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2.72 Elements of Mechanical Design  
Spring 2009

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*2.72*

*Elements of  
Mechanical Design*

*Lecture 02: Review*

# *Intent*

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*High-level review of undergrad material as applied to engineering decision making*

*NOT an ME “redo” or a “how to” recitation*

# *Import*

*Main goal of 2.72 is to teach  
you how to integrate past  
knowledge to engineer a system*

Given this, how do I engineer a mechanical system?  
modular → simple → complex → system

(2.001, 2.002) (2.003, 2.004) (2.005, 2.006) (2.007, 2.008)



# *Impact*

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*Understand why & how we will  
use parts of ME core knowledge*

*Problem set → Engineering*

# *Future help*

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*We can't use lecture time to  
redo the early curriculum*

*BUT*

*We are HAPPY to help outside  
of lecture IF you've tried*

# Schedule and reading assignment

## Reading quiz

## Changing from sponge to active mode

### Lecture

Mechanics  
Dynamics  
Heat transfer  
Matrix math

### Hands-on

Mechanics  
Dynamics  
Heat transfer  
Matrix math



## Reading assignment

### □ Shigley/Mischke

- *Sections 4.1–4.5: 08ish pages & Sections 5.1–5.5: 11ish pages*
- *Pay special attention to examples 4.1, 4.4, 5.3 , and 5.4*

# *Mechanics*

# Free body diagrams

## Useful for:

- ❑ Equilibrium
- ❑ Stress, deflection, vibration, etc...

$$\Sigma \vec{F} = 0 = m\vec{a}$$

$$\Sigma M = 0 = I\vec{\alpha}$$

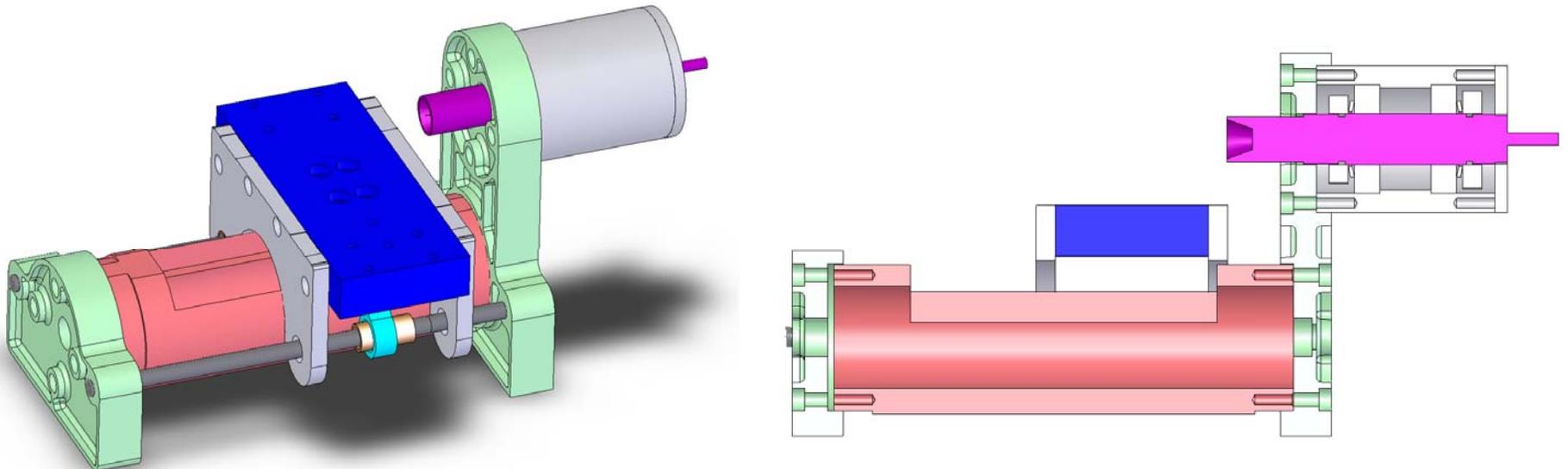
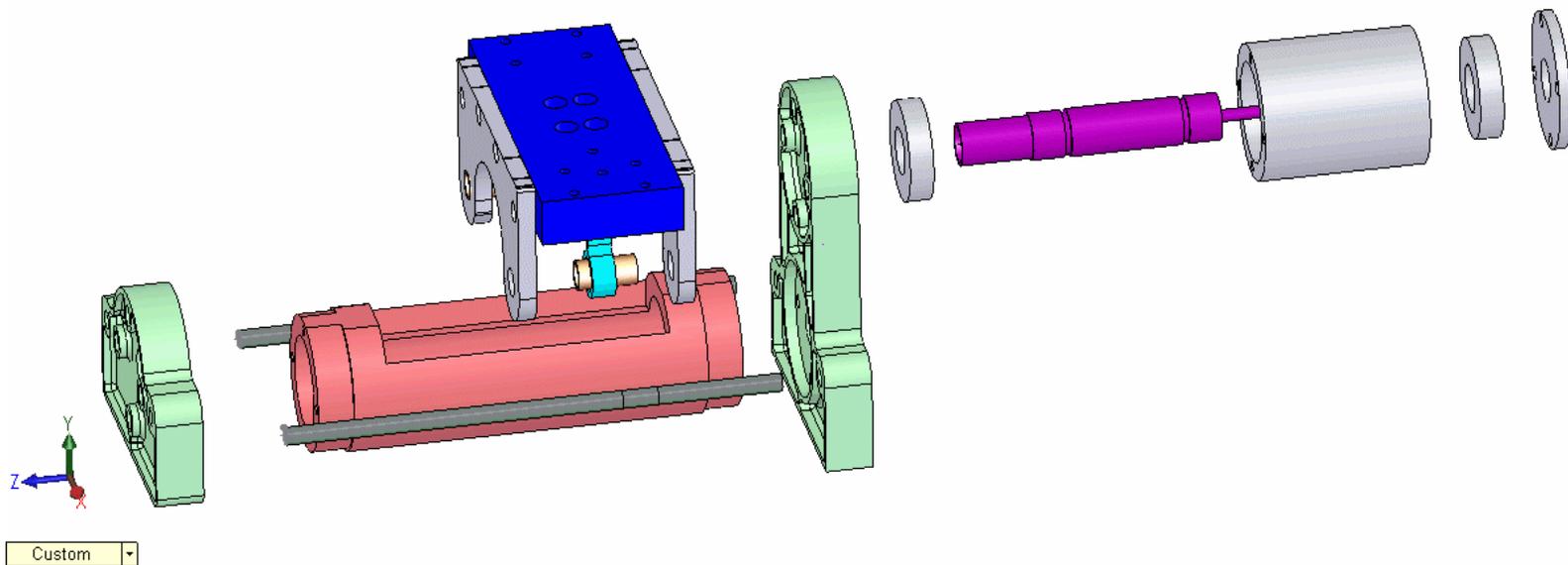
## Why do we ALWAYS use free body diagrams?

- ❑ Communication
- ❑ Thought process
- ❑ Documentation

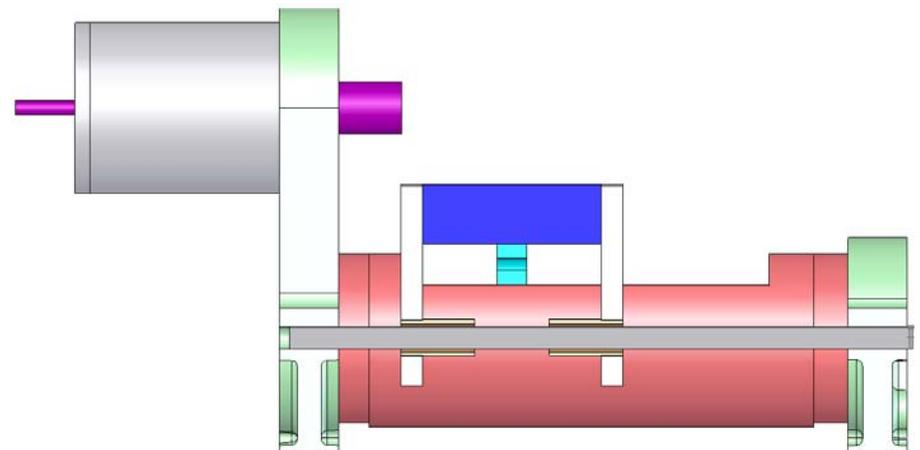
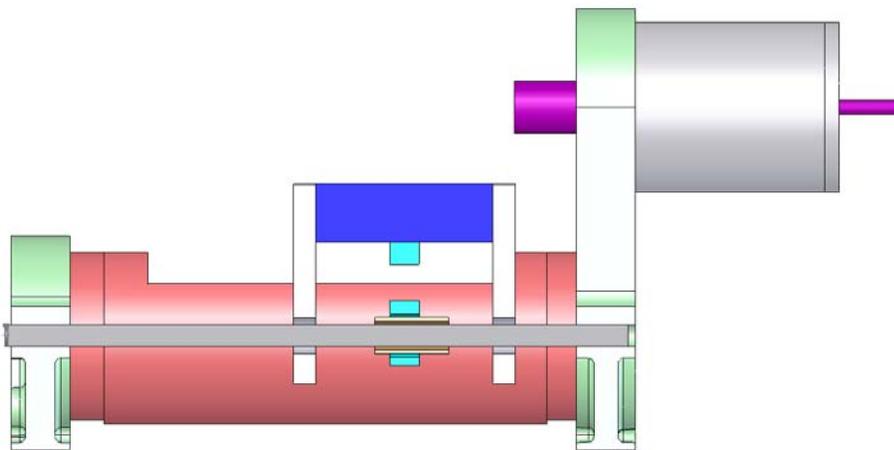
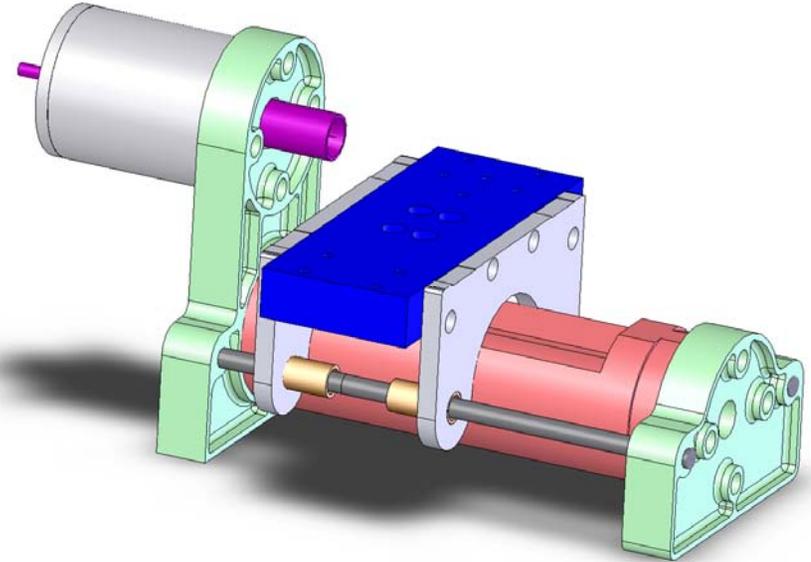
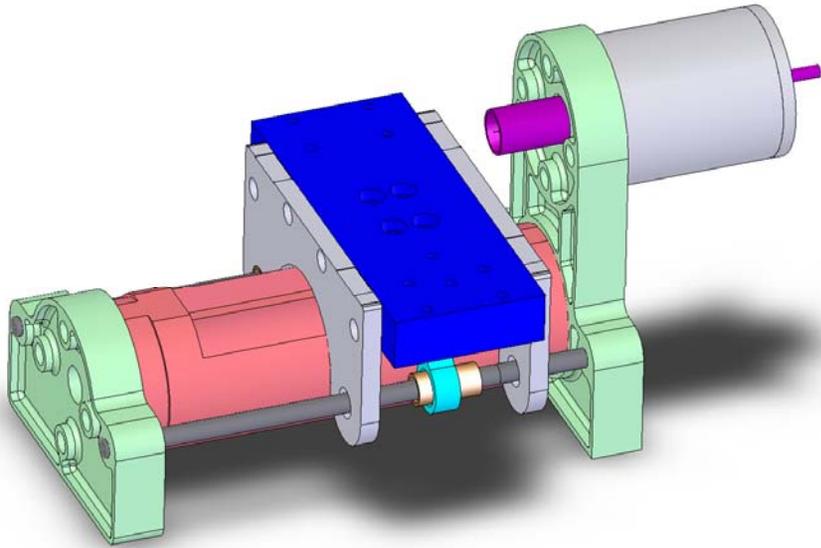
## How will we use free body diagrams?

- ❑ We are dealing with complex systems
- ❑ We will break problem into modules
- ❑ We will model, simulate and analyze mechanical behavior
- ❑ Integrate individual contributions to ascertain system behavior

# Free body diagrams: Bearings/rails

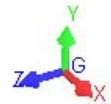
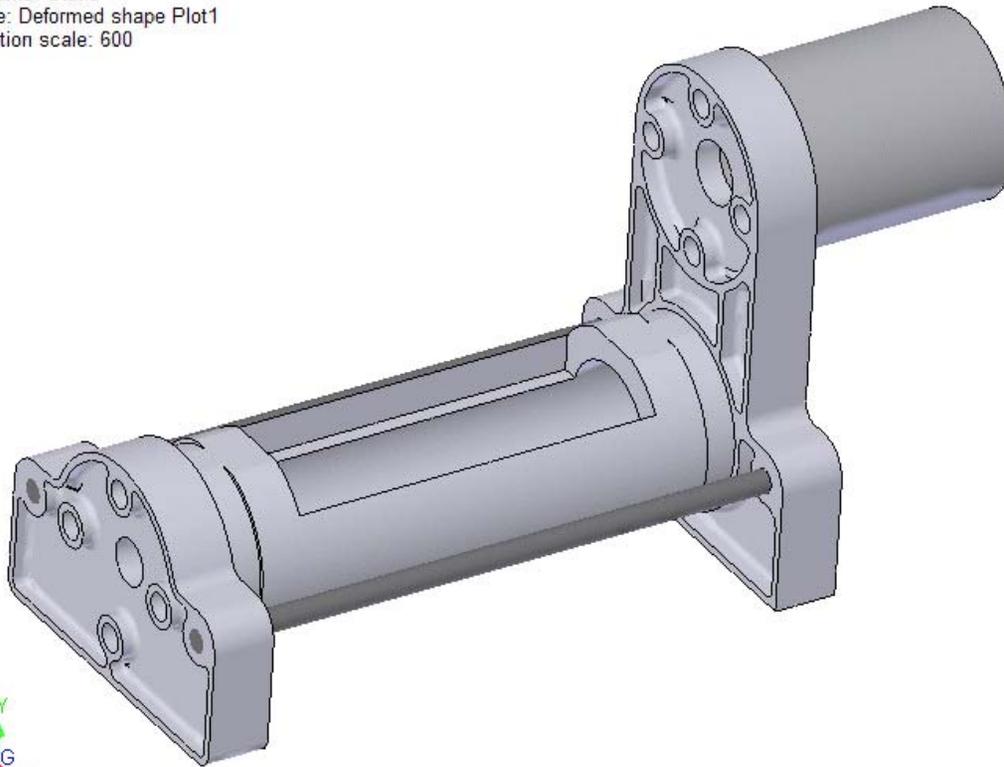


# Free body diagrams: Bearings/rails

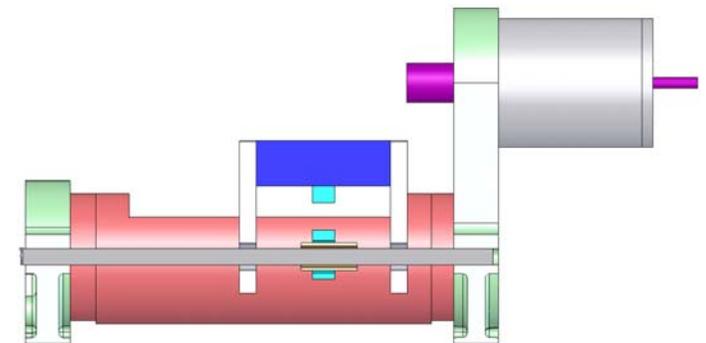
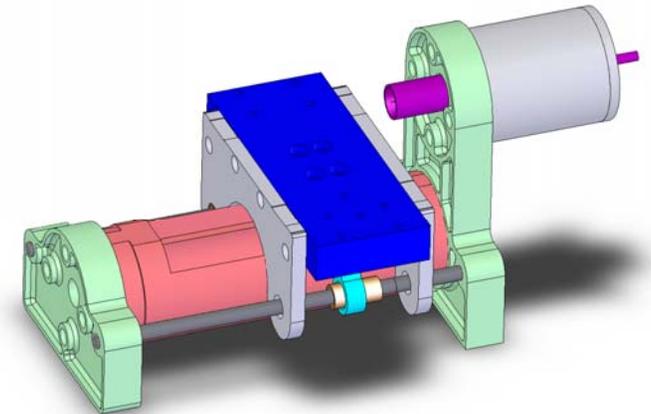


# Static: Head stock deformation

Model name: Lathe\_structure\_dynamics\_example  
Study name: Static  
Plot type: Deformed shape Plot1  
Deformation scale: 600

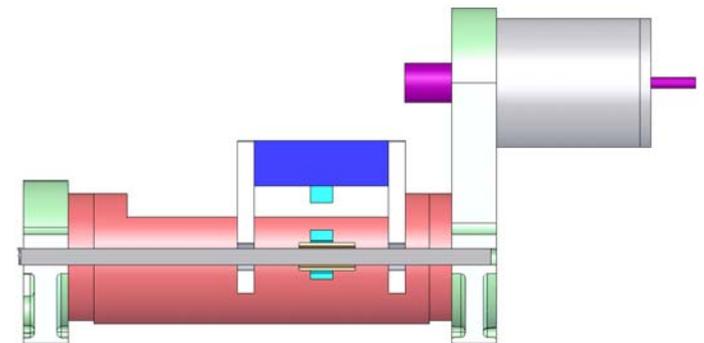
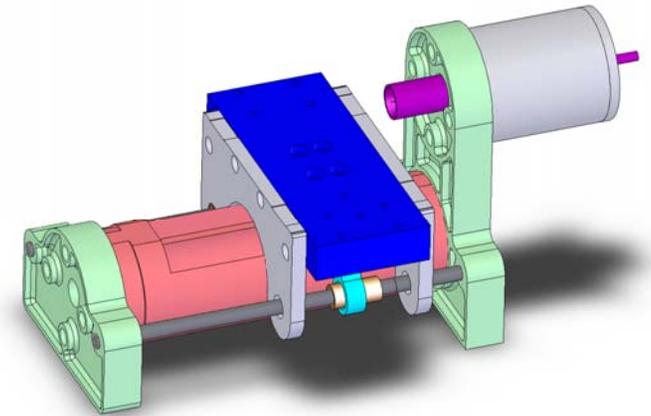
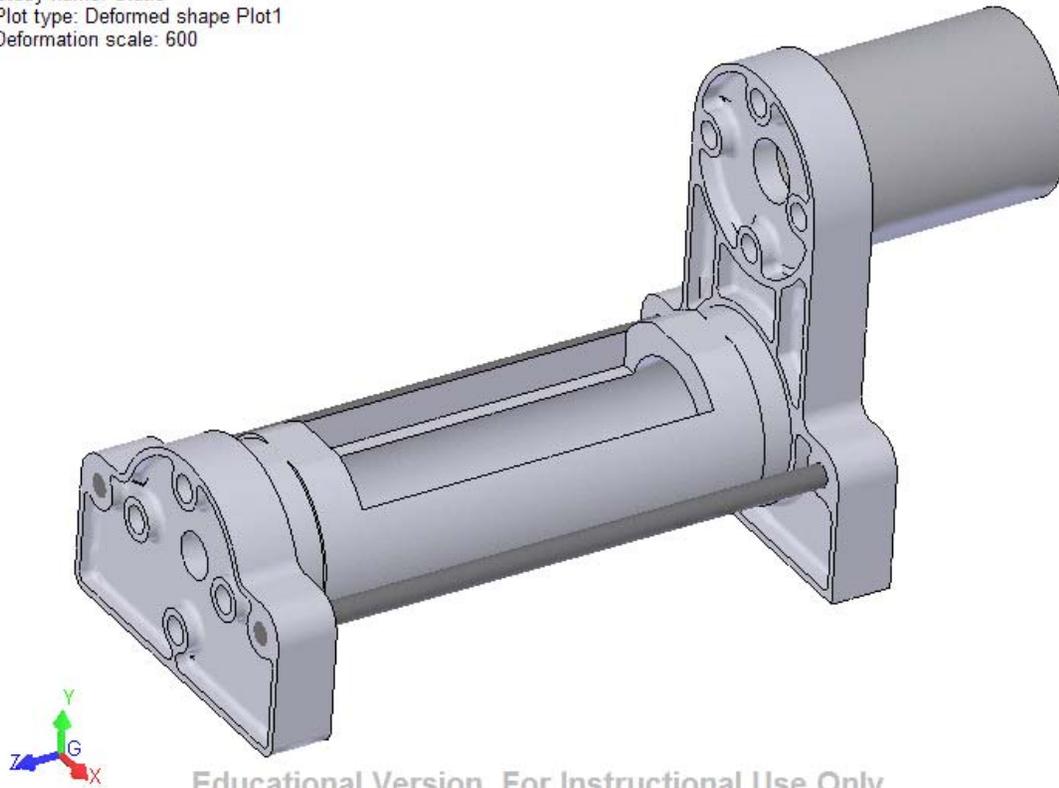


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# Static: Rail deformation

Model name: Lathe\_structure\_dynamics\_example  
Study name: Static  
Plot type: Deformed shape Plot1  
Deformation scale: 600



# Example 1: $0 < x < a$

## Cantilever

- Forces, moments, & torques

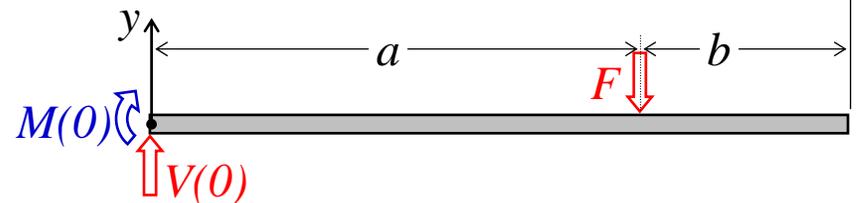
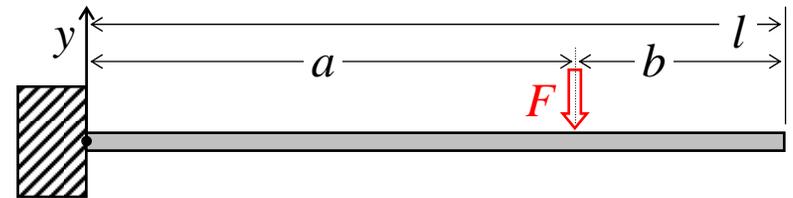
## Why do we care?: Stress

- Shear & normal
- Static failure
- Fatigue failure

## Why do we care?: Stiffness

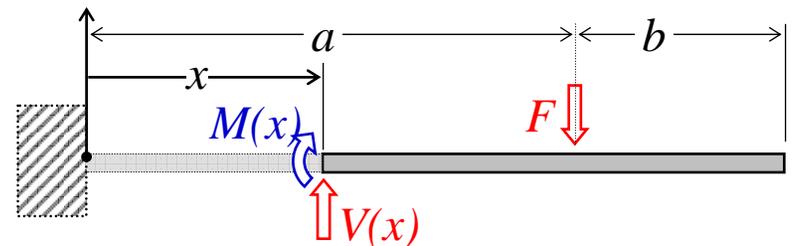
- Displacement
- Rotation
- Vibration  $\rightarrow (k/m)^{1/2}$

But, ends aren't all that matters



$$+\uparrow \sum \bar{F} = 0 = -F + V(0) \rightarrow V(0) = V(x) = F$$

$$\curvearrowleft + \sum \bar{M} = 0 = +F \cdot l + M(0) \rightarrow M(0) = -F \cdot a$$



$$+\uparrow \sum \bar{F} = 0 = -F + V(0) \rightarrow V(0) = V(x) = F$$

$$\curvearrowleft + \sum \bar{M} = 0 = +F \cdot (a - x) + M(x) \rightarrow M(x) = -F \cdot (a - x)$$

# Example 1: $0 < x < a$

But, ends aren't all that matters

## Relating $V(x)$ & $M(x)$

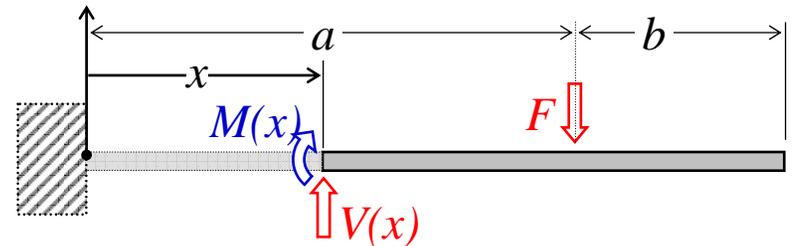
$$\square V(x) = F$$

$$\square M(x) = F \cdot (x - a)$$

$$\square V(x) = \frac{d}{dx} M(x)$$

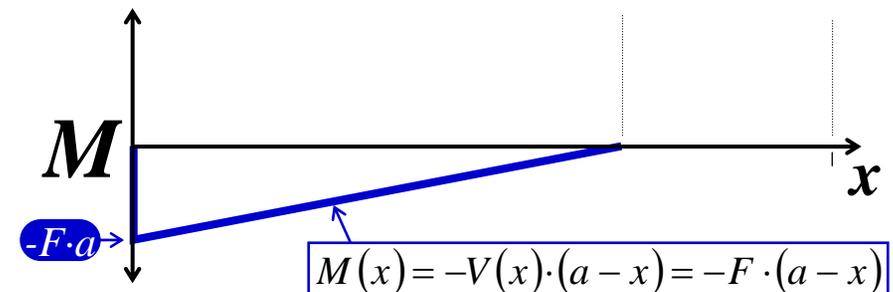
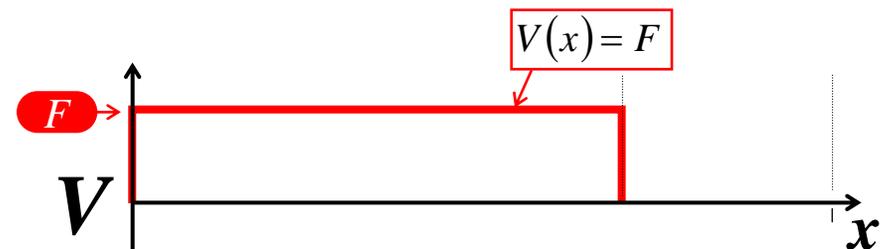
## Shear moment diagrams

- Solve statics equation
- Put point of interest on plots
- Use  $V = dM/dx$  to generate M plot
- Master before spindle materials



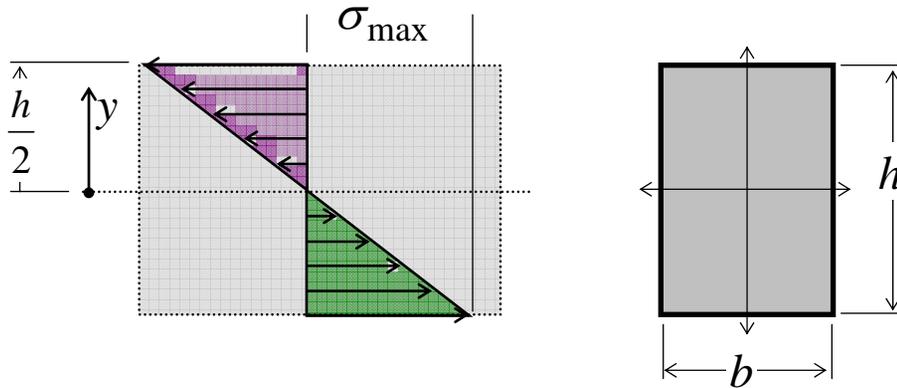
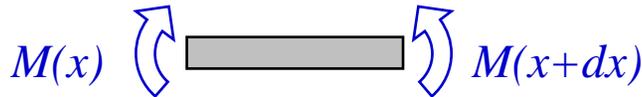
$$+\uparrow \Sigma \bar{F} = 0 = -F + V(0) \rightarrow V(0) = V(x) = F$$

$$\curvearrowright \Sigma \bar{M} = 0 = +F \cdot (a - x) + M(x) \rightarrow M(x) = -V(x) \cdot (a - x)$$



# Example 1: $0 < x < a$

## Stress

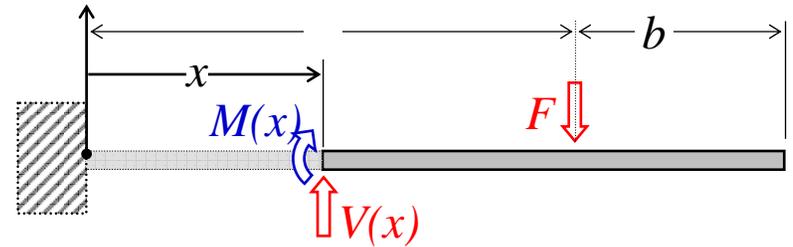


$$I(x) = \frac{1}{12} b(x) [h(x)]^3$$

$$\sigma(y) = M \frac{y}{I(x)} \rightarrow \sigma_{\max} = M \frac{c}{I(x)}$$

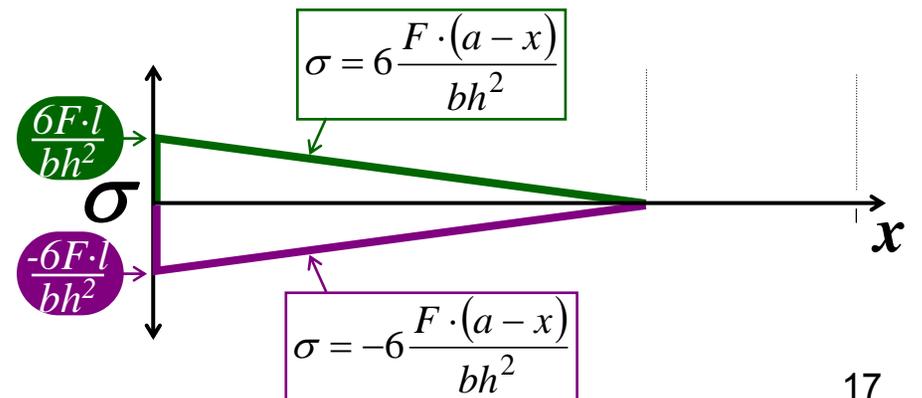
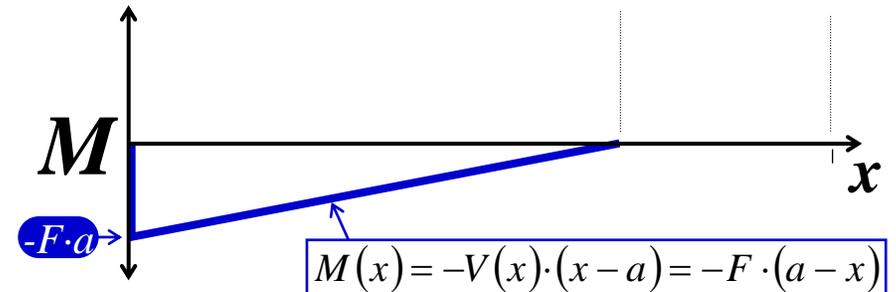
$$|\sigma_{\max}| = F \cdot (a - x) \frac{h(x)}{2} \frac{12}{b(x) \cdot [h(x)]^3}$$

$$|\sigma_{\max}| = 6 \frac{F \cdot (a - x)}{b(x) \cdot [h(x)]^2}$$

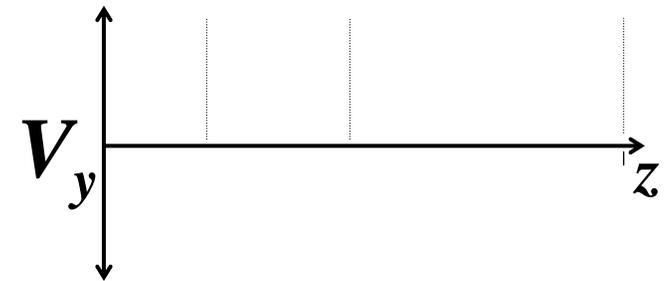
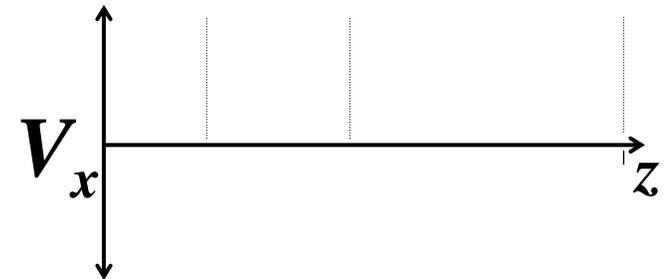
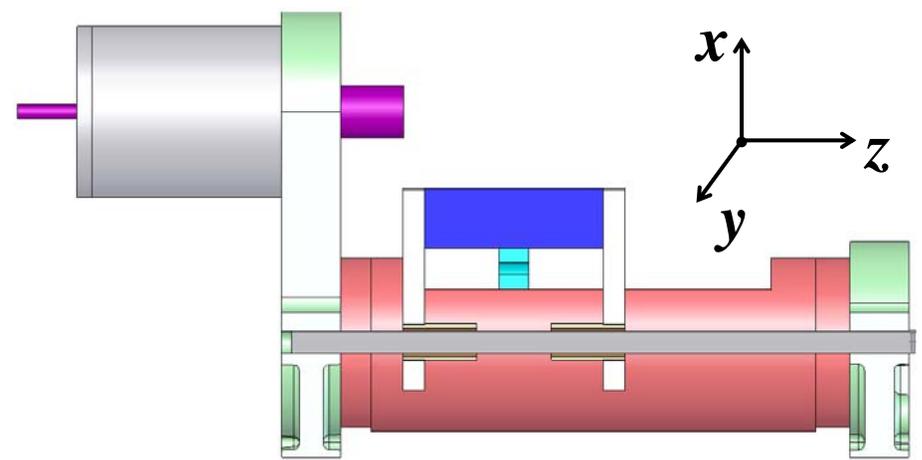
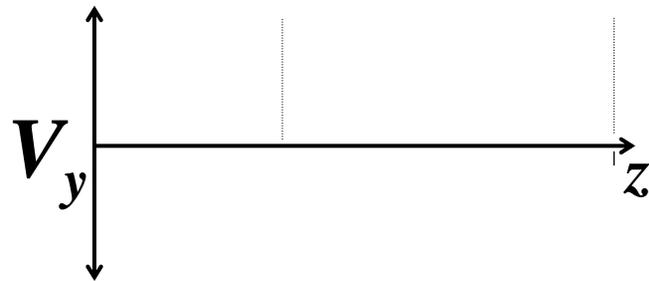
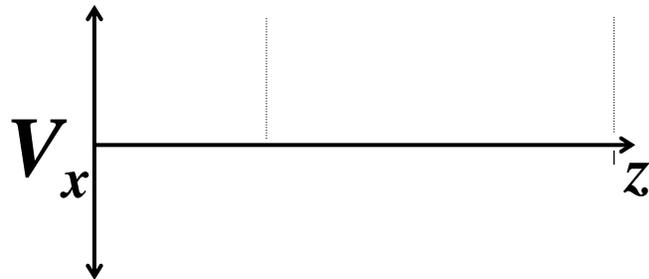
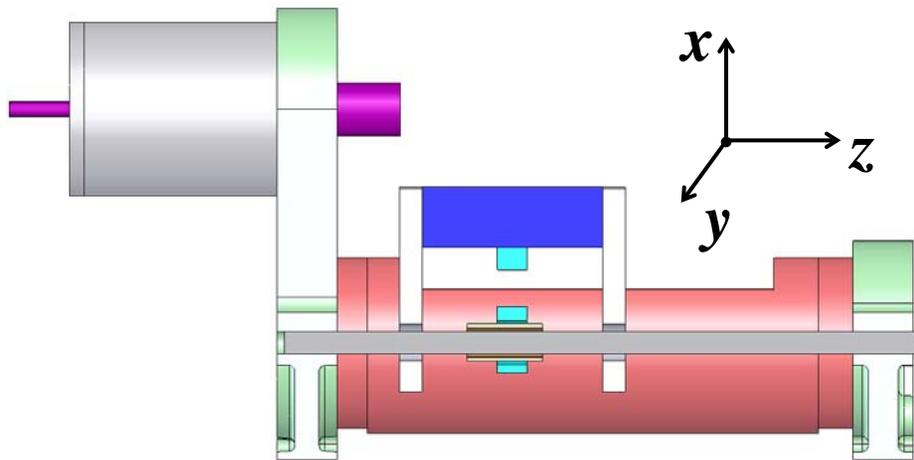


$$+\uparrow \Sigma \vec{F} = 0 = -F + V(0) \rightarrow V(0) = V(x) = F$$

$$\curvearrowright \Sigma \vec{M} = 0 = +V(x) \cdot (a - x) - M(x) \rightarrow M(x) = -V(x) \cdot (a - x)$$



# Group work: Generate strategy for this...



*Dynamics*

# Vibration

## Vibration principles

- ❑ Exchange potential-kinetic energy
- ❑ 2nd order system model

## Blocks and squiggles...

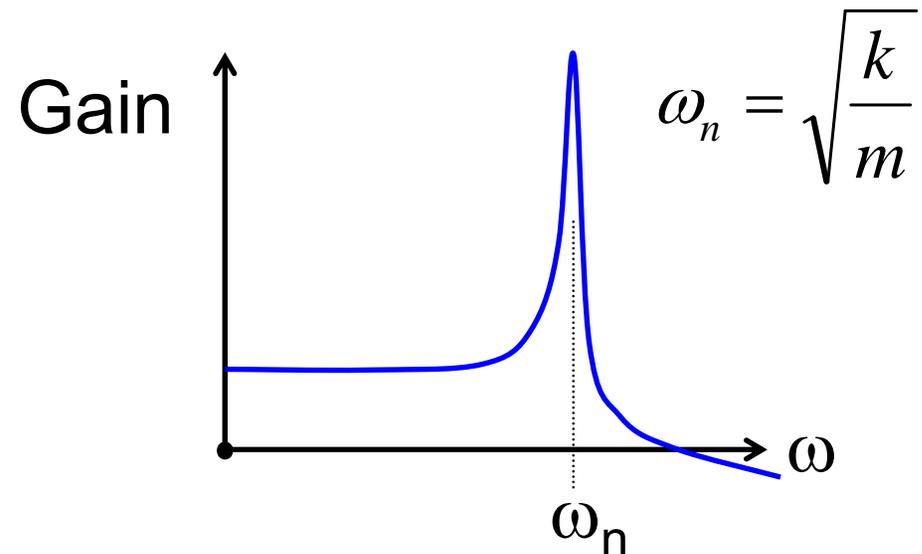
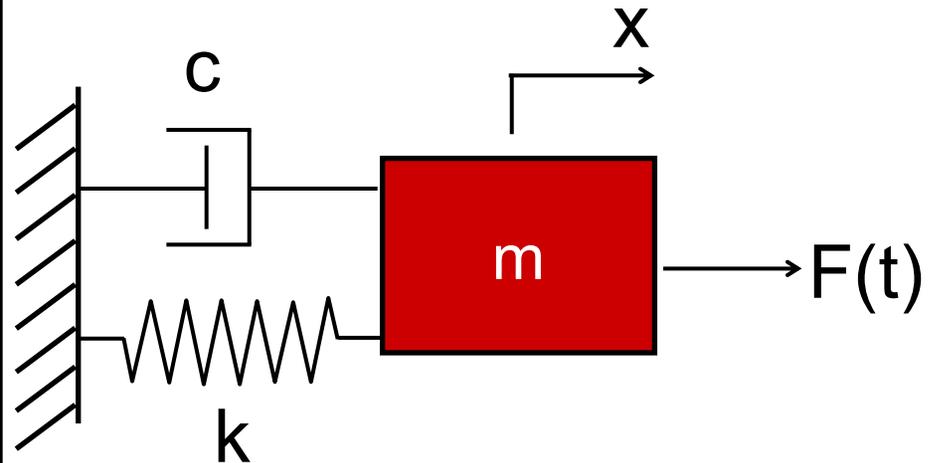
- ❑ What do they really mean?
- ❑ Why are they important?
- ❑ How will we apply this?

## Multi-degree-of-freedom system

- ❑ Mode shape
- ❑ Resonant frequency

## Estimate $\omega_n$ (watch units) for:

- ❑ A car suspension system



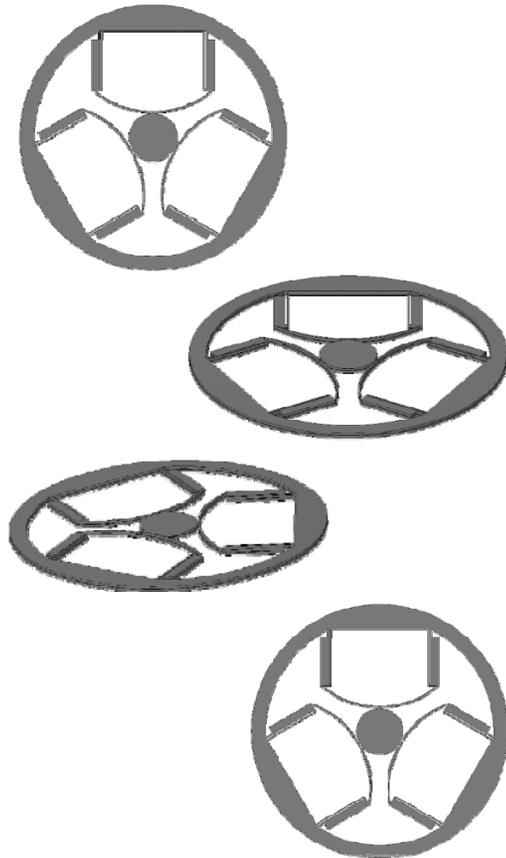
Images removed due to copyright restrictions. Please see:

[http://www.hpiracing.com/graphics/kits/547/\\_MG\\_1962e.jpg](http://www.hpiracing.com/graphics/kits/547/_MG_1962e.jpg)

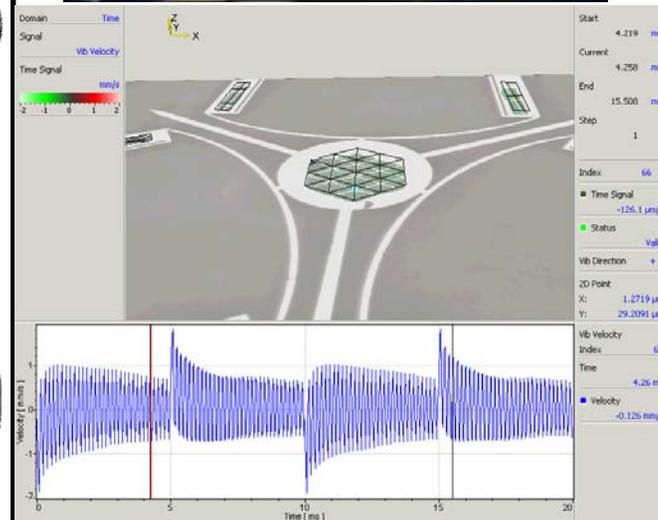
[http://www.societyofrobots.com/images/mechanics\\_suspension\\_honda.gif](http://www.societyofrobots.com/images/mechanics_suspension_honda.gif)

[http://www.bose.com/images/learning/lc\\_susp\\_frontmodule.jpg](http://www.bose.com/images/learning/lc_susp_frontmodule.jpg)

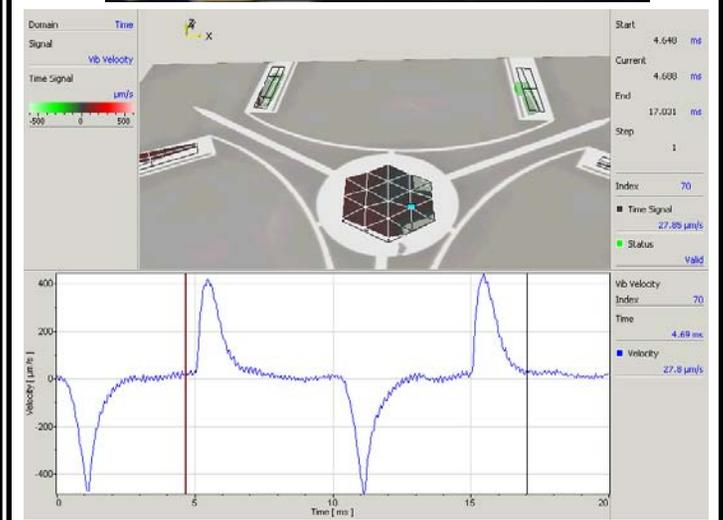
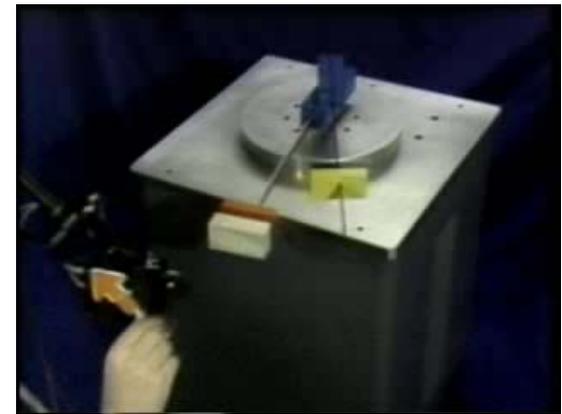
# Vibration: MEMS device behavior



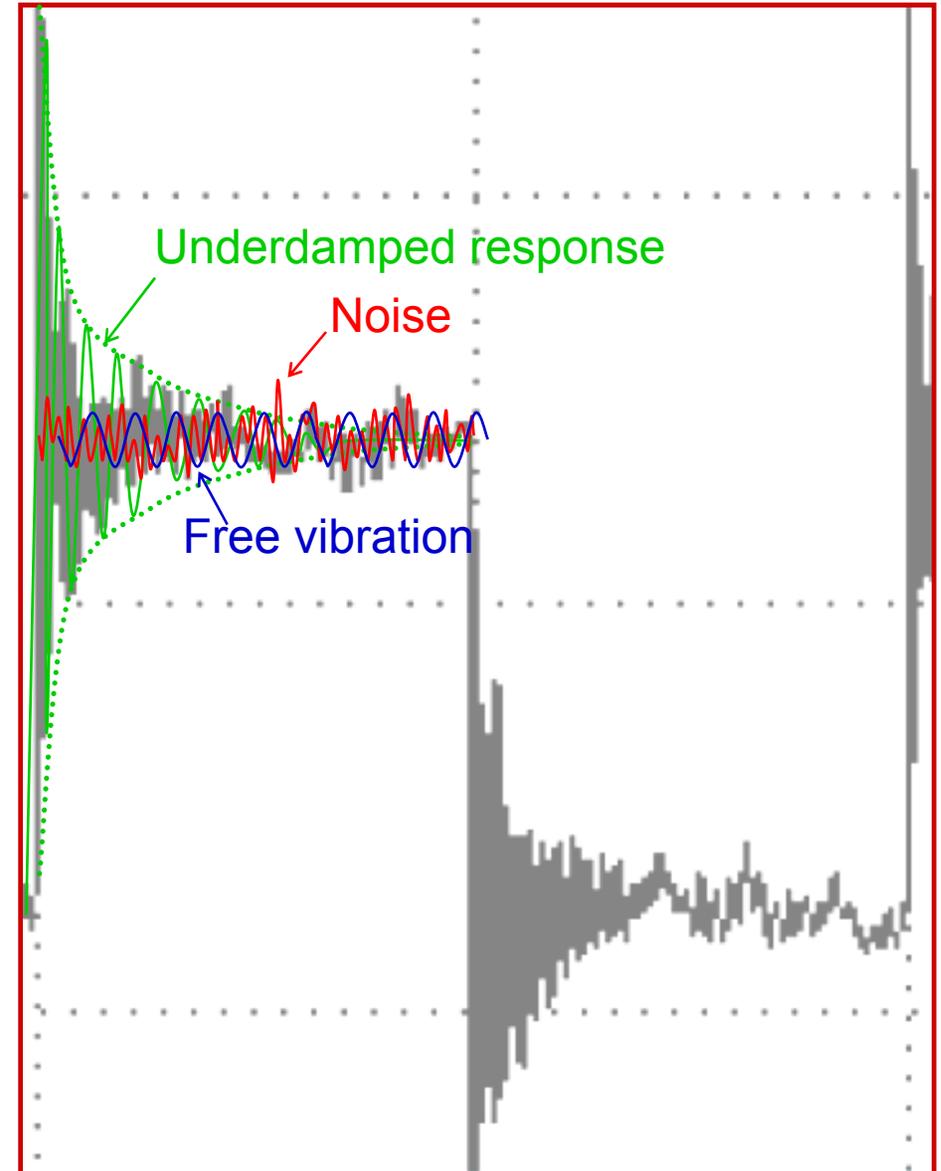
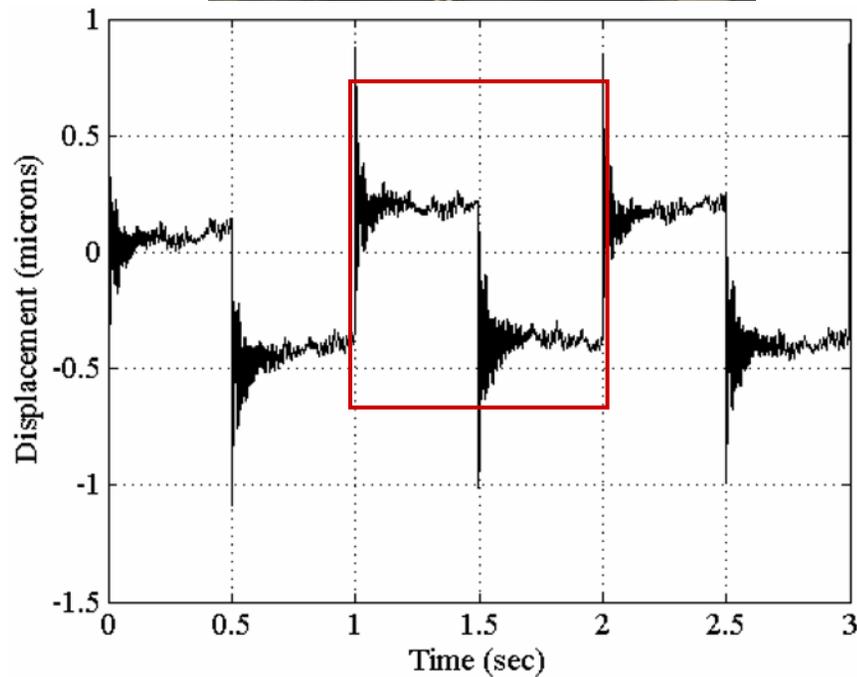
Without input shaping



With input shaping



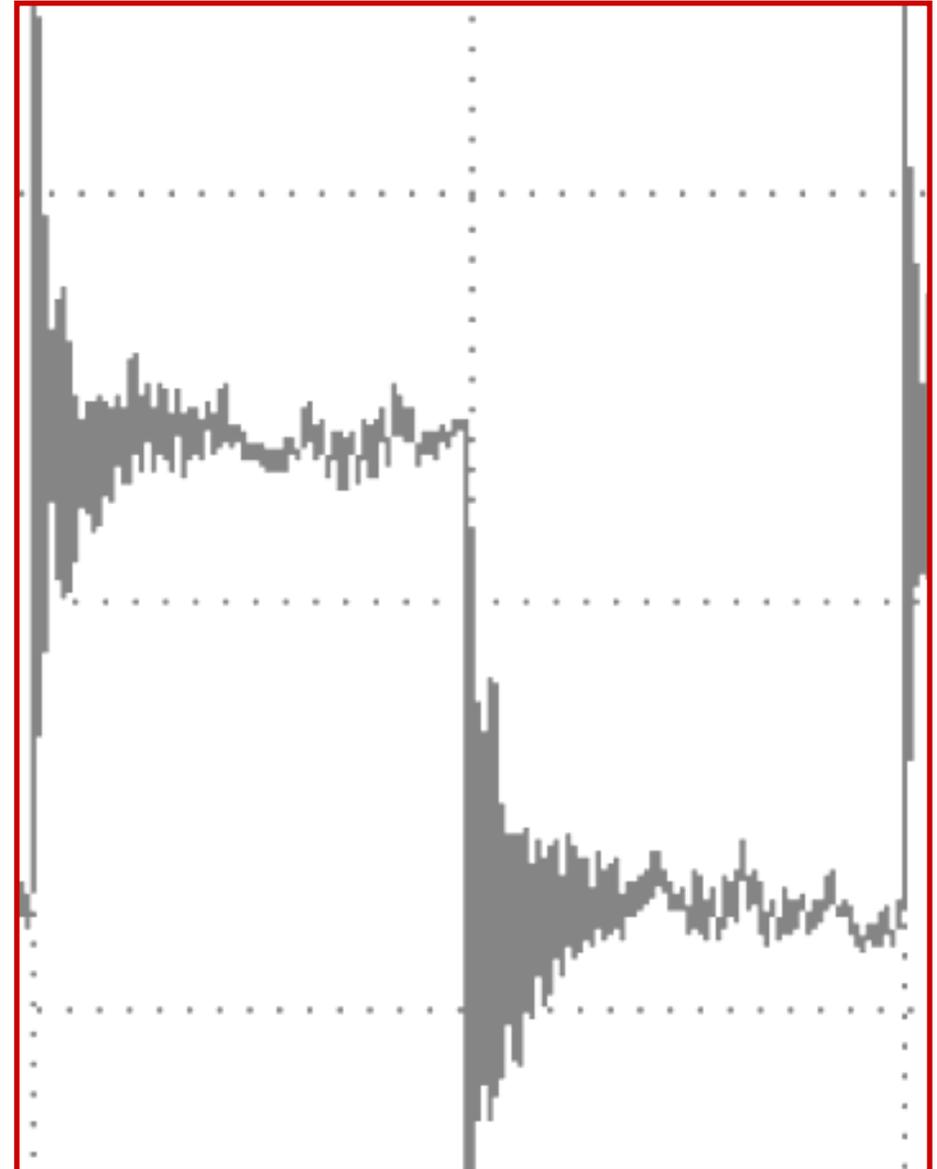
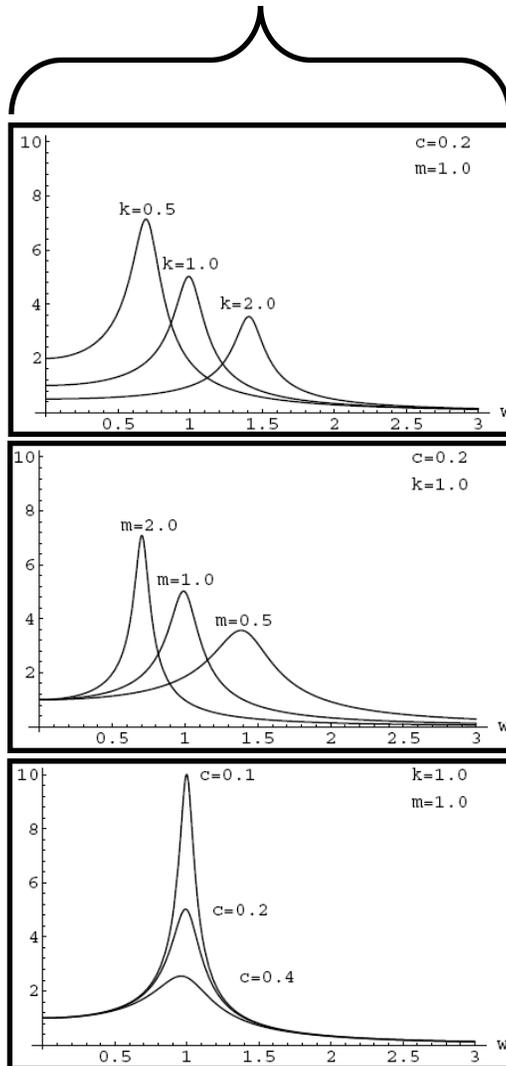
# Vibrations: Meso-scale device behavior



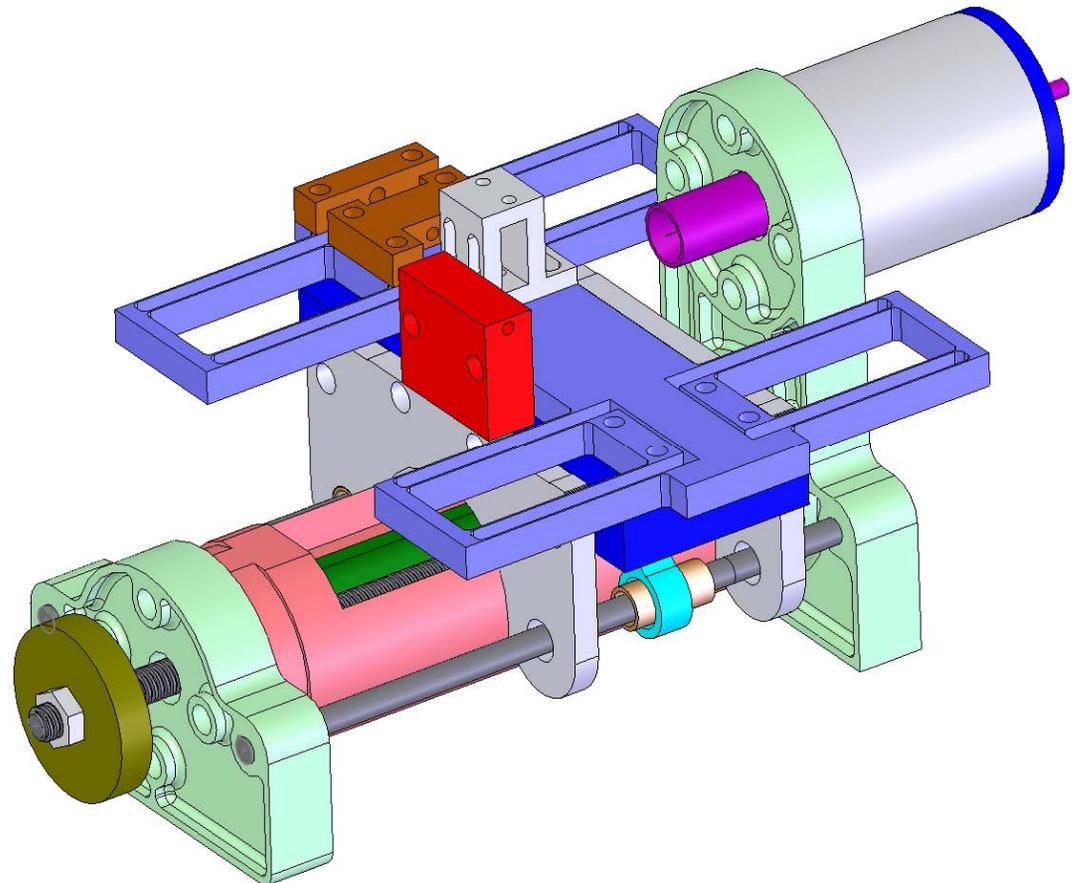
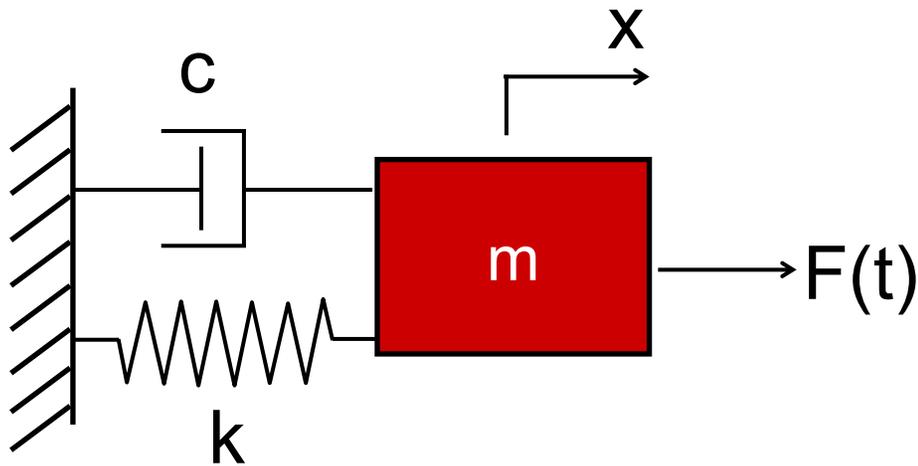
Golda, D. S., "Design of High-Speed, Meso-Scale Nanopositioners Driven by Electromagnetic Actuators," Ph.D. Thesis, Massachusetts Institute of Technology, 2008.

# Vibrations: Reducing amplitude...

How to change  
 $m$ ,  $k$ , and  $c$ ?

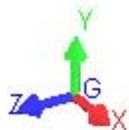
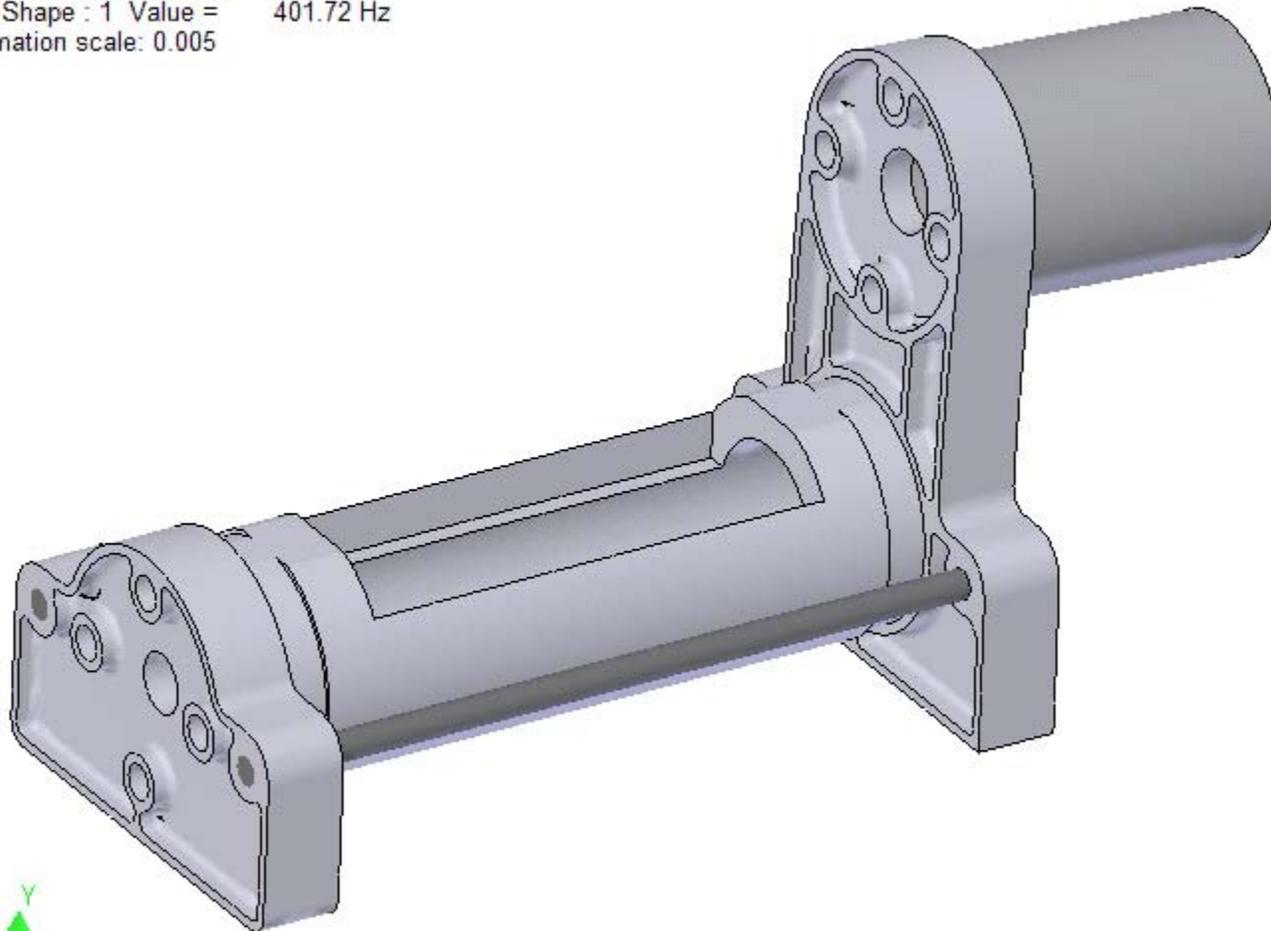


# So... where find in reality... in lathe...



# Vibration: Lathe structure – 1<sup>st</sup> mode

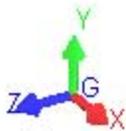
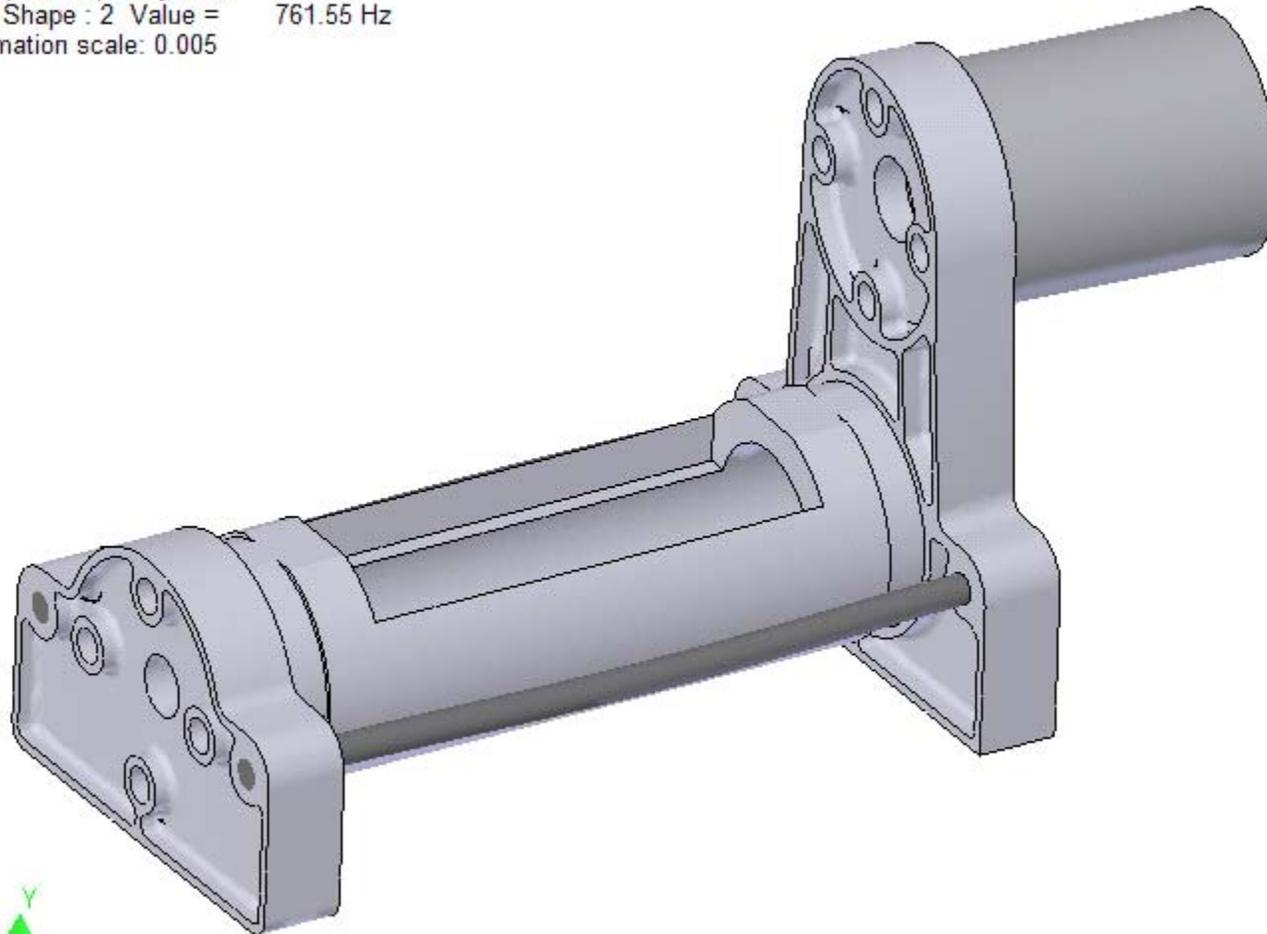
Model name: Lathe\_structure\_dynamics\_example  
Study name: Study 1  
Plot type: Frequency Plot1  
Mode Shape : 1 Value = 401.72 Hz  
Deformation scale: 0.005



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# Vibration: Lathe structure – 2<sup>nd</sup> mode

Model name: Lathe\_structure\_dynamics\_example  
Study name: Study 1  
Plot type: Frequency Plot1  
Mode Shape : 2 Value = 761.55 Hz  
Deformation scale: 0.005



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# *Heat transfer*

# Thermal growth errors

For uniform temperature

$$\Delta L = \alpha L_o \Delta T$$

$$STEEL : 12L14 \rightarrow \Delta L = 11.5 \times 10^{-6} \frac{m}{m^{\circ}C} L_o \Delta T$$

$$ALUMINUM : 6061 T6 \rightarrow \Delta L = 23.6 \times 10^{-6} \frac{m}{m^{\circ}C} L_o \Delta T$$

$$POLYMER : Delrin \rightarrow \Delta L = 100 \times 10^{-6} \frac{m}{m^{\circ}C} L_o \Delta T$$

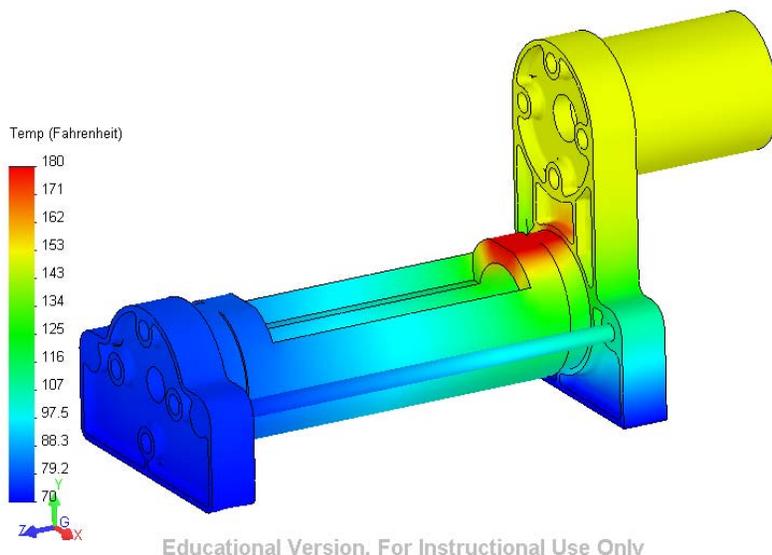
# Convection and conduction

## Convection:

$$\dot{q} = h A_{surface} (T - T_{\infty})$$

## Why do we care?

- ❑ Heat removal from cutting zone
- ❑ Heat generation in bearings
- ❑ Thermal growth errors



## Conduction:

$$\dot{q} = k A_{cross} \frac{dT}{dx}$$

## Why do we care?

- ❑ Heat removal from cutting zone
- ❑ Heat generation in bearings
- ❑ Thermal growth errors

## Common k values to remember

- ❑ Air 0.026 W /(m°C)
- ❑ 12L14 51.9 W /(m°C)
- ❑ 6061 T6 167 W /(m°C)

# Thermal resistance

## Thermal resistance

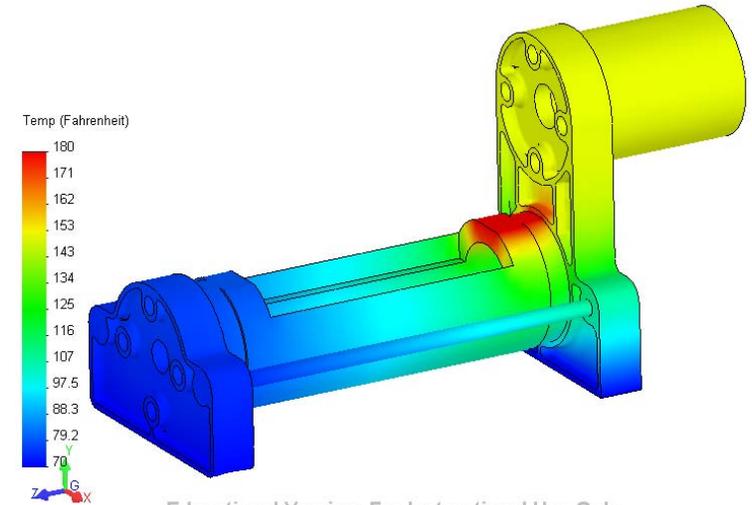
$$\dot{q} = \frac{\Delta T}{R_T}$$

- Convection

$$\dot{q} = \frac{(T - T_\infty)}{(h A_{surface})^{-1}} \mapsto R_T = \frac{1}{h A_{surface}}$$

- Conduction

$$\dot{q} = dT \frac{k A_{cross}}{dx} \mapsto R_T = \frac{dx}{k A_{cross}}$$

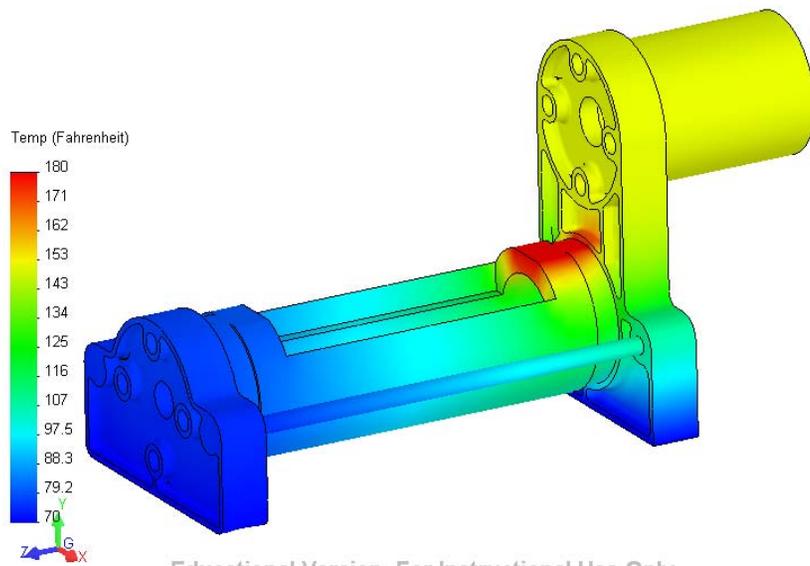


# Biot (Bi) number

Ratio of convective to conductive heat transfer

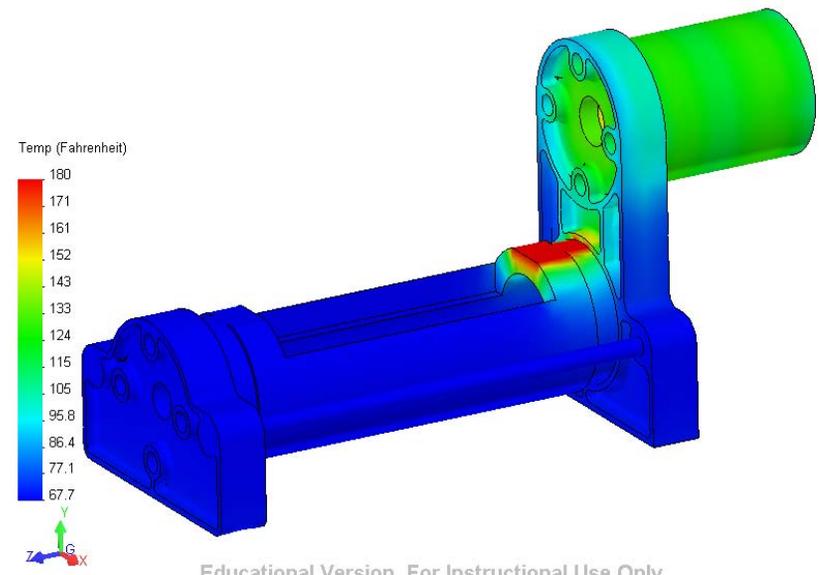
$$\frac{\dot{q}_{convection}}{\dot{q}_{conduction}} \mapsto \frac{(h A \Delta T) \Big|_{convection}}{\left( k A \frac{\Delta T}{L} \right) \Big|_{conduction}} \mapsto Bi = \frac{h L_c}{k}$$

Why do we care?



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Low Bi



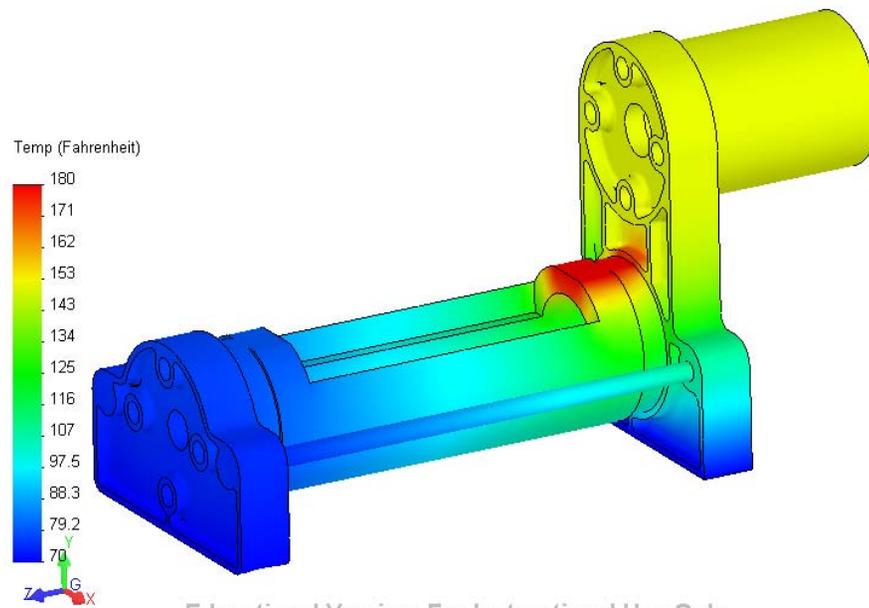
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High Bi

# Example of thermal errors

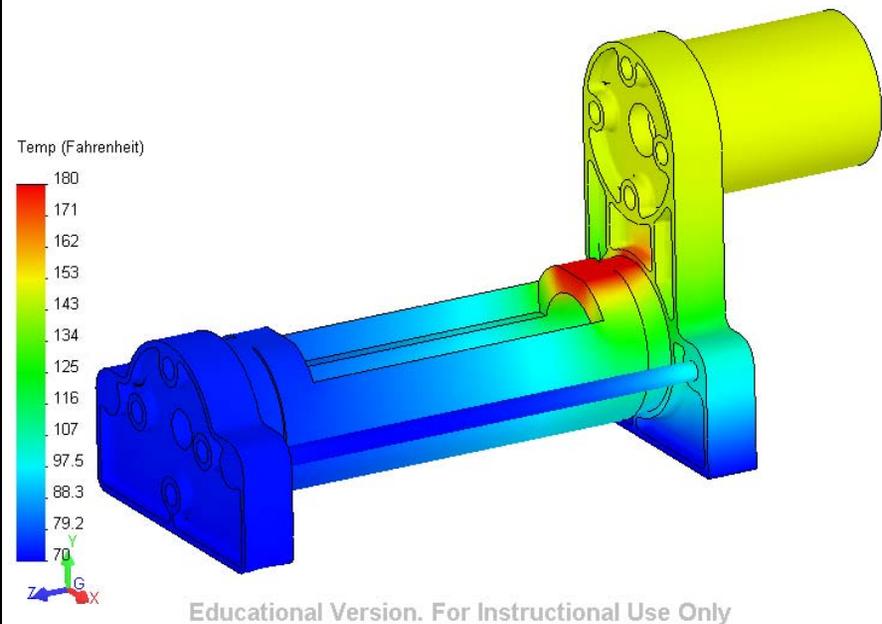
For:

- $h = 0.1 \text{ W}/(\text{m}^2\text{°C})$
- Bearing  $T = 150 \text{ °F}$
- Chip  $T = 180 \text{ °F}$



For:

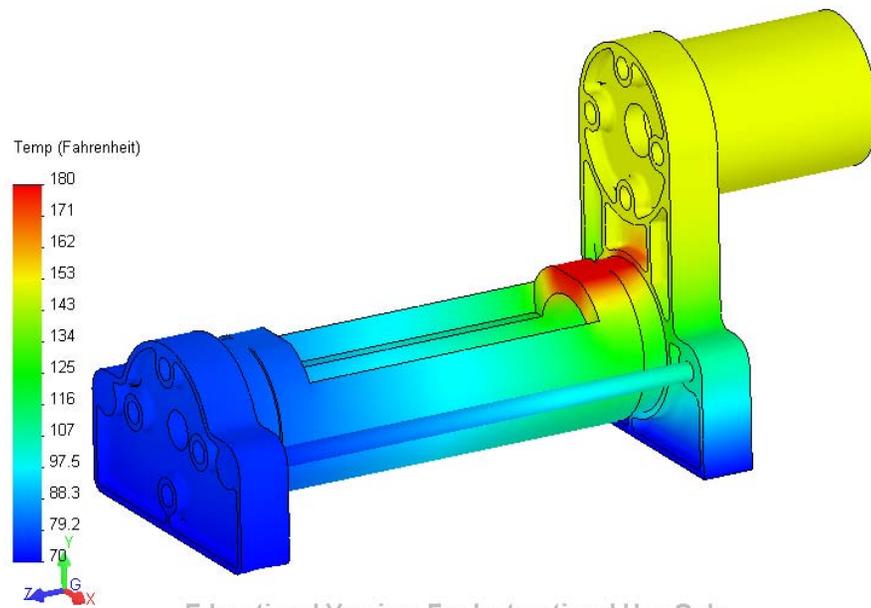
- $h = 50 \text{ W}/(\text{m}^2\text{°C})$
- Bearing  $T = 150 \text{ °F}$
- Chip  $T = 180 \text{ °F}$



# Example of thermal errors

For:

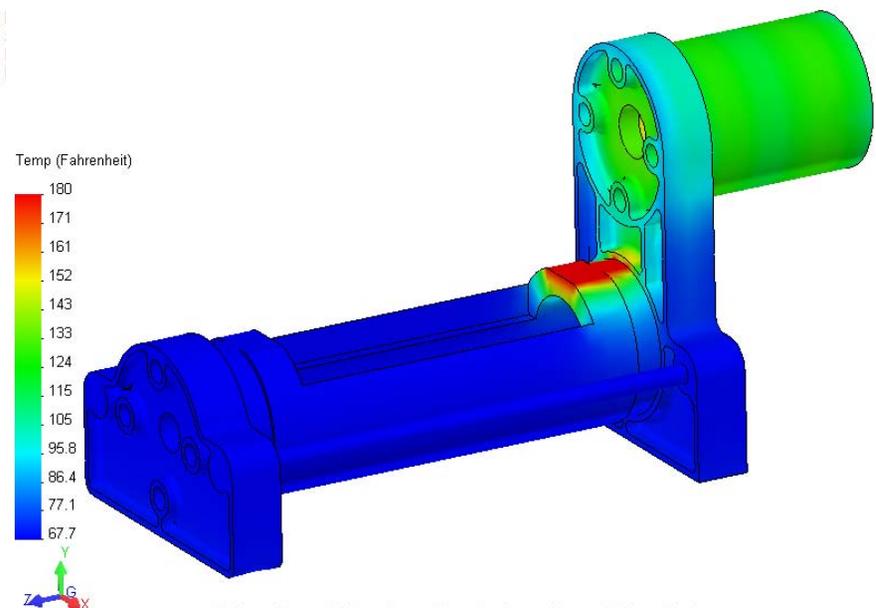
- $h = 0.1 \text{ W}/(\text{m}^2\text{°C})$
- Bearing  $T = 150 \text{ °F}$
- Chip  $T = 180 \text{ °F}$



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For:

- $h = 5000 \text{ W}/(\text{m}^2\text{°C})$
- Bearing  $T = 150 \text{ °F}$
- Chip  $T = 180 \text{ °F}$



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# *Types of errors*

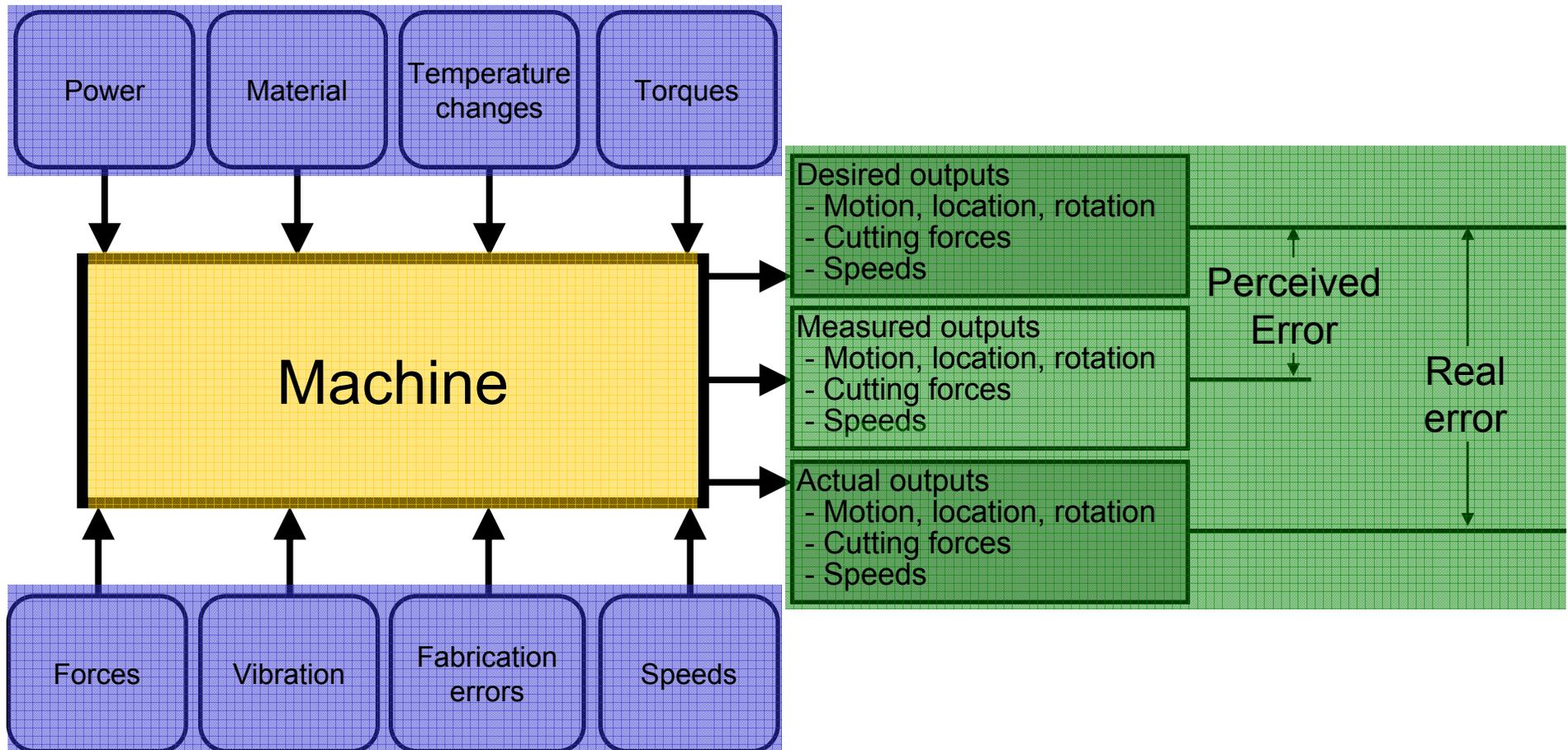
# Machine system perspective

System-level approach

Linking inputs and outputs

Measurement quality

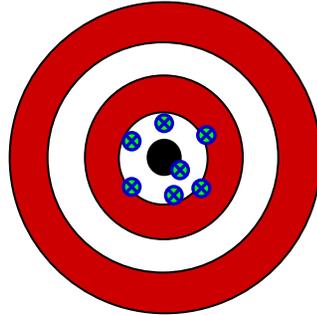
$$[Outputs] = \begin{bmatrix} C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \\ C_7 & C_8 & C_9 \end{bmatrix} [Inputs]$$



# Errors....

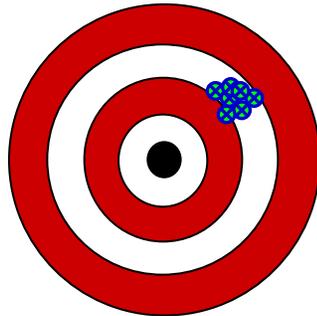
## Accuracy

- ❑ The ability to tell the “truth”



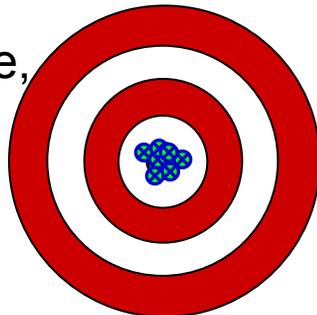
## Repeatability

- ❑ Ability to do the same thing over & over



## Both

- ❑ 1<sup>st</sup> make repeatable, then make accurate
- ❑ Calibrate



## Determinism

- ❑ Machines obey physics!
- ❑ Model → understand relationships

$$[Outputs] = \begin{bmatrix} C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \\ C_7 & C_8 & C_9 \end{bmatrix} [Inputs]$$

- ❑ Understand sensitivity

$$[\Delta Outputs] = J [\Delta Inputs]$$

## Range

- ❑ Furthest extents of motion

## Resolution

- ❑ Smallest, reliable position change

# Categorizing error types

## Systematic errors

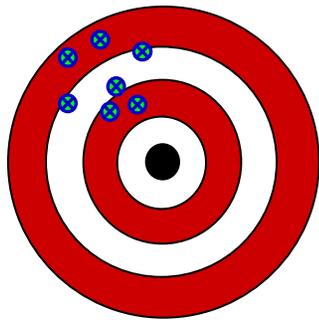
- ❑ Inherent to the system, repeatable and may be calibrated out.

## Non-systematic errors

- ❑ Errors that are **perceived and/or modeled** to have a statistical nature
- ❑ Machines are not “random,” there is no such thing as a random error

## Consider the error for each set below

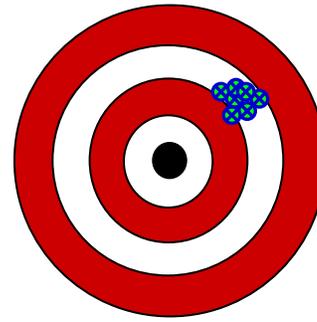
- ❑ Link behavior with systematic and non-systematic errors.



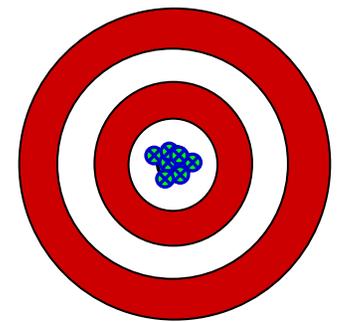
A



B



C



D

*Exercise*

# Exercise

**Due Tuesday, start of class:**

## **Lathe components**

- ❑ Rough sketch(es) of lathe
- ❑ Annotate main components



**1 page bullet point summary of where need to use:**

- ❑ 2.001, 2.002      2.003, 2.004      2.005, 2.006      2.007, 2.008

## **Rules:**

- ❑ You may not re-use examples from lecture!
- ❑ You are encouraged to ask any question!
- ❑ You may work in groups, but must submit your own work

# Group work: Generate strategy for this...

