MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.71/2.710 Optics **Spring '14**

HW6 Posted April 7, 2014, Due April 16, 2014

1. A simplified grating spectrometer setup is illustrated as in **Figure 1**. The spectrometer consists of a sinusoidal amplitude grating with complex amplitude transmission function:

$$t(x) = t_0 + t_1 cos\left(\frac{2\pi}{\Lambda} x\right)$$

The grating is placed at the plane $z = 0$ and illuminated by an off–axis plane wave

$$E(x, z = 0) = E_0 \exp(ikx sin\theta)$$

propagating at angle θ with respect to the optical axis z.

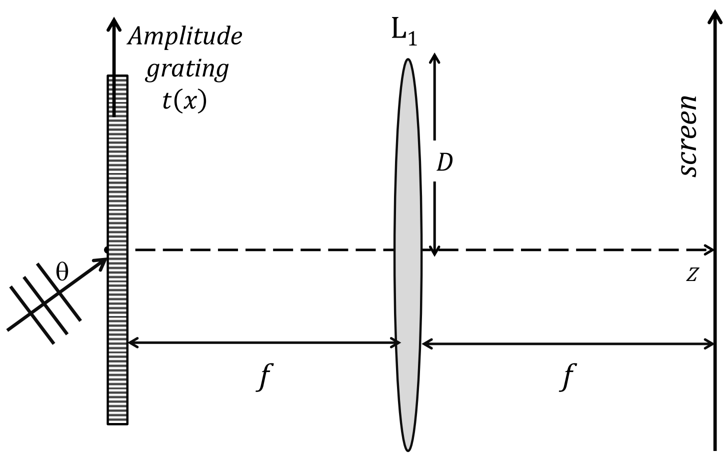


Figure 1. A grating spectrometer.

The diffracted lightwaves are transmitted through a lens L_1 with focal length f, and collected at the back focal plane for wavelength analysis. In all problems below, let's assume the grating is sufficiently large and paraxial approximation is valid across the whole system.

a) With a **coherent** monochromatic illumination at wavelength λ, derive an expression for the **Fresnel** diffraction pattern right behind the grating, and argue that the expression that you derived physically represents a coherent superposition of three plane waves of different orientation.

b) Using **geometric** optics, determine the location of the resulting spots on the back focal plane, neglecting for now the diffraction effect due to finite beam width and aperture of the lens. Use your result to show that the wavelength detection limit, $\Delta\lambda$, of this spectrometer is inversely proportional to grating period Λ.

c) (**2.710 only**) Now taking into account of the diffraction effect due to finite beam width W, estimate the resulting spot width of the diffracted order at the back focal plane at monochromatic wavelength λ. Evaluate again the minimum spectral resolution $\Delta\lambda$ of the grating spectrometer.

2. **(Goodman 5-9)Effect of defocusing**: A unit-amplitude, normally incident monochromatic plane wave illuminates an object of maximum linear dimension D, situated immediately in front of a larger positive lens of focal length f, as shown below. Due to a positioning error, the intensity distribution is measured across a plane at a distance $d - \Delta$ behind the lens. How small must Δ be if the measured intensity distribution is to accurately represent the Fraunhofer diffraction pattern of the object?

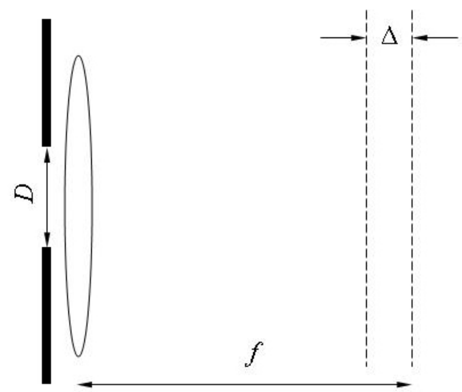


Figure 2(Goodman 5.18). Schematic of a measurement of defocusing effect.

3. As shown in **Figure 3**, the input images $t_1(x, y)$and a phase mask $t_2(x', y')$ were placed on the surface of the lens L_1 and L_2. The focal length of the lens L1 is $f_1 = 2a$ and that of L_2 is $f_2 = a$. The spacings between L_1 and L_2 and the screen are all $2a$.
    a. Derive an expression for the light distribution on the screen.
    b. (**2.710 only**) Can you suggest a possible application of such an arrangement?

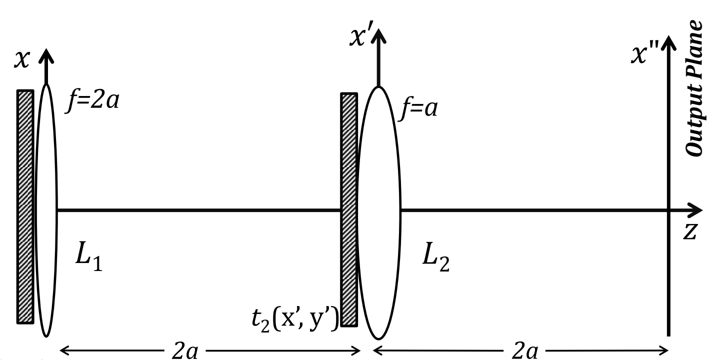
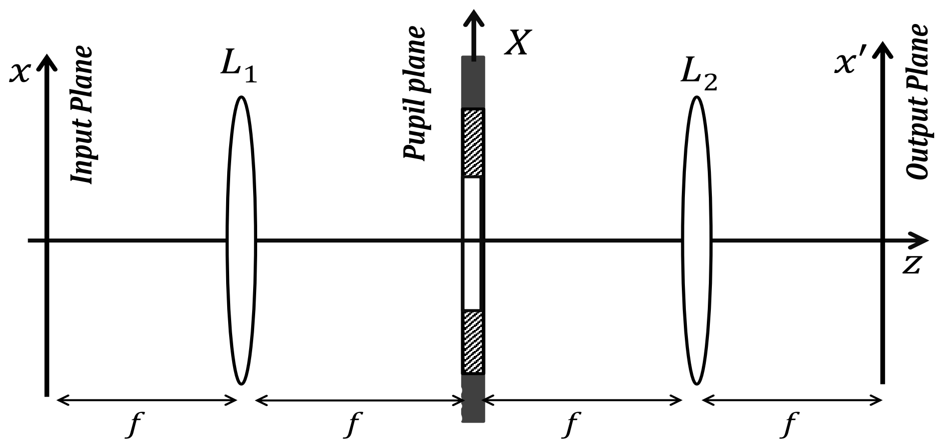


Figure 3. Optical information processing using 2 convex lenses

4. We are given a 4F imaging system consisting of two identical lenses L_1, L_2 with focal length f=10cm. As an experiment, you attached a T shaped amplitude mask at the pupil plane of the imaging system.



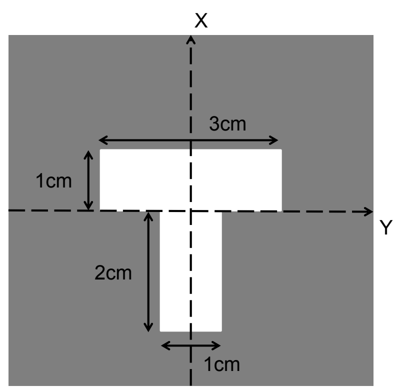


Figure 4. 4F imaging system with a T-shaped amplitude mask (dark area is completely opaque, and white area is transparent) placed at pupil plane(X, Y).

a) A quasi–monochromatic, coherent point source at wavelength $\lambda = 0.5\mu\text{m}$, is placed at the origin of the input plane (x = 0, y=0). **calculate** and sketch the Point Spread Function of this lens system at output plane.

b) A coherent illumination field pattern at the input plane of the system is given by

$$E(x,y) = \frac{E_0}{2}\left[3 + cos\left(\frac{2\pi x}{4\mu m}\right) + cos\left(\frac{2\pi x}{2\mu m}\right) + cos\left(\frac{2\pi x}{1\mu m}\right)\right]$$

Calculate the output field and the contrast of the output intensity pattern for wavelength λ=0.547 um(green) and 0.633 um (red).

5. We are given a 4F imaging system consisting of two identical lenses L1, L2 with identical focal length f. A glass prism of refractive index n, base length w, and apex angle α is placed at the pupil plane of the telescope. In all questions below, the illumination is assumed to be monochromatic at wavelength λ.

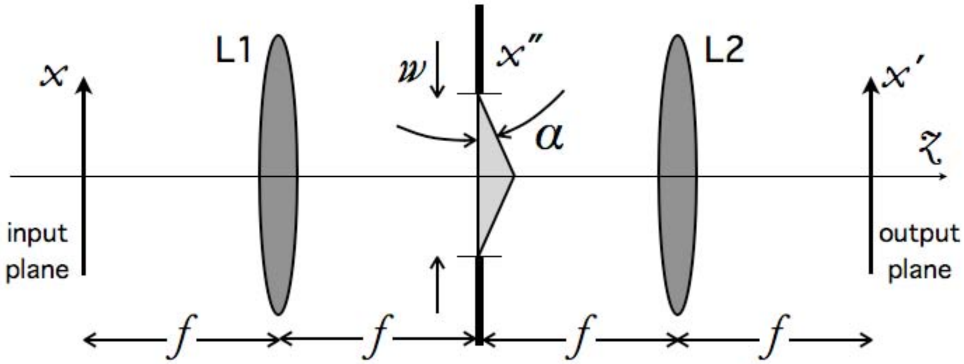


Figure 4. Schematic of a 4F imaging system with a biprism placed at pupil plane(X",Y").

a) Consider first a point source placed at the origin $x = 0$ at the input (object) plane, and a ray departing at angle θ from the point source. What is the angle with respect to the optical axis z that the same ray emerges from the prism?

b) Use the answer to the previous question to determine the 1D point–spread function (PSF) of this system, neglecting for now any diffraction effects due to the finite base length w of the prism.

c) Now modify the 1D PSF, taking into account the finite base length w of the prism. You may assume that the prism acts like a thin transparency and $\alpha \ll 1$ radian for this calculation.

d) (2.710 only) Now consider the rotationally symmetric version of the prism, also known as an "axicon." What is the 2D PSF of the rotationally symmetric system?

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