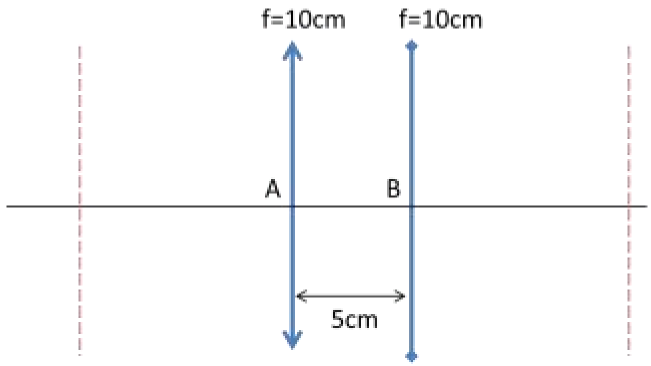
Massachusetts Institute of Technology

2.71/2.710 Optics **Spring 2014**

Solution for HW2

1. Modified from Pedrotti 18-9

a) The schematic of the system is given below.



b) Using matrix method from point A to B,

$$M_{2lens} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} = \begin{pmatrix} 1 - d/f_1 & d \\ d/f_1 f_2 - 1/f_1 - 1/f_2 & 1 - d/f_2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 1/10 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/10 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 5 \\ -1/20 & 3/2 \end{pmatrix}$$

From the above matrix, we see that A and D are unitless, B is in the unit of length, and C has the unit of 1/length. The effective focal length and location of the principle planes can be calculated from Table 18-2 of Pedrotti:

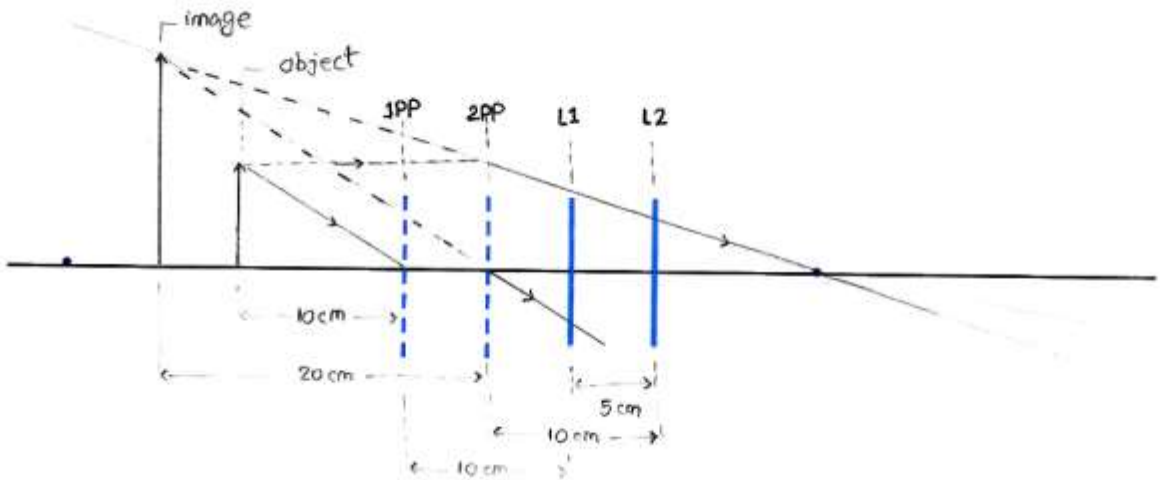
$$EFL: f_{eq} = -1/C = 20cm,$$

$$FFP: p = D/C = -30cm, \; BFP: q = -A/C = 10cm,$$

$$PP_1: r = (D-1)/C = -10cm, \;\; PP_2: s = (1-A)/C = -10cm.$$

Here, PP1 represents the location of the starting plane of the system with respect to the original frontal plane A. PP2 represents the location of the modified ending plane of the system with respect to the original ending plane B.

c) An example is sketched with the object placed at 20cm to the left of the first lens. The object is located 10cm left to PP1.



From the lens law, we get:

$$\frac{1}{10}+\frac{1}{s_i}=\frac{1}{20}, \quad s_i=\frac{20\times 10}{-10}=-20cm.$$

We find that the image is located 20cm left to the 2nd principal plane, which is 25cm left to L1.

2. Plano-convex doublet lens

In the visible spectrum, the refractive index of glass is close to a constant as shown below in the spectrum provided by Newport.

a) Using the matrix method to calculate the overall optical response, we can get

$$\begin{aligned} M_{total} &= (M_{R3})(M_{t2})(M_{R2})(M_{t1})(M_{R1}) \\ &= \begin{pmatrix} 1 & 0 \\ \dfrac{n_2 - n_0}{R_3} & \dfrac{n_2}{n_0} \end{pmatrix} \begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{n_1 - n_2}{R_2} & \dfrac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{n_0 - n_1}{R_2} & \dfrac{n_0}{n_1} \end{pmatrix} \\ &= \begin{pmatrix} 0.968 & 4.026 \\ -0.00950 & 0.994 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \end{aligned}$$

and the optical power of the composite lens can be calculated from the equivalent focal length.

$$P = \frac{1}{f_{eq}} = -C = 0.00951 mm^{-1}$$

b) If a plane wave is incident from the left, it focuses on the BFP.

$$BFP = -\frac{A}{C} = 101.97 mm$$

Therefore the plane wave focuses 102mm after the right surface of the lens.

3. A Telephoto Lens

a) The focal length of each individual lens can be determined as the following.

The ABCD matrix for two thin lens is:

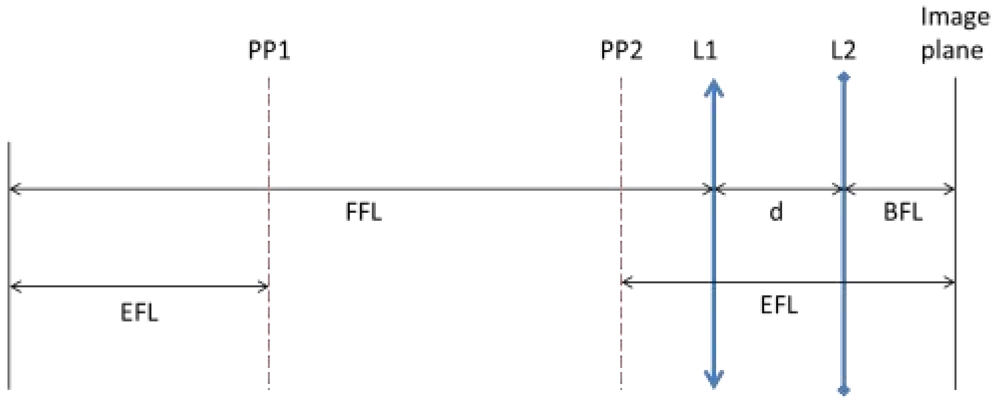
$$M_{2lens} = \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} = \begin{pmatrix} 1 - d/f_1 & d \\ d/f_1 f_2 - 1/f_1 - 1/f_2 & 1 - d/f_2 \end{pmatrix}.$$

From the matrix we can set the following parameters to be:

$$EFL: f_{eq} = -1/C = 300m,$$
$$BFL: q = -A/C = 100mm.$$

Plugging in the two values and using d=120mm, we get

$$f_1 = 180mm,$$
$$f_2 = -150mm.$$



b) the principal planes can be obtained from:

$$M_{2lens} = \begin{pmatrix} 1/3 & 120 \\ -1/300 & 9/5 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$PP_1 = \frac{D - n_0/n_f}{C} = -240mm,$$
$$PP_2 = (1-A)/C = -200mm.$$

PP1 is 240mm left of L1 and PP2 is 200mm left of L2.

c) The entrance pupil is identical to the aperture stop. The diameter can be obtained from the equation for F-number:

$$N = f/D = 4,\ D = f/N = 300mm/4 = 75mm.$$

d) The most efficient way to predict the answer is to graphically think about how much light propagates with the given angle variation. We can convert the input angle into output vertical distance by recognizing that the effective distance of propagation until the focus is EFL:

$$\Delta x_{approx} = EFL \cdot \tan(\Delta\theta) \approx EFL \cdot \theta = 0.3mm\,.$$

Is it really true? Let's check mathematically with the rigorous matrix method. We can construct a new system matrix that includes the propagation to the BFP:

$$\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = \begin{pmatrix} 1 & BFL \\ 0 & 1 \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix} = \begin{pmatrix} 0 & B - D\cdot BFL \\ C & D \end{pmatrix}\begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix},$$

since BFL=-A/C. This can be further simplified by:

$$B - D\cdot BFL = B - D\cdot\frac{A}{C} = B - (1 + B\cdot C)\frac{1}{C} = -\frac{1}{C} = EFL\,.$$

Indeed! Here, $AD - BC = 1$ comes from the fact that the determinant of the system matrix is equal to the ratio between the initial and final refractive indices, n_0 / n_f. Therefore the distance between the two focuses is 0.3mm.

4. Modified from Pedrotti 3-5

Among elements A, L1, and L2, the one whose image through the elements to its left subtends the smallest angle at the center of the EnP is the field stop. Aperture A in this case is our entrance pupil.

The rim of A at the EnP subtends 90°.

The rim of L1 at the EnP subtends $\tan^{-1}(3/3) = 45°$

L2 should be imaged towards the left, through L1. This iamge, labeled L2', can be calculated as:

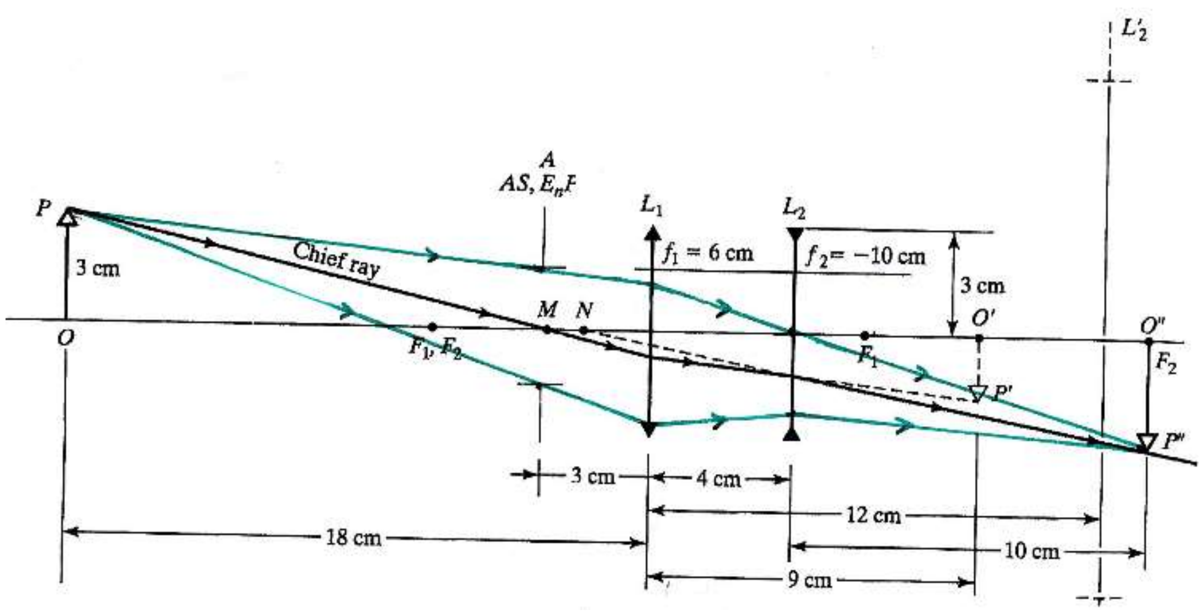
$$\frac{1}{s_i} + \frac{1}{4} = \frac{1}{6},\ \ s_i = -12cm;$$

$$M = -\frac{s_i}{s_o} = 3 = \frac{y_i}{3},\ \ y_i = 9cm.$$

Therefore, the rim of L2' at the EnP subtends $\tan^{-1}(9/15) = 31°$, which is the smallest angle, making L2 the field stop of the system, and L2' the EnW.

b) The EnW is the image of the FS through the optical components to its left. In a) we calculated this to be L2'. The ExW is the image of the FS through the optical components to its right. Since there are no components to the right, the ExW is the L2 itself.

c) The schematic of the system is given below. The marginal rays in this illustration start from P and toucnes the edges of the EnP. After hitting L1, they aim toward P', and after hitting L2, they are redirected towards P'', which is the final image. (Image Credit: © 2007 Pearson Prentice Hall, Inc)



5. Camera lens and Image Stabilizer system

a)

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.5)\left(\frac{1}{50}-\frac{-1}{50}\right)=\frac{1}{50mm}.$$

The transfer matrix from A to A' can be found as:

$$\begin{pmatrix}1 & s\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -1/f & 1\end{pmatrix}\begin{pmatrix}1 & d-s\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1-s/f & s+(d-s)(1-s/f)\\ -1/f & 1-\dfrac{d-s}{f}\end{pmatrix}=\begin{pmatrix}A & B\\ C & D\end{pmatrix}$$

The distance is d=36nm. The imaging condition is B=0, which can be solved for s to yield $(s-30)(s-6)=0$. Which is the correct answer? To have a demagnified image (s<d-s), the valid answer is s=6cm.

b) After the rotation of the camera,

$$x_o=(d-s)\sin\theta\approx 5.24mm$$

Then we move the lens up by distance δ: $x_o'=x_o-\delta$, $x_i'=-\delta$,

$$-\frac{x_i'}{x_o'}=\frac{s}{d-s}\ .$$

Therefore the lens must be moved by $\delta=0.873mm$ upward to compensate for the shaking.