MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.71/2.710 Optics Spring '14
Practice Problems Posted May 11, 2014

1. **(Pedrotti 13-21)** A glass plate is sprayed with uniform opaque particles. When a distant point source of light is observed looking through the plate, a diffuse halo is seen whose angular width is about 2^o. Estimate the size of the particles. (Hint: consider Fraunhoffer diffraction through random gratings)

2. **(Adapted from Pedrotti 16-1 and 16-12)**

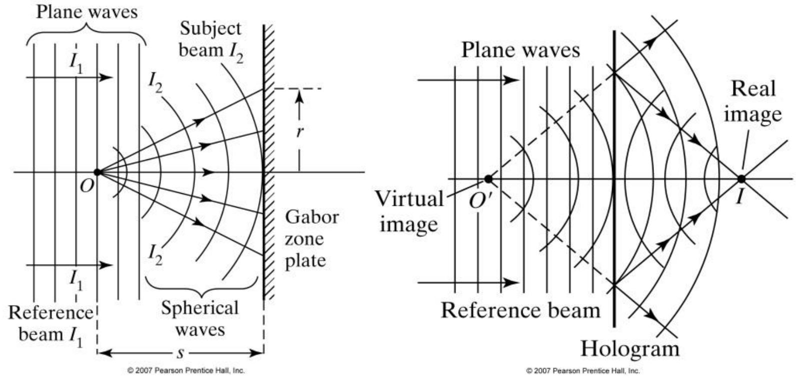


Figure A. Recording(Left) and Reconstruction(Right) of a Gabor Hologram

a) Use the superposition of two beams to show that the recorded intensity pattern on a Gabor zone-plate (the hologram of a point source) is given approximately by

$$I = A + Bcos^2(ar^2)$$

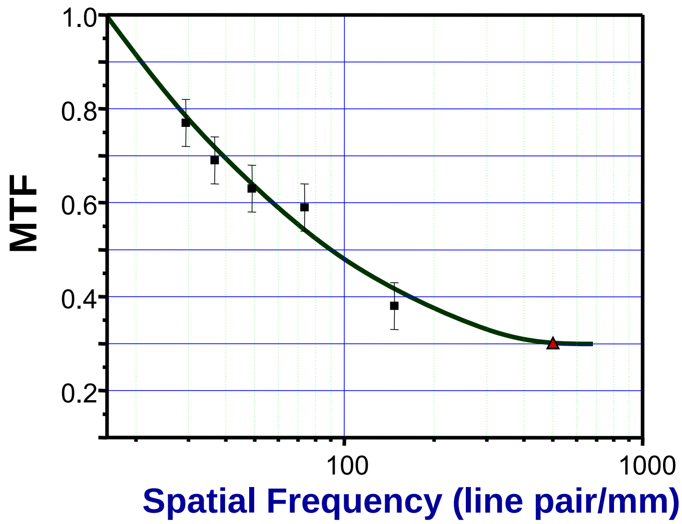
Where $A = I_1 + I_2 - 2\sqrt{I_1 I_2}$, $B = 4\sqrt{I_1 I_2}$, and $a = \pi/(2s\lambda)$. Here I_1 and I_2 are the intensity due to the reference and signal beams, respectively, s is the distance of the object point from the film, and λ is the wavelength of the light. For the approximation, assume the path difference between the two beams is much smaller than s, so we are looking at the inner zones of the hologram.

b) (**2.710 only**) Show that the phase delay of the diverging subject beam, at a point on the film at distance r from the axis, is given by $\pi r^2/ls$. This results follows when r<<*s*. Show also that the amplitude of the light transmitted by the film under illumination of the reference beam produces converging spherical wavefront, thus a real image on reconstruction.

3. **Modulation Transfer Function:**

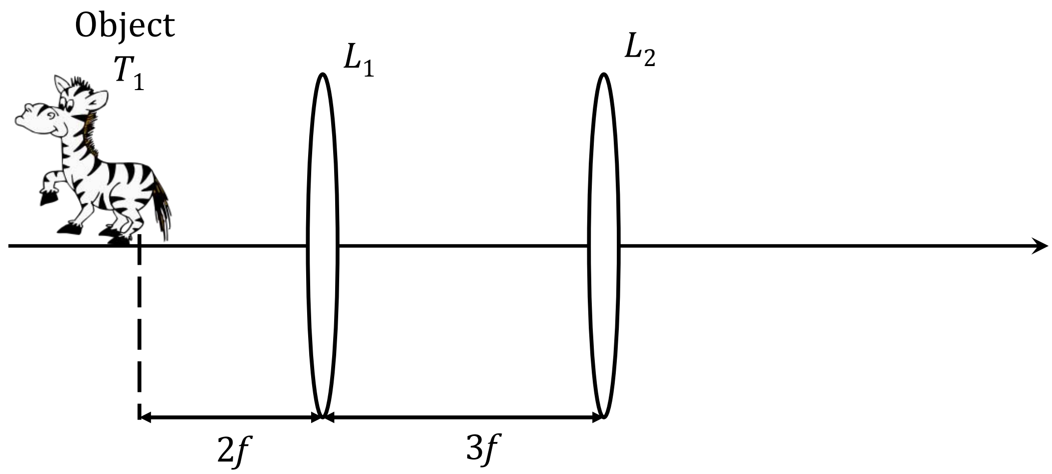
The measured modulation transfer function (MTF) of an optical lithography system is given in the figure below. The system is illuminated with spatially incoherent light at 400nm wavelength. The intensity pattern at the input plane of the system is given by

$$I(x) = \frac{1}{2}\left[1 + cos\left(\frac{2\pi x}{50\mu m}\right) + cos\left(\frac{2\pi x}{10\mu m}\right)\right]$$



a) What is the contrast of the intensity pattern at the input plane?
b) Plot the intensity pattern formed at the output plane, and calculate the image contrast.
c) Can you guess the coherent transfer function and cut-off spatial frequency for this imaging system?

4. Consider the optical system shown the following schematic, where lenses L_1, L_2 are identical with focal length f and diameter $2a$. A thin-transparency object T_1 is placed at distance $2f$ to the left of L_1.



a) Where is the image formed? Use geometrical optics, ignoring the lens apertures for the moment.

b) If the object T_1 is an on-axis point source, describe the Fraunhofer diffraction pattern of the field to the right of L_2.

c) How are your two previous answers consistent within the approximations of paraxial geometrical and wave optics?

d) The point source object T_1 is replaced by a clear aperture of full width w and a second thin transparency T_2 is placed between the two lenses, at distance f to the left of L_2. The system is illuminated coherently with a monochromatic on-axis plane wave at wavelength λ. Write an expression for the field at distance $2f$ to the right of L_2 and interpret the expression that you found.

e) Derive and sketch approximately, with as much quantitative detail as you can, the intensity observed at distance $2f$ to the right of L_2 when T_2 is an infinite sinusoidal amplitude grating of period Λ, such that $\Lambda \ll a$.

f) (**2.710 only**) Now we want to place a picture of zebra as T_1, and make a copy of the zebra image that removes the black and white stripes. A careful examination of the picture T_1 shows the stripe patterns are approximately periodical with center to center spacing of 0.5mm and black stripe width of 0.1mm. The picture T_1 is 20mm wide and 10 mm tall. The lens L_2 is moved to a distance of $3.5f$ to the right of L1. The focal length f=250mm for both lenses L_1 and L_2. Can you design a transparency mask T_2 for this filtering application for coherent and incoherent illumination? T_2 is placed at distance f to the left of L_2.

5. **(2.710 only) Zernicke phase mask.** You are given an imaging system which consists of two thin transparencies T1, T2 and two thin lenses L1, L2 arranged as shown in Figure A. The shapes and dimensions of T1, T2 are shown immediately below in Figure B.

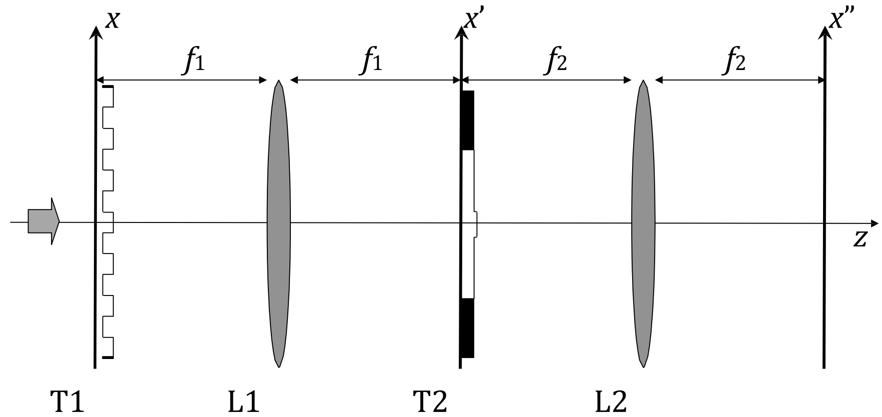


Figure A *(not to scale)*

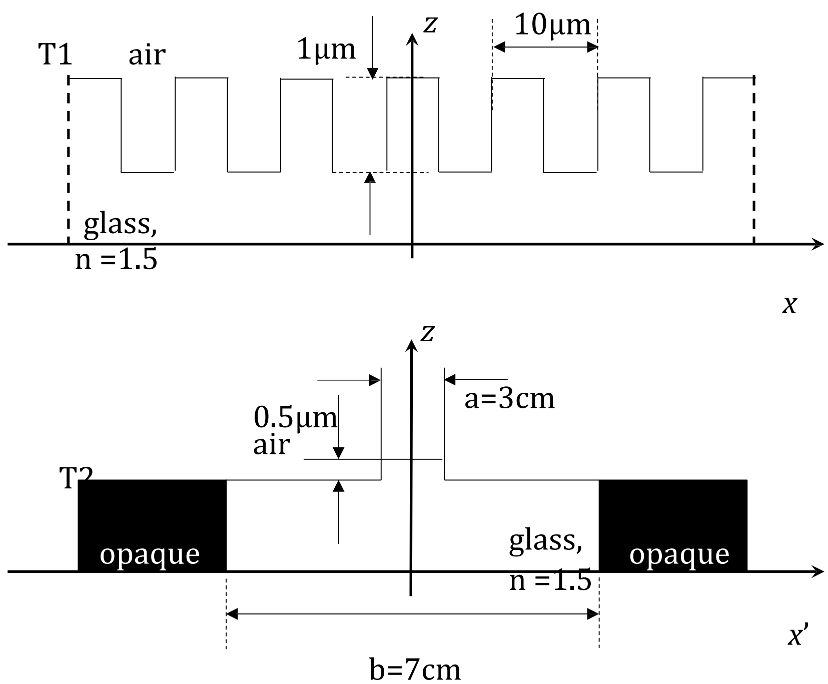


Figure B *(not to scale)*

Transparency T1 is infinitely large in the *x* dimension. Lenses L1, L2 are identical, with infinitely large apertures and focal lengths $f_1 = f_2 = 10\text{cm}$. Numerical values for the symbols *a*, *b* denoting lateral feature sizes of T2 are defined in Figure B.

The illumination is an on–axis plane wave at wavelength $\lambda = 1\mu$m. The observation plane is located one focal distance behind L2.

a) What is the intensity immediately after T1?

b) What is the optical field immediately before T2?

c) What is the intensity measured at the observation plane?

d) Comparing your answers (a) and (c), how is T2 helpful in imaging the phase object T1?

e) Consider the limit $b \to \infty$. How does then answer (c) change? Is the larger aperture helpful in this case?

f) If a = 0.5cm and $b \to \infty$, is your answer (d) still valid? If yes, why? If not, what has gone wrong?