Review on Geometrical Optics (02/26/14)

2.71/2.710 Introduction to Optics –Nick Fang

Reminder: Quiz 1 (closed book, Monday 3/3, in class)

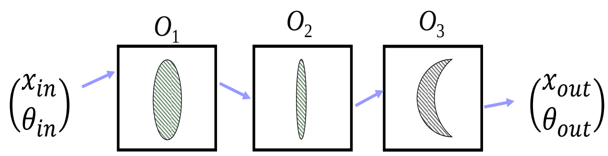
Topics Covered: (Pedrotti Chapter 2, 3, 18)

- **Reflection, Refraction, Fermat's Principle,**
- **Prisms, Lenses, Mirrors, Stops**
- **Lens/Optical Systems**
- **Analytical Ray Tracing, Matrix Methods**

Outline:

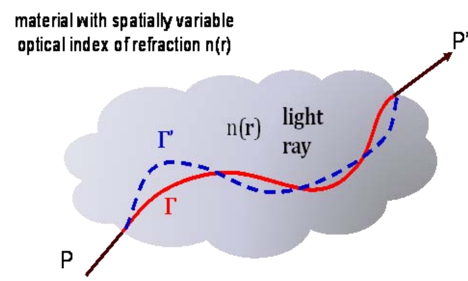
A. Two General Approaches in Geometrical Optics
B. From "Perfect" imager to paraxial rays, effects of aperture and stops
C. Thin lenses and imaging condition
D. Composite lenses

A. Two General Approaches in Geometric Optics



$$\begin{pmatrix}x_{out}\\\theta_{out}\end{pmatrix} = [O_3][O_2][O_1]\begin{pmatrix}x_{in}\\\theta_{in}\end{pmatrix}$$

vs



Both approaches of GO Focused on the **ray vector** (x, θ) (which carry the power flux (e.g. unit: W/cm^2), and its change with distance and as the ray propagates through optics.

Approach 1. Tracing rays through interfaces (e.g. Matrix Methods)

Approach 2: Given the source and destination, what are the possible paths of shortest distance? (e.g. Eikonal Equations, Langrangian and Hamiltonian Optics)

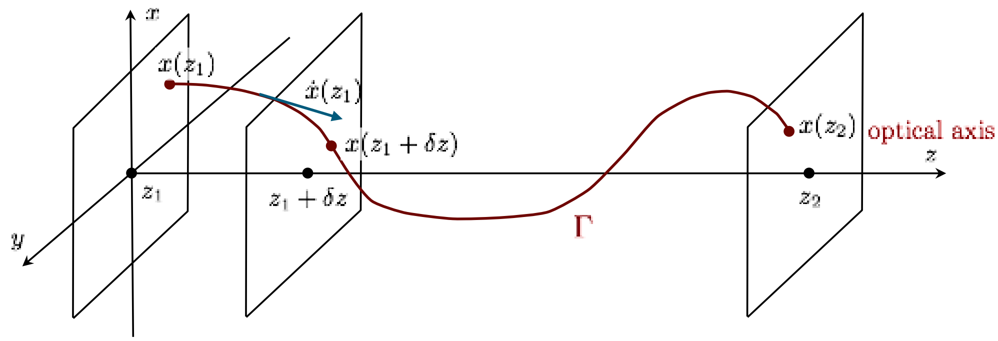
-The connection: θ (momentum) describe the slope of change of the ray in position **x** at different time interval (but here we see them in steps along the optical axis z).

Therefore, the matrix method is similar to Newtonian equations:

$$x(t+\Delta t) = x(t) + v\Delta t$$

$$v(t+\Delta t) = v(t) + a\Delta t$$

$$(a\ is\ acceleration;\ in\ a\ potential\ field\ U(x), a = -\frac{dU}{dx})$$



We may define Optical Lagrangian: $\mathcal{L} = n(x,z)\sqrt{\theta^2+1}$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dz}\left(\frac{\partial \mathcal{L}}{\partial \theta}\right)$$

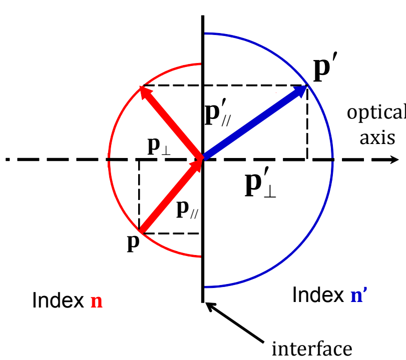
LHS: "Potential force"
RHS: "Acceleration"

Or

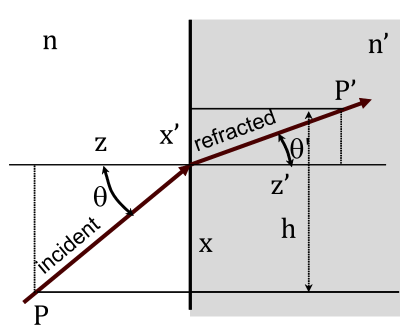
$$\frac{\partial n}{\partial x}\sqrt{\theta^2+1} = \frac{d}{dz}\frac{n\theta}{\sqrt{\theta^2+1}}$$

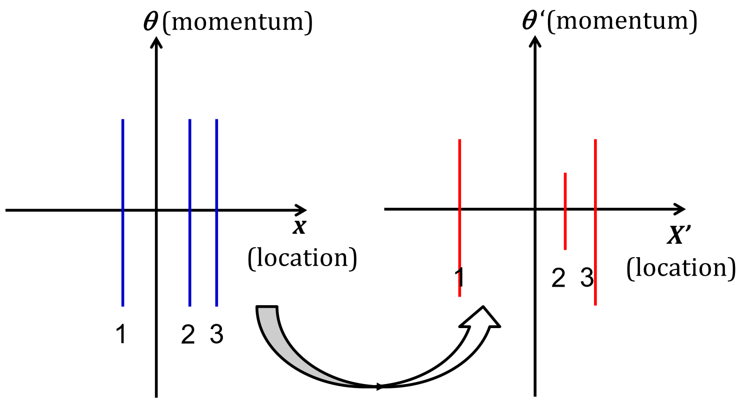
$$\frac{\partial n}{\partial x} = \frac{1}{\sqrt{\theta^2+1}}\frac{d}{dz}\frac{n\theta}{\sqrt{\theta^2+1}}$$

Example: Two Interpretation of *Refraction*



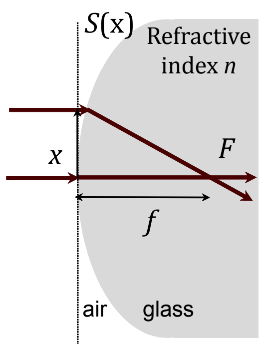
vs



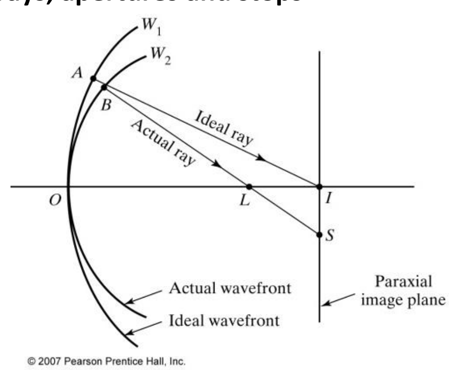


Imaging as a mapping in ray vector spaces

B. From "Perfect" imager to paraxial rays, apertures and stops



"Ideal" lens: aspherical!

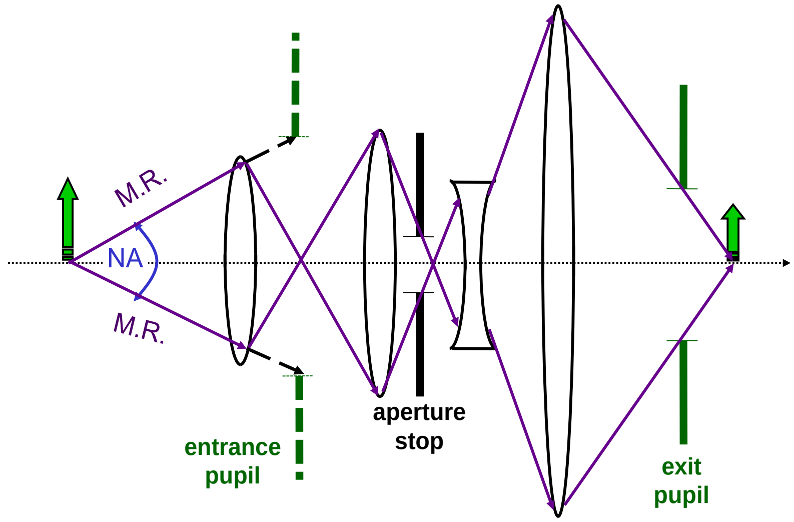


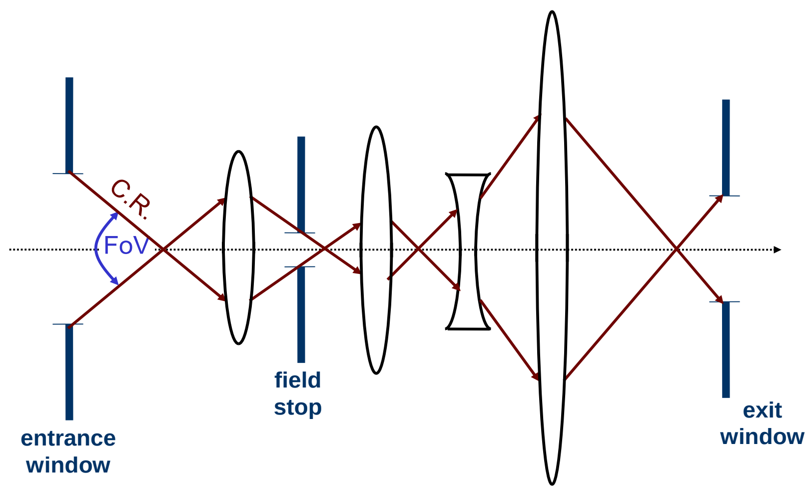
Spherical lens: only good for small θ

Problem: *spherical aberration* ($\cos\theta \approx 1 - \frac{\theta^2}{2} + \frac{\theta^2}{4!} + \cdots$);

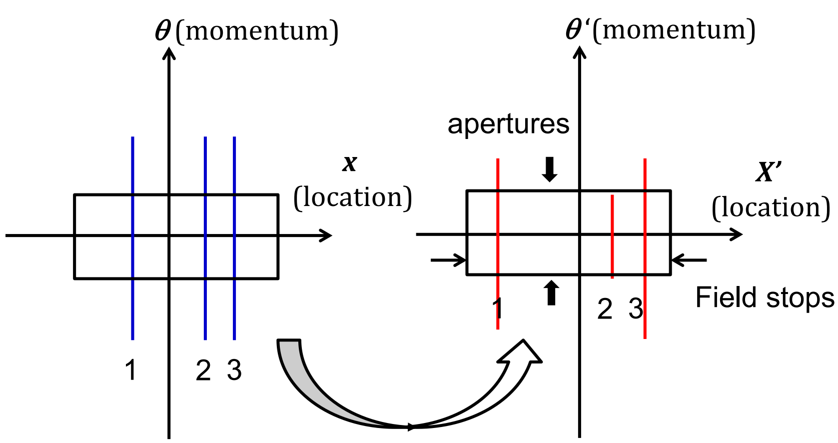
Mitigation: *use proper apertures to reduce NA* (θ_{max})

- ***Effect of Aperture and field stops***

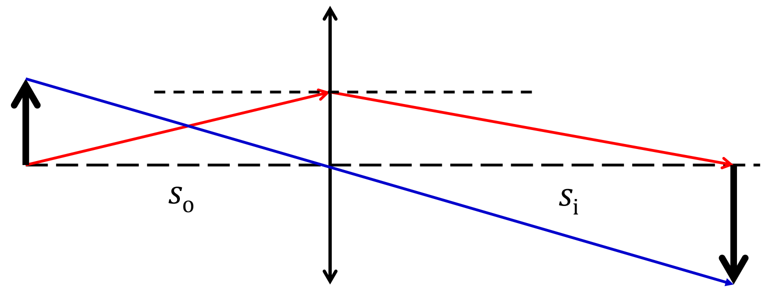




Effect of Apertures and stops



C. Thin Lens and Imaging Condition



Imaging condition:

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

Spatial Magnification:

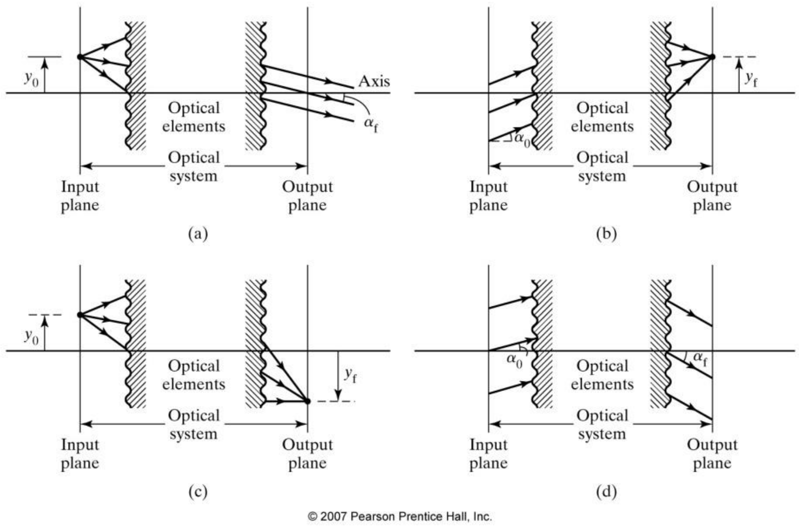
$$M \equiv \frac{h_i}{h_o} = A = 1-\frac{s_i}{f}$$

Angular Magnification:

$$M_\alpha \equiv \frac{\theta_{in}}{\theta_{out}} \approx \frac{x_{in}/s_i}{x_{in}/s_0} = D = 1-\frac{s_o}{f}$$

D. Composite lens and Principal Planes

E. Typical function of composite lens, and corresponding matrix elements



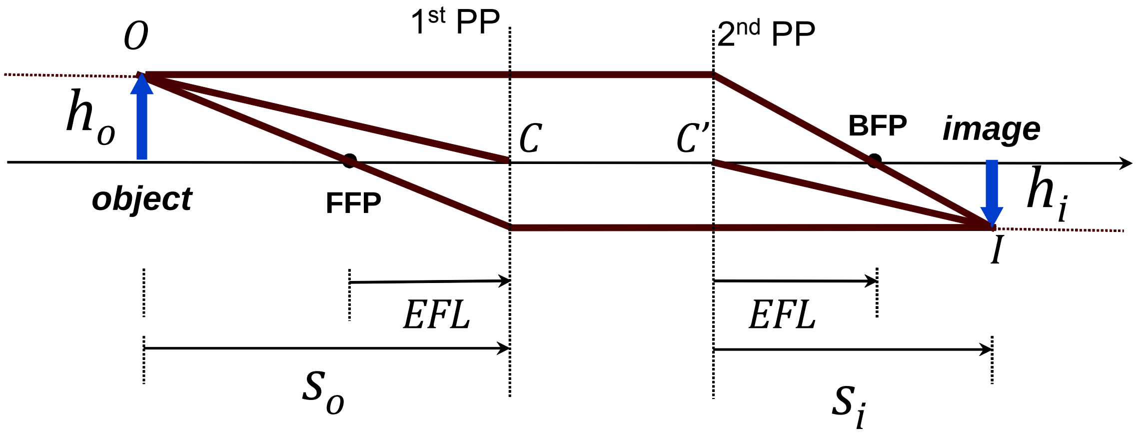
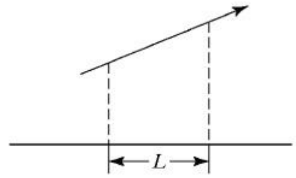
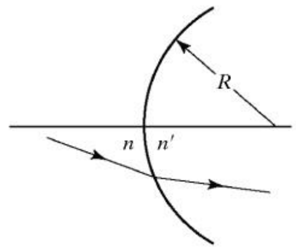
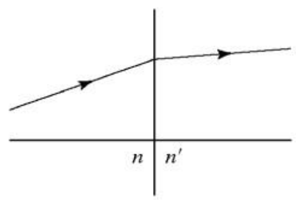
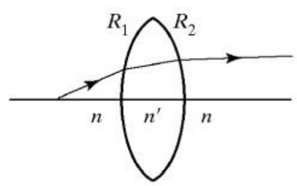
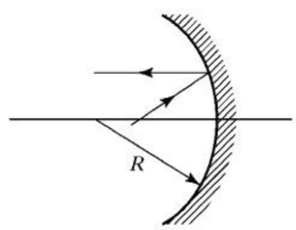
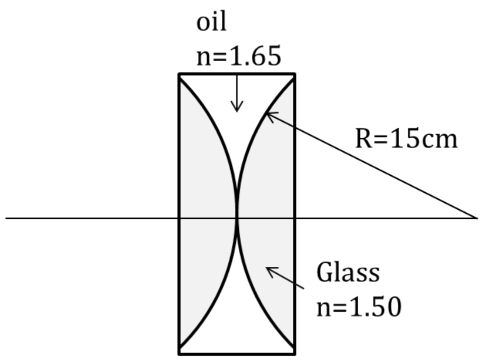

O
1st PP
2nd PP
h_o
C
C'
BFP
image
object
FFP
h_i
I
EFL
EFL
S_o
S_i

TABLE 18-1 SUMMARY OF SOME SIMPLE RAY-TRANSFER MATRICES

Translation matrix:	$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} = \mathfrak{T}$	
Refraction matrix, spherical interface:	$M = \begin{bmatrix} 1 & L \\ \frac{n-n'}{Rn'} & \frac{n}{n'} \end{bmatrix} = \mathfrak{R}$	 $(+R)$: convex $(-R)$: concave
Refraction matrix, plane interface:	$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n}{n'} \end{bmatrix}$	
Thin-lens matrix:	$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$ $\frac{1}{f} = \frac{n'-n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	 $(+f)$: convex $(-f)$: concave
Spherical mirror matrix:	$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$	 $(+R)$: convex $(-R)$: concave

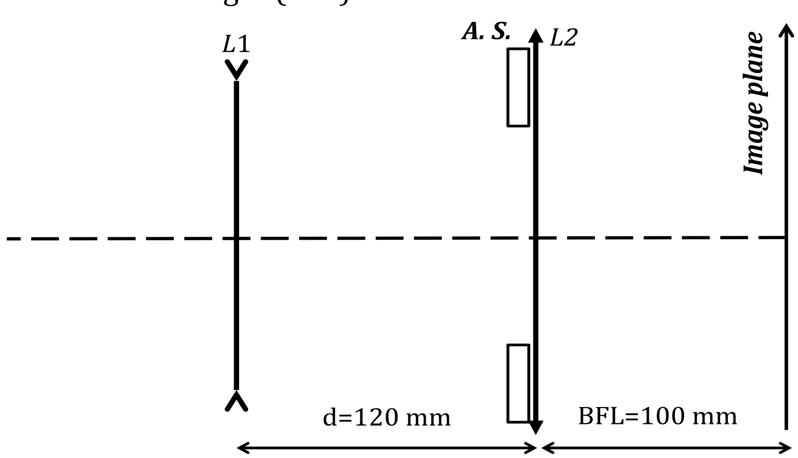
Note: when prisms and mirrors are used in the optical train, consider "unfold" the optical axis first!

Example 1: Two identical, thin, plano-convex lenses with radii of curvature of 15cm are situated with their curved surfaces in contact at their centers. The intervening space is filled with oil, of refractive index n=1.65. The index of the glass is n=1.50. Determine the focal length of the combination.



Example 2: A Retrofocus Lens. For an object placed at infinity, we need to design a composite lens system with the following specifications:

- The spacing from the front lens to the rear lens is 120 mm.
- The working distance (from the rear lens to image plane) is 100 mm.
- The effective focal length (EFL) is 60mm.



a) Assuming all elements are thin lenses, determine the focal length of each individual lens.

b) Locate the principal planes of the lens system.

c) The aperture *stop* (A. S.) of this system is located at the rear lens. In order for the system to operate at Numerical Aperture =1/8, what should be the diameter of the aperture?