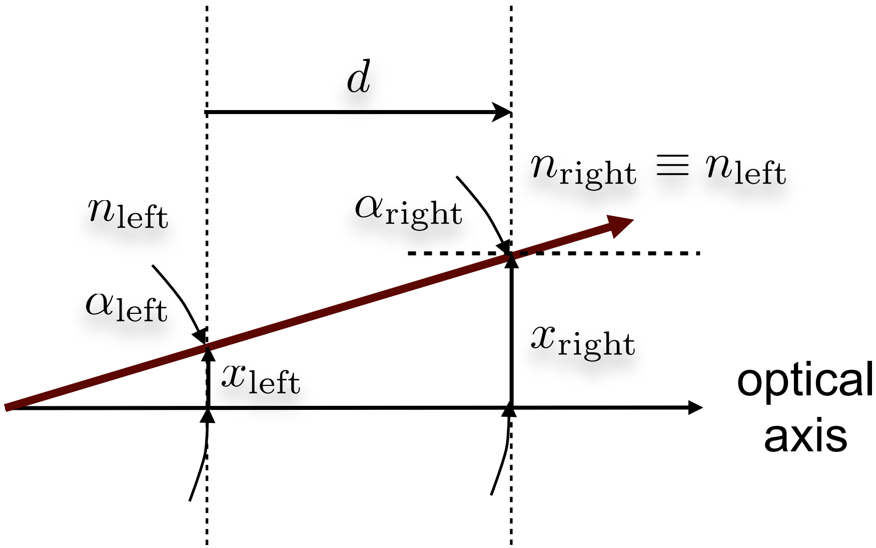
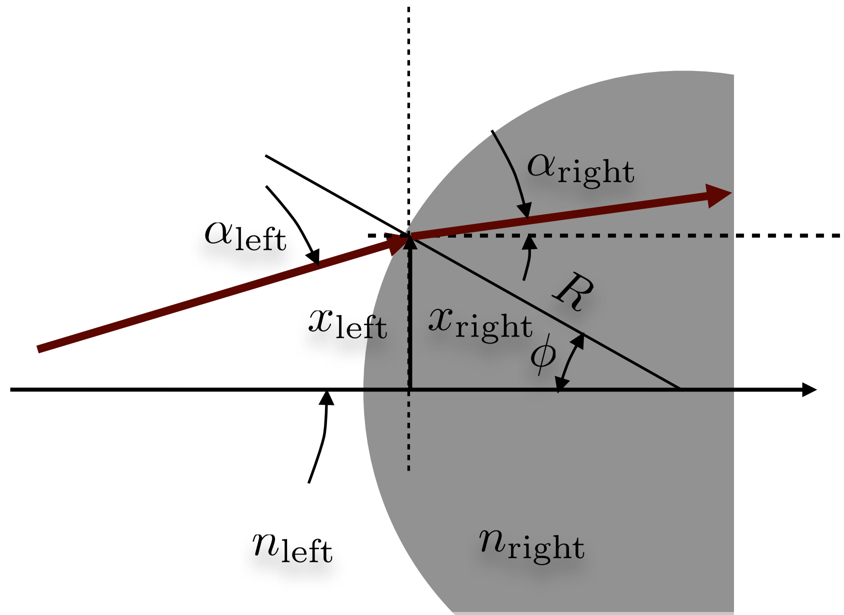
# Overview: ray transfer matrices




$$\begin{pmatrix} n_{\text{right}}\alpha_{\text{right}} \\ x_{\text{right}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \dfrac{d}{n_{\text{left}}} & 1 \end{pmatrix} \begin{pmatrix} n_{\text{left}}\alpha_{\text{left}} \\ x_{\text{left}} \end{pmatrix}$$

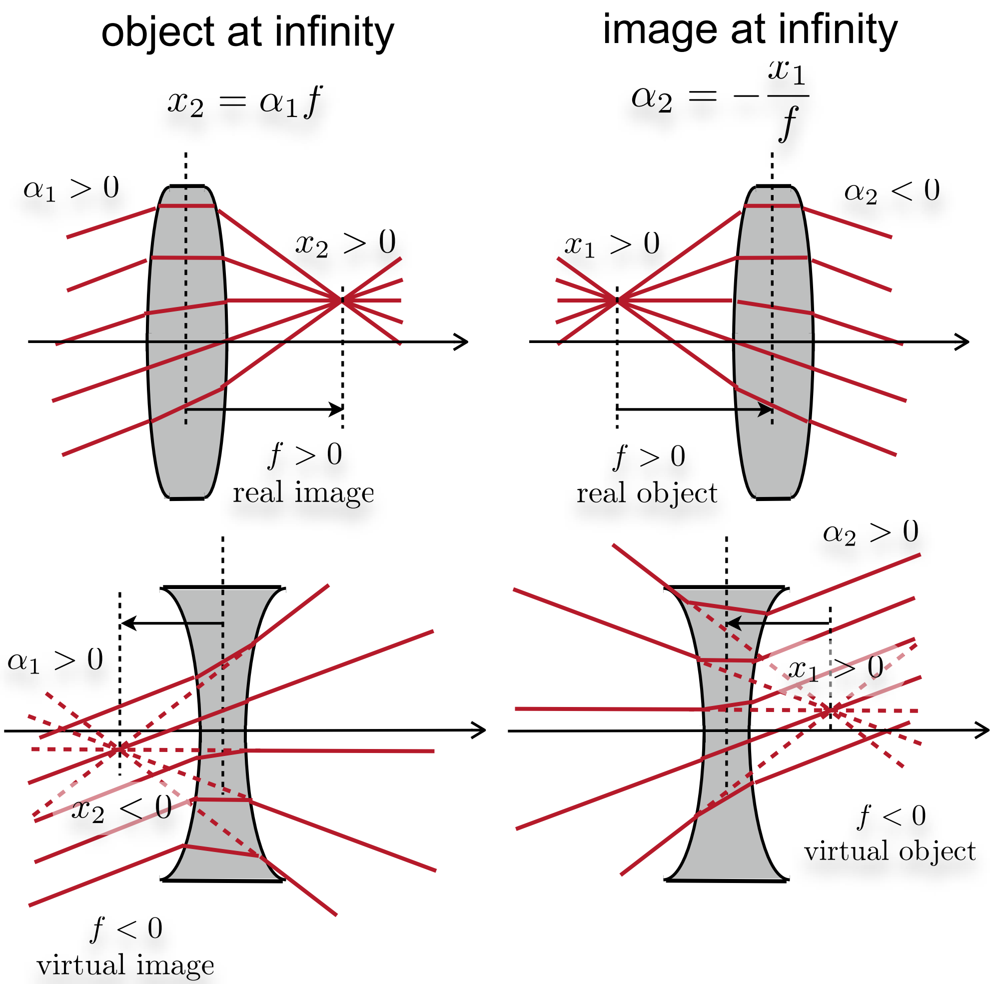
Propagation through uniform space:
distance $d$, index of refraction $n_{\text{left}}$

$$\begin{pmatrix} n_{\text{right}}\alpha_{\text{right}} \\ x_{\text{right}} \end{pmatrix} = \begin{pmatrix} 1 & -\dfrac{n_{\text{right}} - n_{\text{left}}}{R} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} n_{\text{left}}\alpha_{\text{left}} \\ x_{\text{left}} \end{pmatrix}$$

Refraction at spherical interface:
radius $R$, indices $n_{\text{left}}$, $n_{\text{right}}$

By using these elemental matrices, we may ray trace through an arbitrarily long cascade of optical elements *(provided the paraxial approximation remains valid throughout.)*

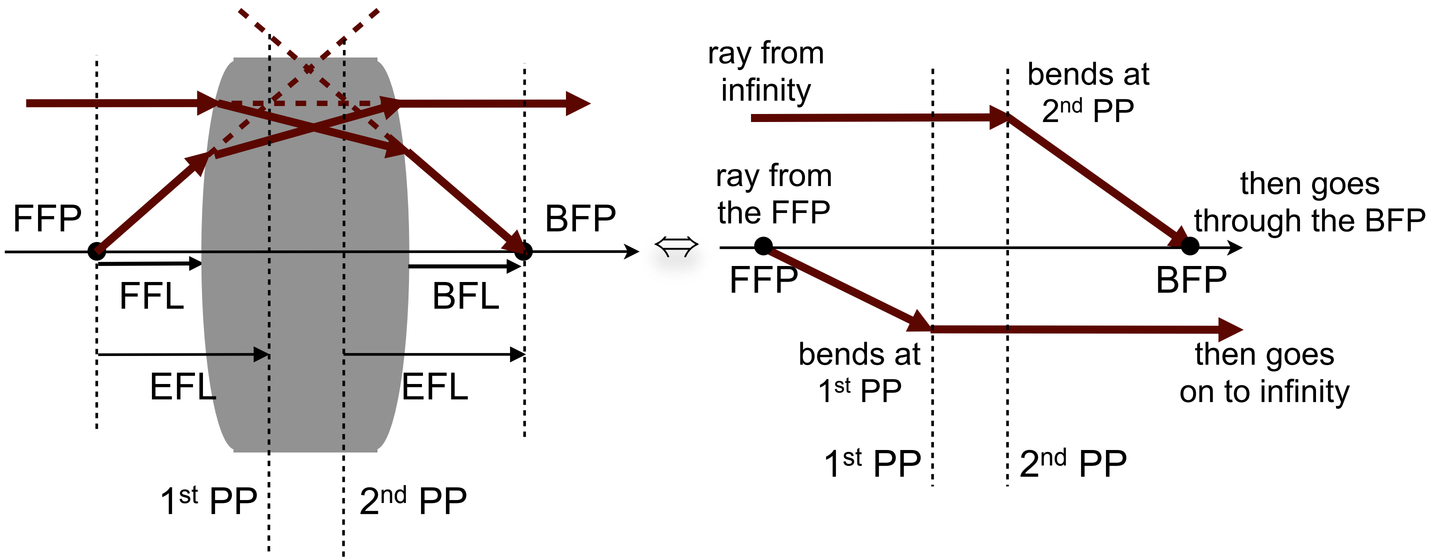
# Overview: thin lens and object/image at infinity

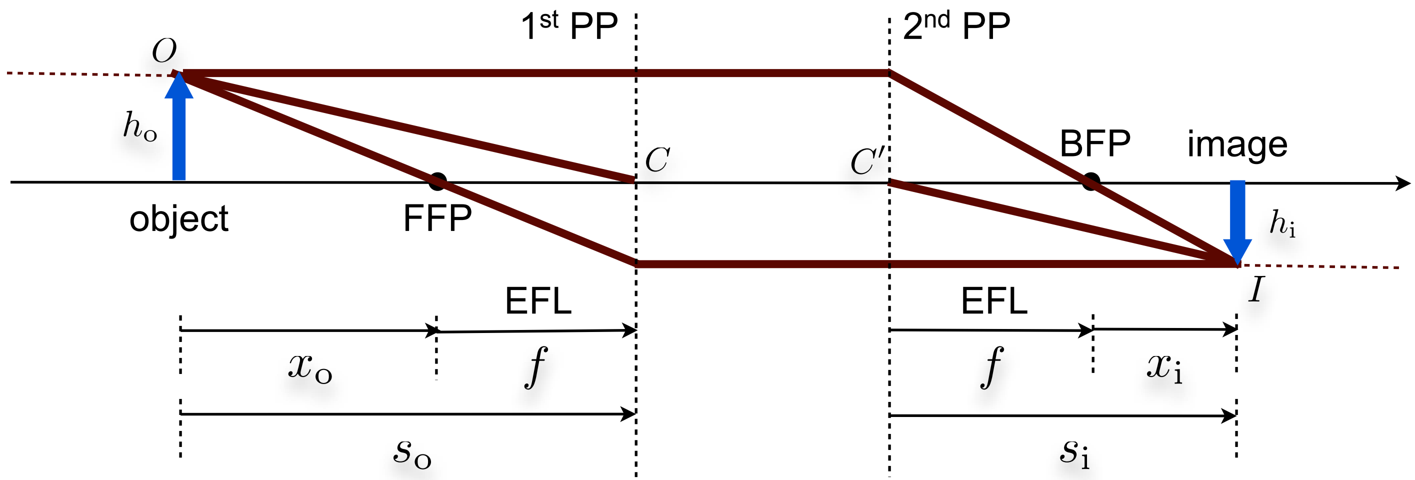



$$\begin{pmatrix} \alpha_{\text{right}} \\ x_{\text{right}} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{\text{left}} \\ x_{\text{left}} \end{pmatrix} \quad \text{where}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_{\text{left}}} - \frac{1}{R_{\text{right}}}\right) \quad \text{Lens maker's Equation}$$

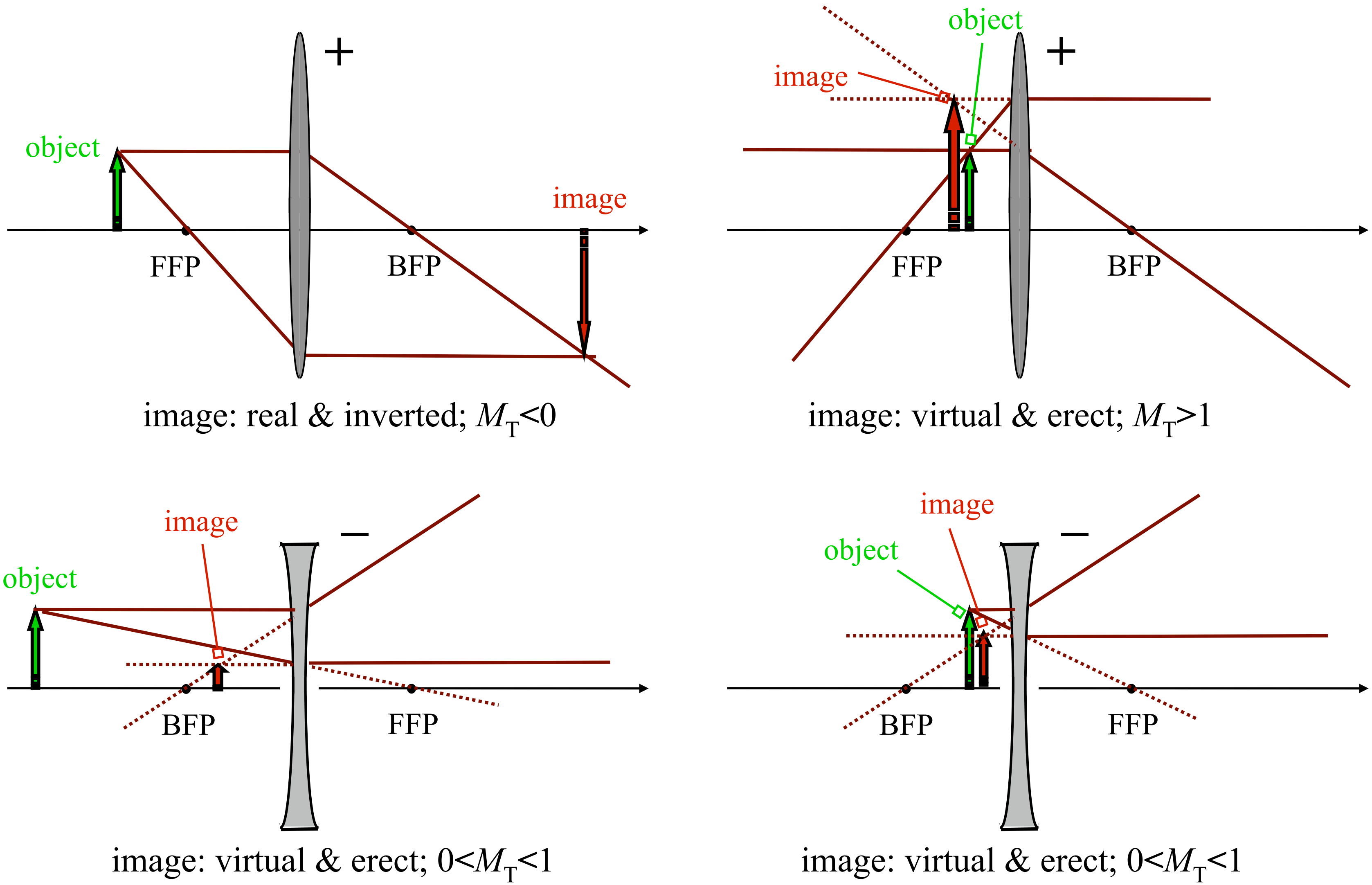
# Overview: composite optical elements



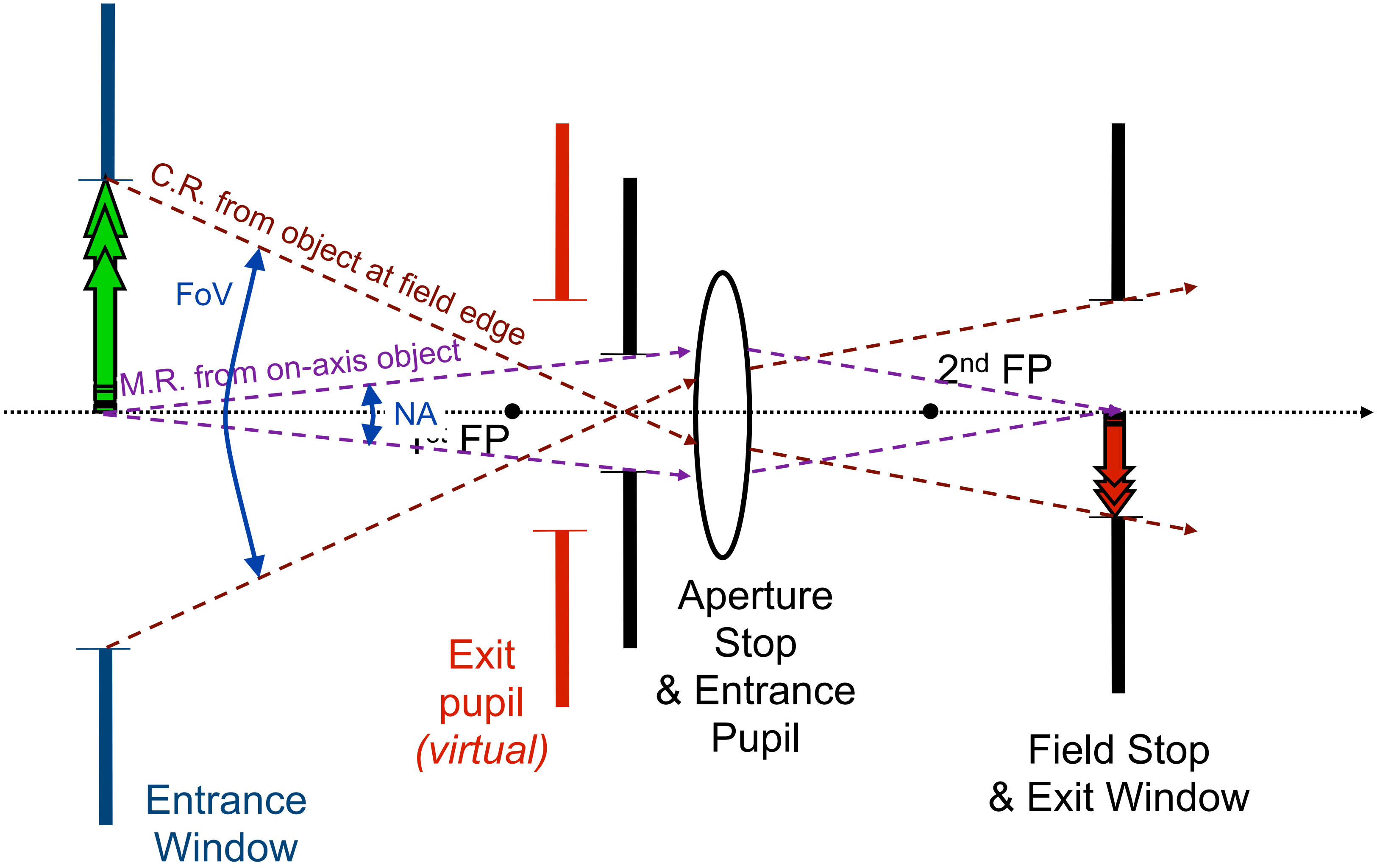




$$x_o x_i = f^2; \qquad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}; \qquad M_T = -\frac{x_i}{f} = -\frac{f}{x_o} = -\frac{s_i}{s_o}; \qquad M_A = \frac{1}{M_T}.$$

# Overview: real and virtual images



# Overview: apertures, stops, pupils, windows

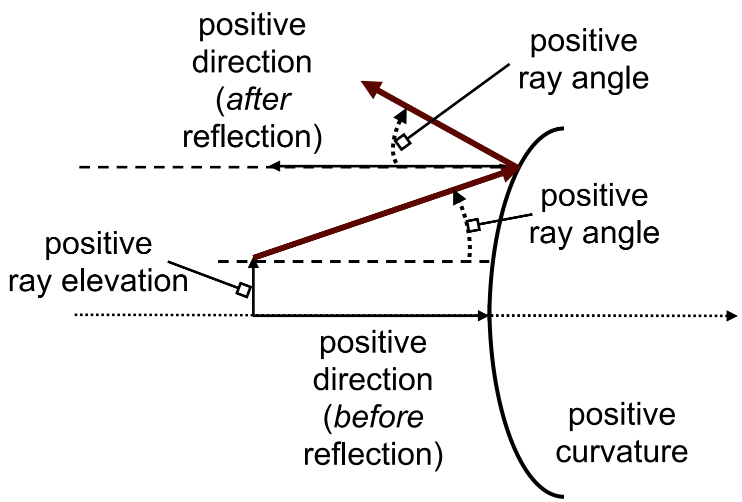


# Today

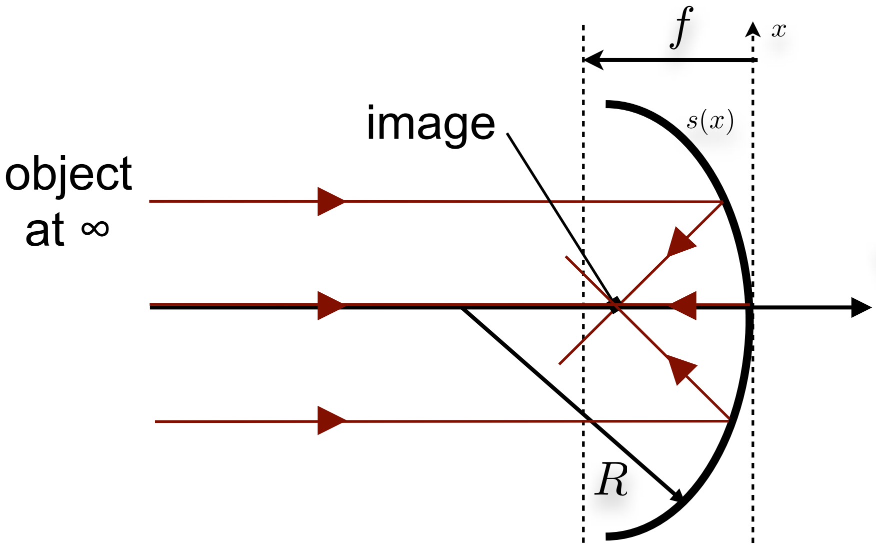
- Ray optics of mirrors
    - [[ Some terminology:
        - **catoptric** ≡ utilizing mirrors
          κάτοπτρον (kátoptron) = mirror
        - **dioptric** ≡ utilizing refractive lenses
          δίοπτρον (díoptron) = lens
        - **catadioptric** ≡ utilizing both mirrors and refractive lenses]]
- The basic optical imaging systems
    - magnifier lens
    - eyepiece
    - microscope
    - telescope
        - refractive: Keppler (dioptric)
        - reflective: Cassegrain (catoptric)
        - Schmidt (catadioptric)

# Sign conventions for catoptric optics

- Light travels from left to right *before reflection and from right to left after reflection*
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances *before reflection* are positive if pointing to the right; *longitudinal distances after reflection are positive if pointing to the left*
- Longitudinal distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the $+z$ axis counterclockwise through an acute angle



# Object/image at infinity with spherical mirror




Object at infinity

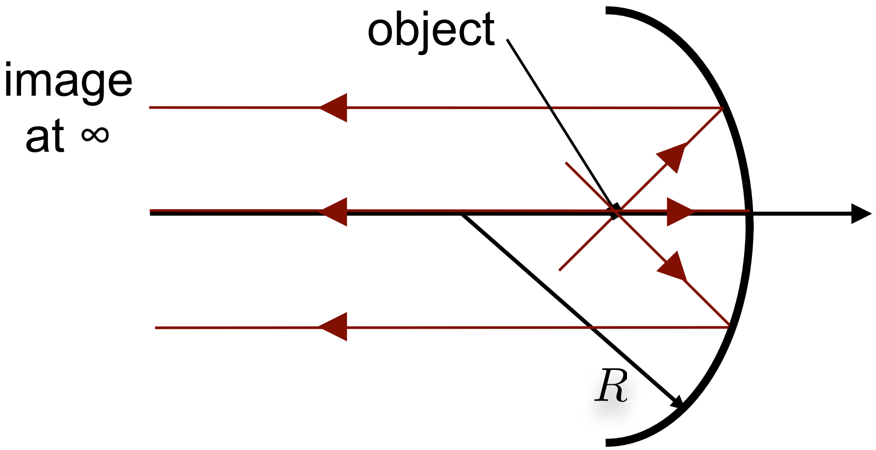



Image at infinity

Recall that to bring a plane wave (object at $\infty$) to focus, the perfect reflector shape is paraboloidal:

$$s = \frac{x^2}{4f}$$

We seek the sphere that best matches the paraboloidal curvature in the paraxial region:

$$(s+R)^2 + x^2 = R^2 \Rightarrow$$

$$\begin{aligned} s &= -R + \sqrt{R^2 - x^2} = R\left[-1 + \sqrt{1 - \frac{x^2}{R^2}}\right] \\ &\approx R\left[-1 + \left(1 - \frac{x^2}{2R^2}\right)\right] = -\frac{x^2}{2R}. \end{aligned}$$

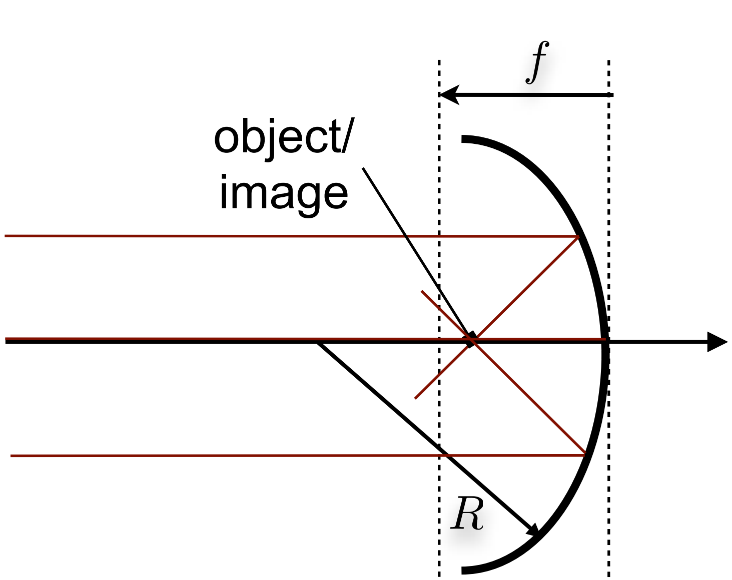
We can see that the surfaces match if

$$R = -2f$$

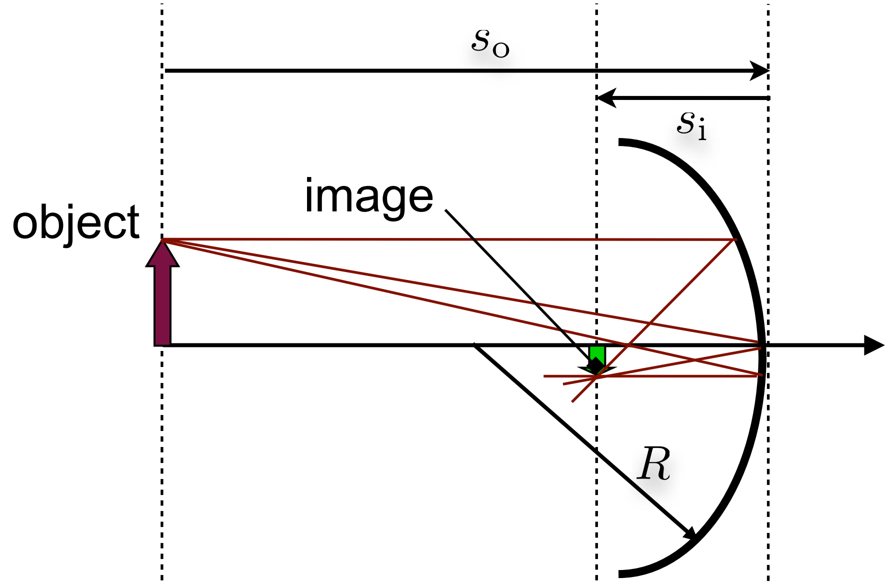
Note $R < 0$ because the surface is concave toward the left. Therefore, $f > 0$; *i.e.*, this is a positive lens.

⇒Focal length $\quad f = -\dfrac{R}{2}$

# Catoptric imaging formulae




Object/image at infinity




Object, image at finite distances

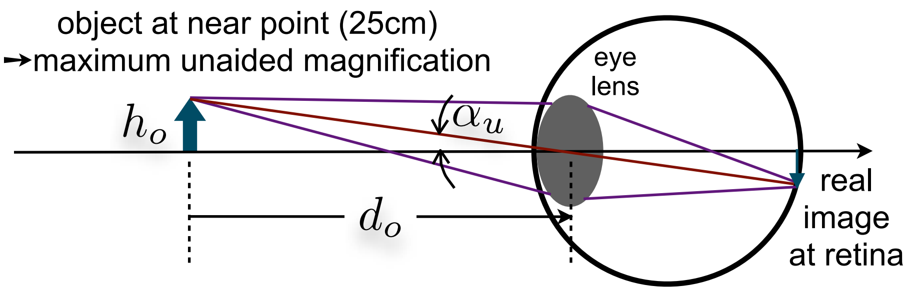
Focal length $$f = -\frac{R}{2}$$

Ray transfer matrix
$$\begin{pmatrix} \alpha_\mathrm{right} \\ x_\mathrm{right} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_\mathrm{left} \\ x_\mathrm{left} \end{pmatrix} = \begin{pmatrix} 1 & \frac{2}{R} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_\mathrm{left} \\ x_\mathrm{left} \end{pmatrix}$$

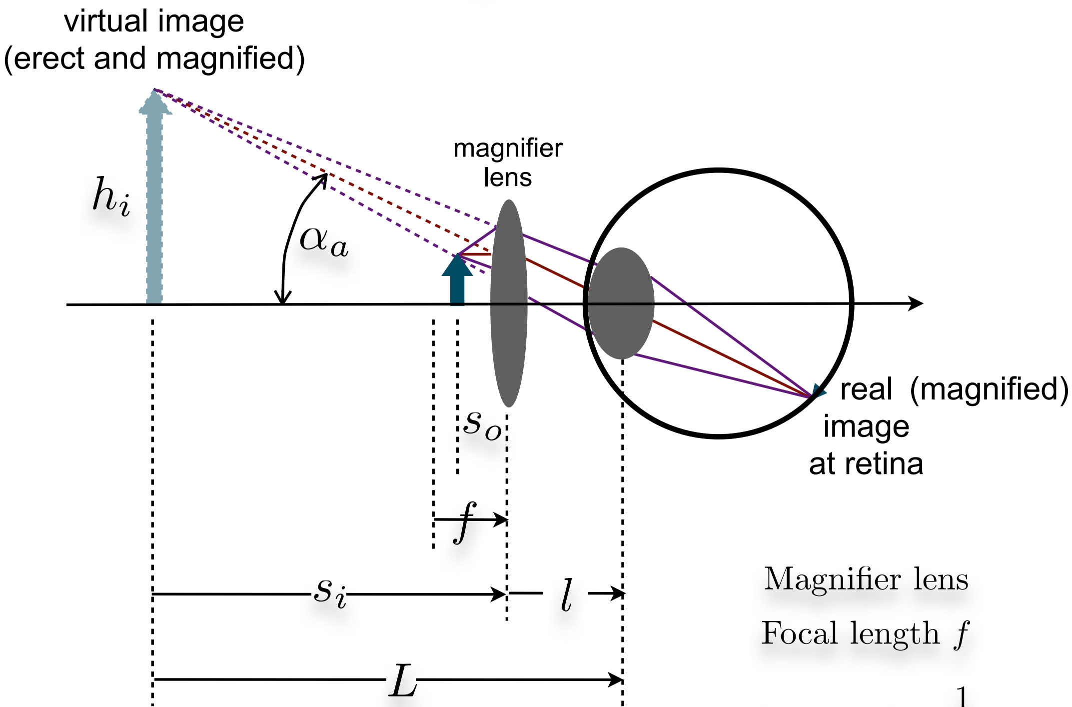
Imaging condition $$\frac{1}{s_\mathrm{o}} + \frac{1}{s_\mathrm{i}} = \frac{1}{f} = -\frac{2}{R}$$

Magnification $$M_\mathrm{A} = -\frac{s_\mathrm{o}}{s_\mathrm{i}} \qquad M_\mathrm{T} = -\frac{s_\mathrm{i}}{s_\mathrm{o}}$$

# The single lens magnifier




Near point $d_o \equiv 25\text{cm}$




Magnifier lens

Focal length $f$

Power $P \equiv \frac{1}{f}$

Magnifying power $\text{MP} \equiv \frac{\alpha_a}{\alpha_u}$

$$\text{MP} = \frac{h_i/L}{h_o/d_o} = \frac{h_i}{h_o} \times \frac{d_o}{L}$$

$$\text{but} \quad \frac{h_i}{h_o} = M_\text{T} = -\frac{s_i}{s_o} = 1 - \frac{s_i}{f}$$

$$\Rightarrow \text{MP} = \left(1 - \frac{s_i}{f}\right) \times \frac{d_o}{L} = \left(1 - \frac{L-l}{f}\right) \times \frac{d_o}{L}$$

$$\Rightarrow \text{MP} = \Big(1 - P\,(L-l)\Big) \times \frac{d_o}{L}$$

(1) $l = f \Rightarrow \text{MP} = d_o P$

(2) $l = 0 \Rightarrow \text{MP} = d_o \left(\frac{1}{L} + P\right)$

Largest MP for smallest permissible L, *i.e.*

$$L = d_o \Rightarrow \text{MP} = 1 + d_o P$$

(3) $s_i = f \Rightarrow s_i = \infty$

Virtual image at $\infty \Rightarrow$
unaccommodated (relaxed) eye, and

$$\text{MP} = d_o P$$

# Eyepiece

- also known as "ocular"
- Magnifier meant to look at the intermediate image formed by the preceding optical instrument:
  - eye looks into eyepiece
  - eyepiece "looks" into optical system (microscope, telescope, etc.)
- Ideally should
  - produce a virtual image at infinity ($\Rightarrow$ MP=$d_o P$)
    - the final image is viewed with relaxed (unaccommodated) eye
  - center the exit pupil (eye point) where the observer's eye is placed at 10mm (eye relief) from the instrument

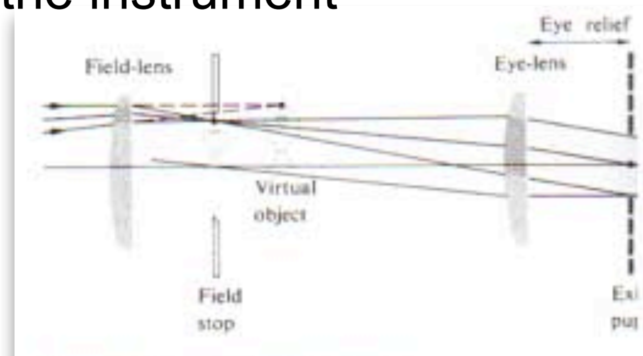
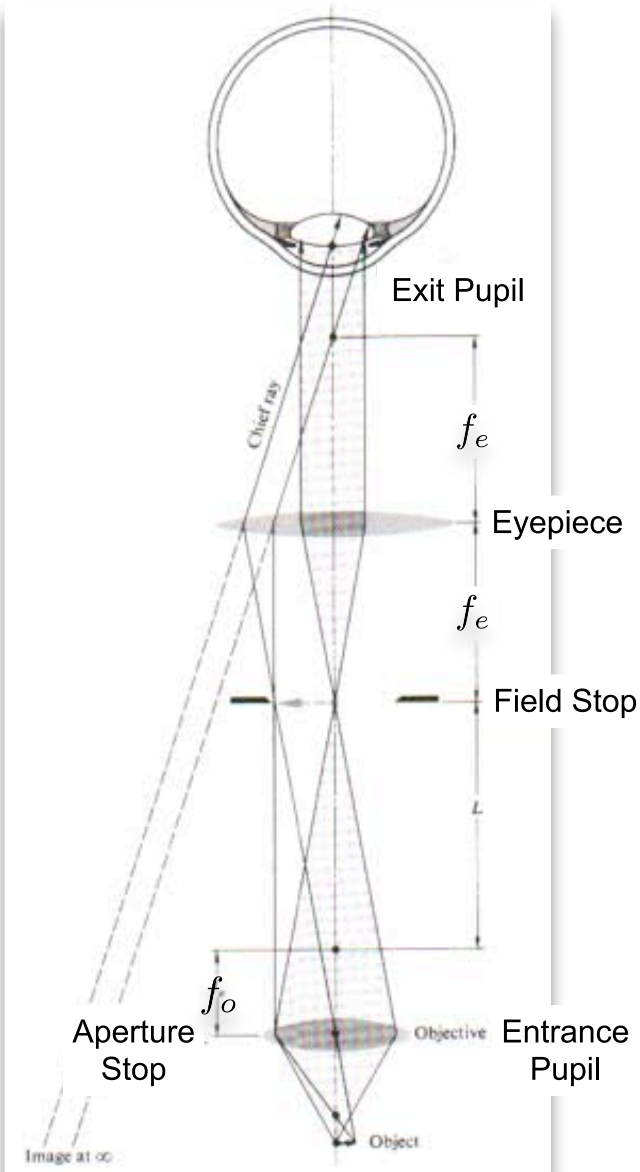


**Figure 5.93** The Huygens eyepiece.



Fig. 5.93 in Hecht, Eugene. *Optics*. Reading, MA: Addison-Wesley, 2001.

# Microscope




- Purpose: to “magnify” thereby providing additional detail on a small, nearby object
- Objective lens followed by an eyepiece
  - Objective: forms real, magnified image of the object at the plane where the instrument’s field stop is located
  - Eyepiece: its object plane is the objective’s image plane and forms a virtual image at infinity
    - ➡ magnified image can be viewed with relaxed (unaccommodated) eye
    - ➡ compound magnifying power is the product of the magnifications of the two elements, i.e.

$$\mathrm{MP} = M_{\mathrm{T}}^{\mathrm{objective}} \times M_{\mathrm{A}}^{\mathrm{eyepiece}}$$

- The distance from the BFP of the objective to the FFP of the eyepiece is known as tube length and is standardized at 160mm.
- The near point used as $d_o$ is standardized at 254mm (10 inches.)

$$\mathrm{MP} = \left(-\frac{160}{f_{\mathrm{objective}}}\right) \times \left(\frac{254}{f_{\mathrm{eyepiece}}}\right)$$



Fig. 5.99 in Hecht, Eugene. *Optics*. Reading, MA: Addison-Wesley, 2001.