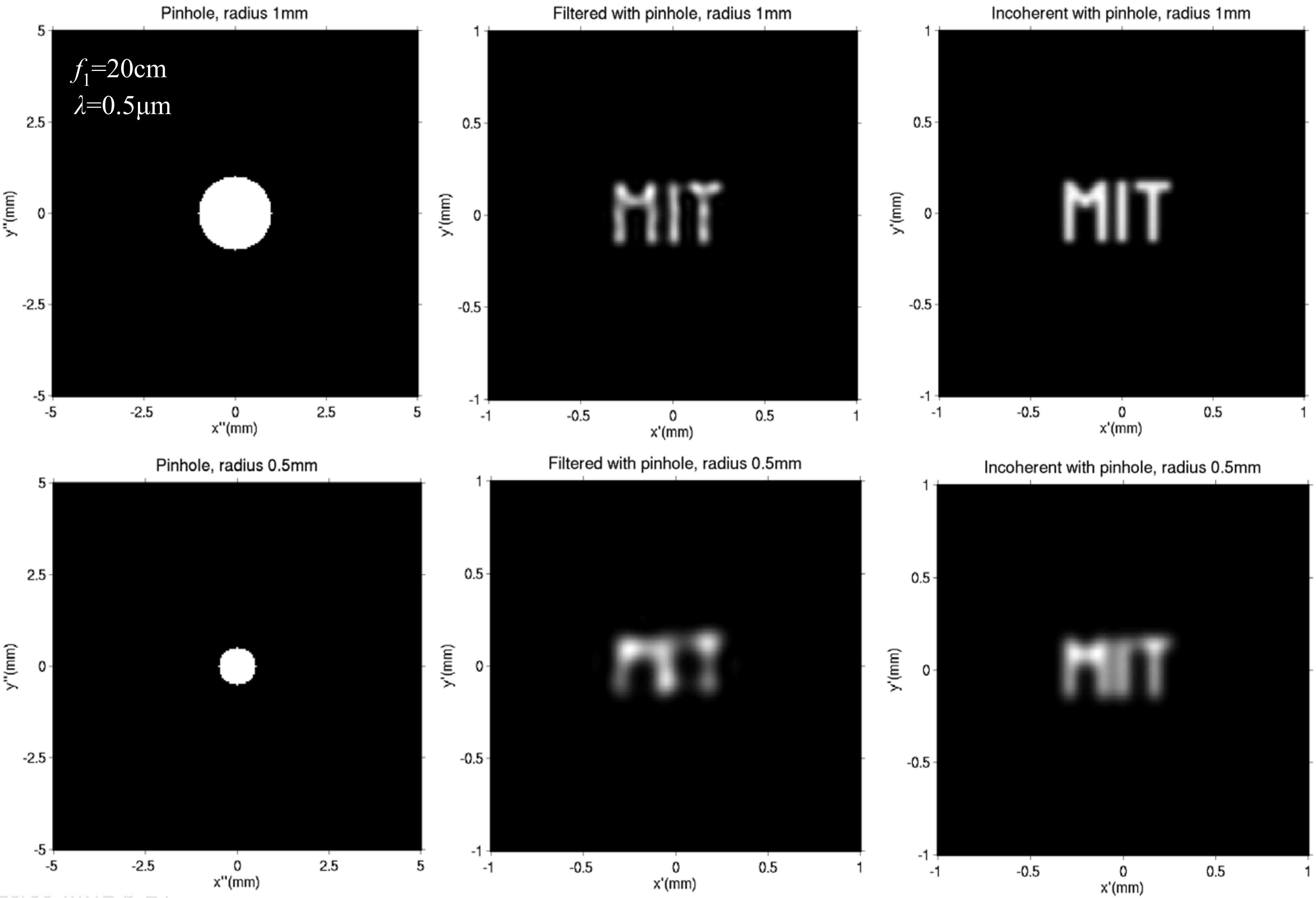
$$H(u) = g_{\mathrm{PM}}(x''/\lambda f_1)$$

$$\mathcal{H}(u) = H(u) \star H(u)$$

$$\begin{aligned} I_{\text{out}}(x') &= \frac{1}{3} + \frac{1}{2} \times \left(\frac{\sqrt{3}}{4\pi} \exp\left\{ i2\pi \frac{2x'}{10\mu\text{m}} \right\} + \frac{\sqrt{3}}{4\pi} \exp\left\{ -i2\pi \frac{2x'}{10\mu\text{m}} \right\} \right) \\ &= \frac{1}{3} + \frac{\sqrt{3}}{4\pi} \cos\left(i2\pi \frac{2x'}{10\mu\text{m}} \right). \end{aligned}$$

Numerical comparison of spatially coherent *vs* incoherent imaging

physical aperture | coherent imaging | *in*coherent imaging



Qualitative comparison of spatially coherent *vs* incoherent imaging

- Incoherent generally gives better image quality:
  - no ringing artifacts
  - no speckle
  - higher bandwidth (even though higher frequencies are attenuated because of the MTF roll-off)
- However, incoherent imaging is insensitive to phase objects
- Polychromatic imaging introduces further blurring due to chromatic aberration (dependence of the MTF on wavelength)