2.72 Elements of Mechanical Design
Spring 2009

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Elements of Mechanical Design
Lecture 03: Shafts

Schedule and reading assignment

Reading quiz

Hand forward lathe exercise quiz

Topics

- Finish matrices, errors
- Shaft displacements
- Stiffness exercise

Reading assignment

- Shigley/Mischke
  - *Sections 6.1–6.4: 10ish pages & Sections 6.7–6.12: 21ish pages*
  - *Pay special attention to example 6.12 (modified Goodman portion)*

Deflection within springs and shafts

Shafts, axles and rails

Shafts

- Rotating, supported by bearings/bushings
- Dynamic/fluctuating analysis

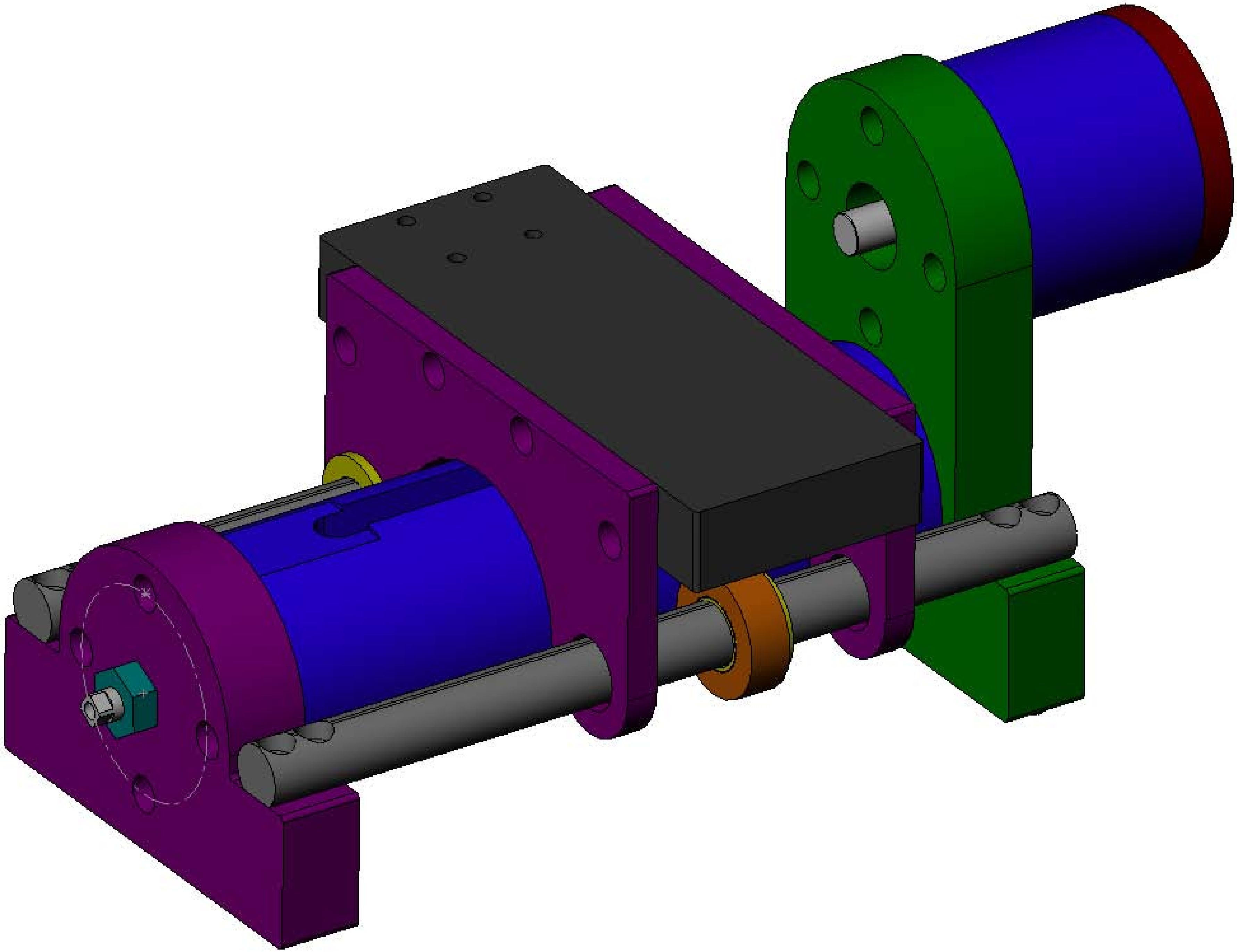
Axles

- Non-rotating, supported by bearings/bushings
- Static analysis

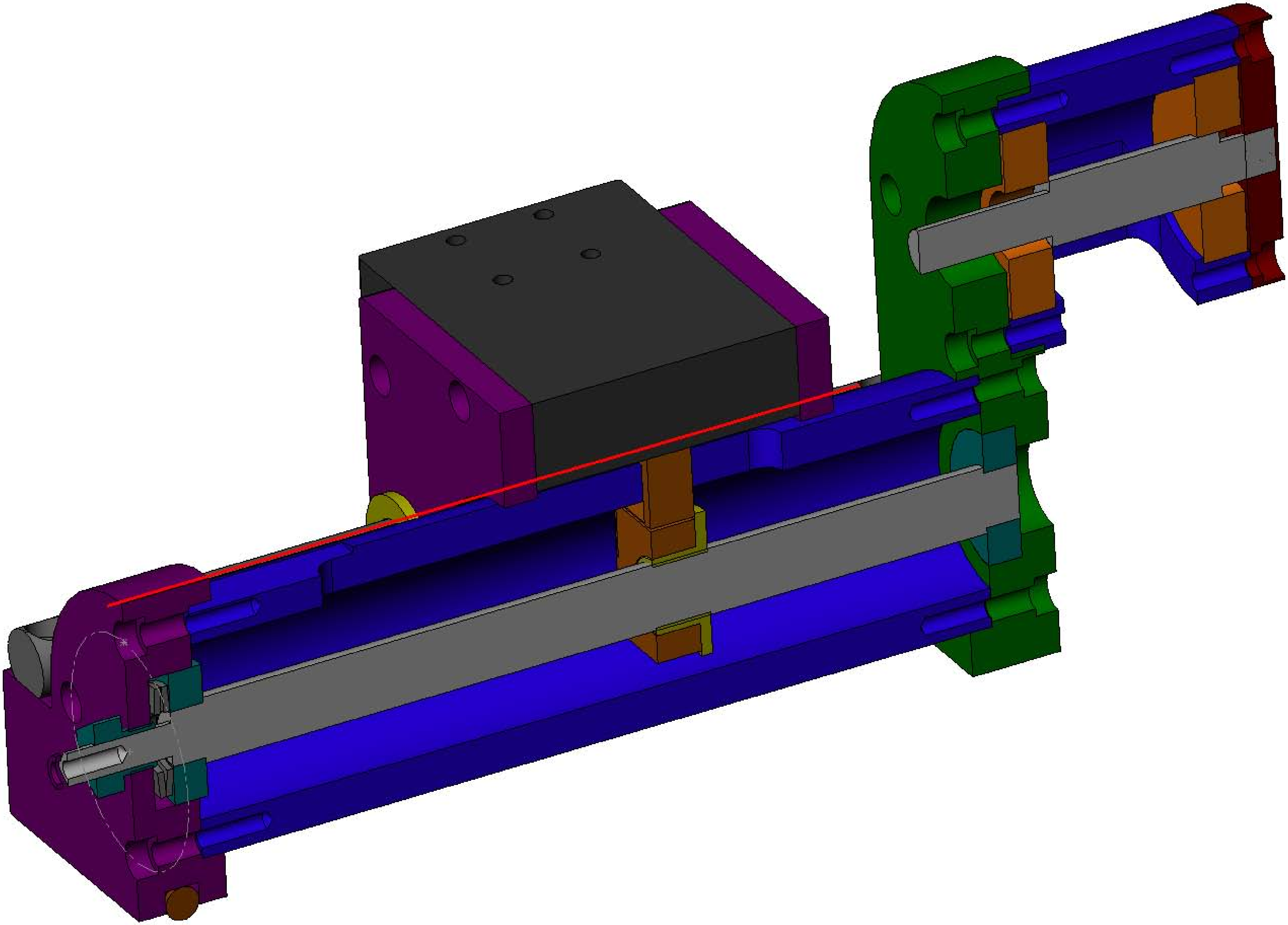
Rails

- Non-rotating, supports bearings/bushings
- Static analysis

Examples drawn from your lathe



Examples drawn from your lathe



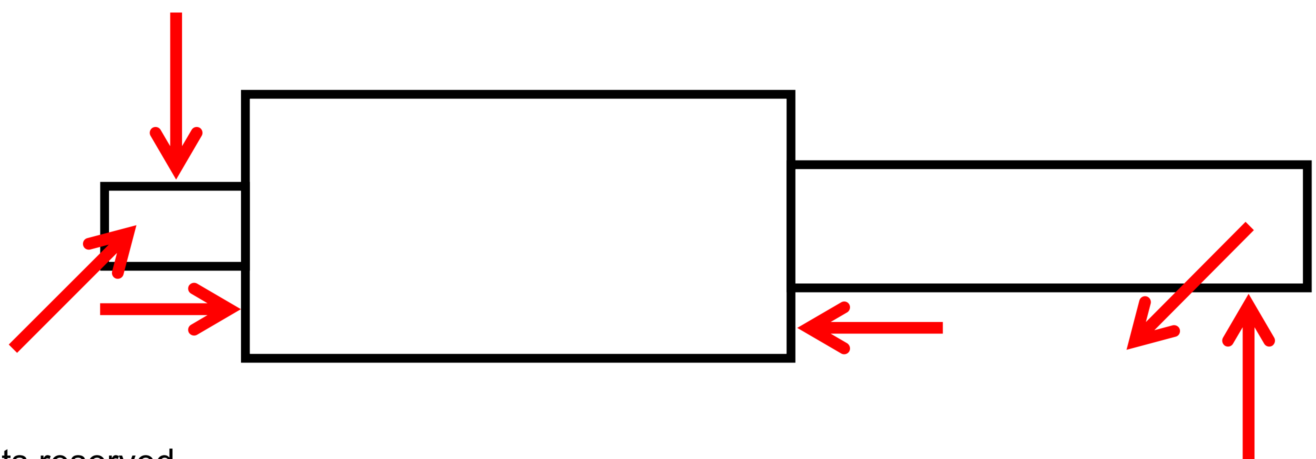
In practice, we are concerned with

Deflection

- Stiffness
- Bearings and stiffness of connectivity points
- Function of global shaft geometry, sometimes adjacent components

Stress

- Catastrophic failure: Ductile Brittle Fatigue
- Function of local shaft geometry



What is of concern?

Deflection and stiffness

- Beam bending models
- Superposition

Load and stress analysis

- Bending, shear & principle stresses
- Endurance limit
- Fatigue strength
- Endurance modifiers
- Stress concentration
- Fluctuating stresses

Failure theories

- Von Mises stress
- Maximum shear stress

Materials

Steel vs. other materials

- Aluminum
- Brass
- Cast iron

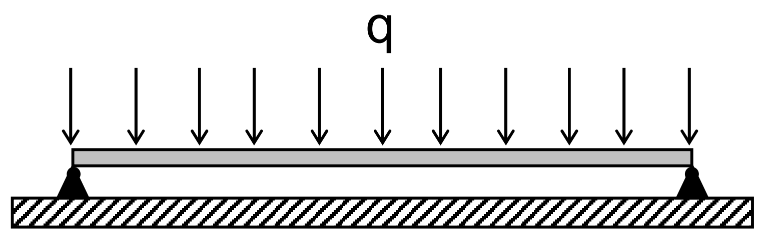
Important properties

- Modulus Yield stress Fatigue life CTE
- Is density important?

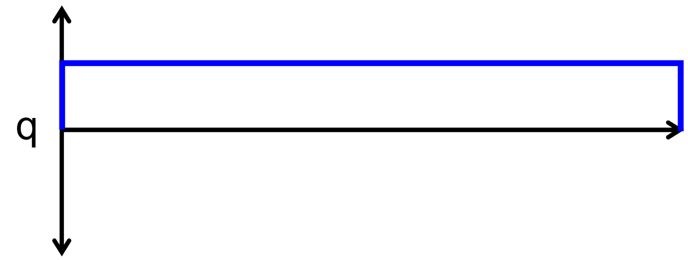
Material treatment – Hardening

- What does hardening do the material properties
- It is expensive
- Affects final dimensions
- You can usually design without this

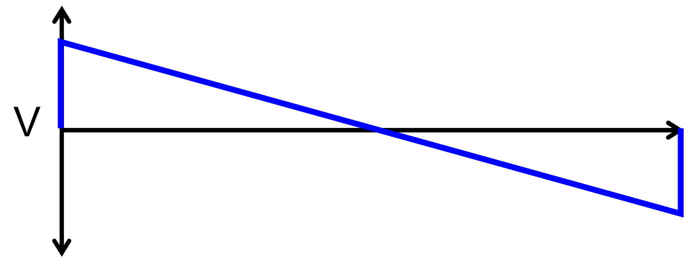
Principles of stiffness: Relationships



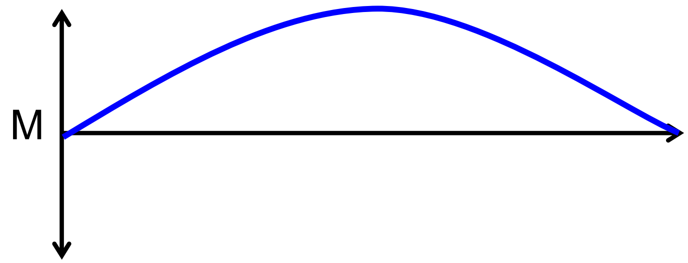
$$\frac{q}{EI} = \frac{d^4 y}{dx^4}$$



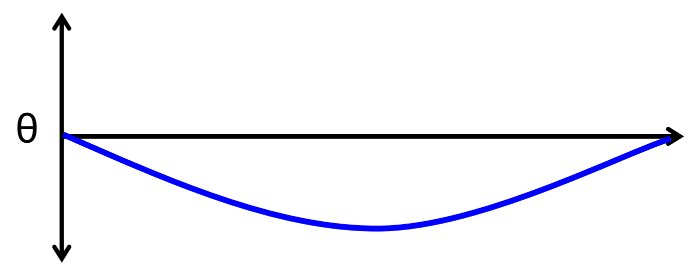
$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$



$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$



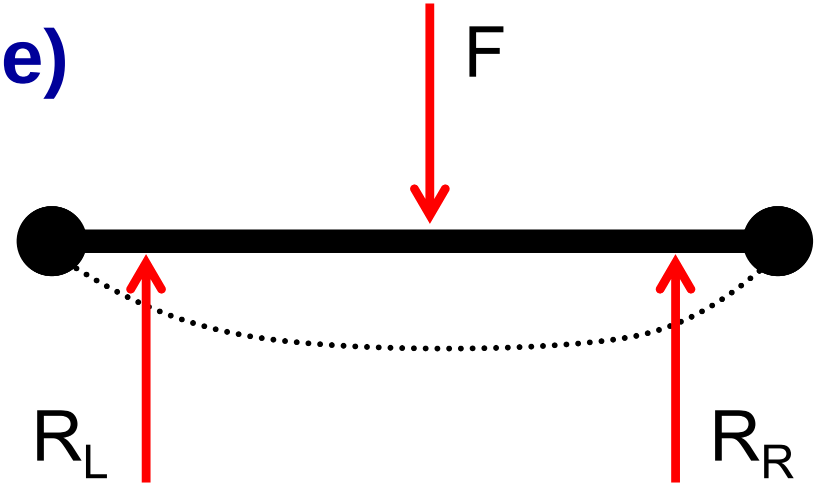
$$\theta = \frac{dy}{dx}$$



Modeling: General forms of equations

Lateral bending deflection (middle)

$$\delta = \frac{F\,L^3}{48E\,I}\,Const\,\frac{F\,L^3}{E\,I}$$



Axial deflection

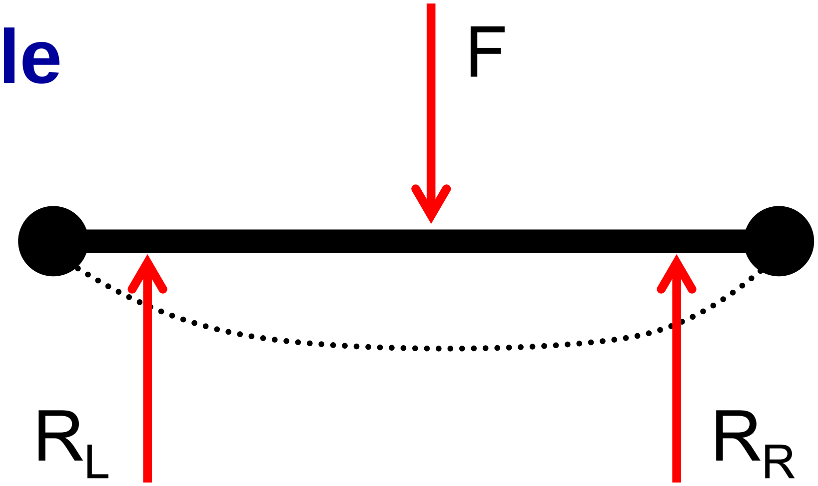
$$\delta = \frac{F\;\;L}{A\;E}$$

Lateral bending angles (at ends)

$$\delta = \frac{F\,L^2}{6\;E\,I} = \frac{F\,L^2}{Const\;\;E\,I}\;or\;\frac{M\;L}{Const\;\;E\,I}$$

Modeling: Stiffness

Lateral bending stiffness at middle



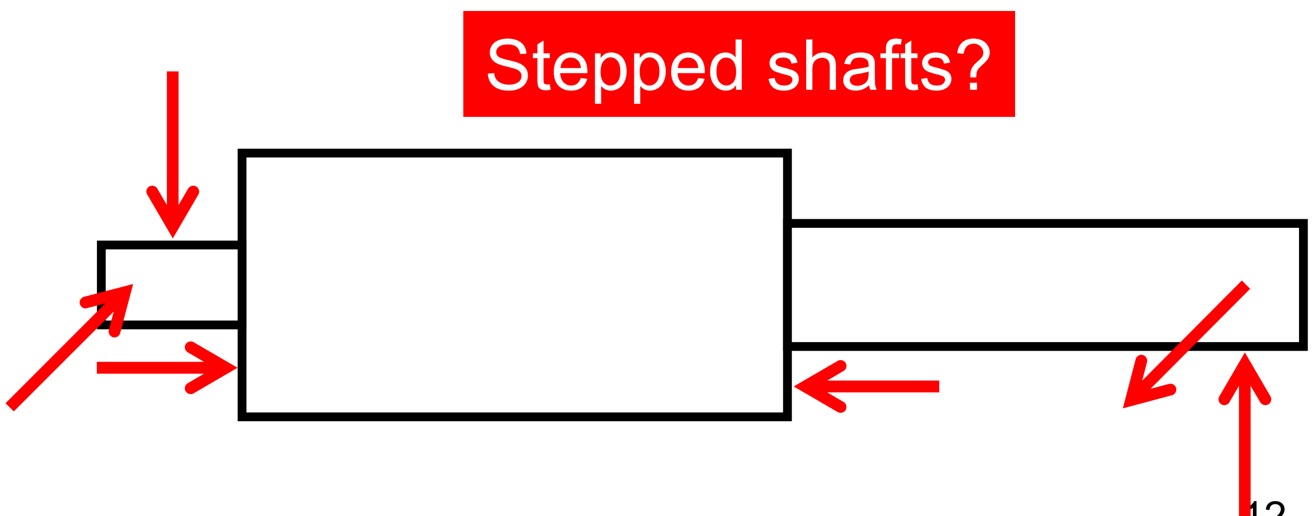
$$k_b = 48\frac{(E\,I)}{L^3} = Const\,(E\,I)\,L^n$$

Axial stiffness

$$k_A = \frac{A\,E}{L}$$

Torsional stiffness

$$k_\theta = \frac{J\,G}{L}$$



Modeling: Stiffness

These pop up in many places, memorize them

- Square cross section

$$I = \frac{1}{12} b h^3$$

- Circular cross sections

$$I = \frac{\pi}{64}\left[\left(d_{outer}\right)^4 - \left(d_{inner}\right)^4\right]$$

$$J = \frac{\pi}{32}\left[\left(d_{outer}\right)^4 - \left(d_{inner}\right)^4\right]$$

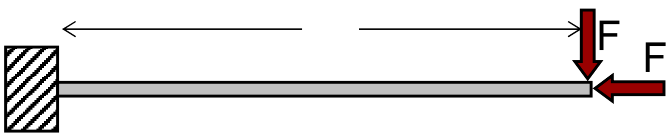
Principles of stiffness: Ratios

Everything deforms

- Impractical to model the stiffness of everything
- Mechanical devices modeled as high, medium & low stiffness elements
- Stiffness ratios show what to model as high-, medium, or low stiffness

Stiffness ratio

$$R_k = \frac{k_{1st}}{k_{2nd}}$$



$$k_{axial} = \frac{AE}{l}$$

$$k_{lateral} = \frac{3EI}{l^3}$$

$$R_k = \frac{\frac{AE}{l}}{\frac{3EI}{l^3}} = 4\frac{l^2}{h^2}$$

Building intuition for stiffness

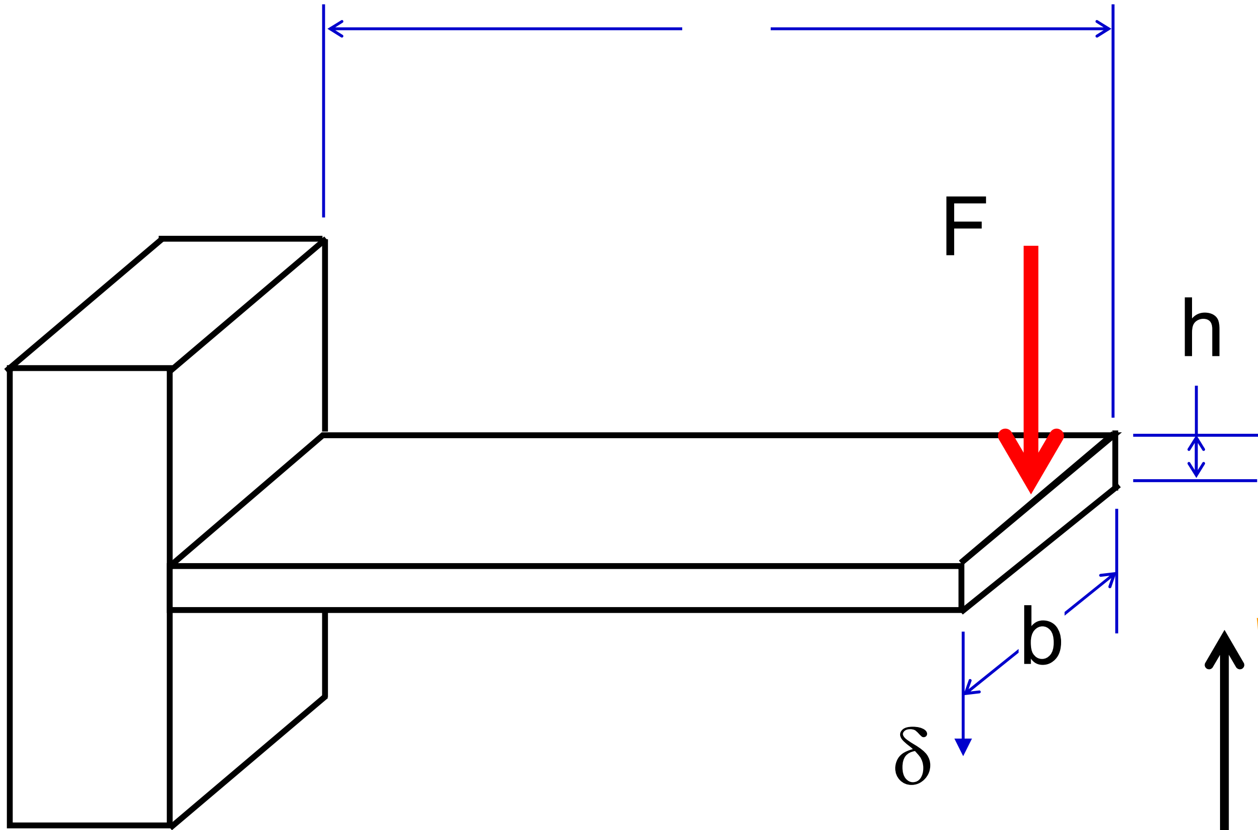
- You can’t memorize/calculate everything
- Engineers must be reasonable “instruments”
- Car suspension is easy, but flexed muscle vs. bone?

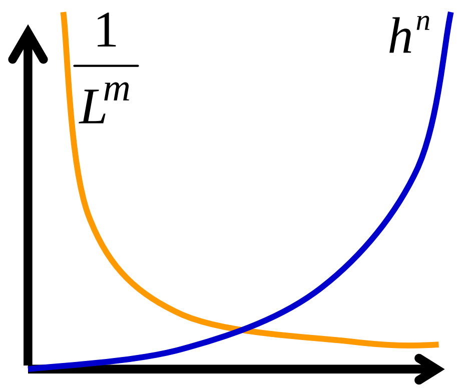
Principles of stiffness: Sensitivity

Cantilever

$$\delta = \frac{F \cdot L^3}{3 \cdot E \cdot I}$$

$$I = \frac{1}{12} \cdot b \cdot h^3$$





$$F = \left(\frac{E \cdot b}{4} \cdot \left[\frac{h}{L} \right]^3 \right) \cdot \delta$$

$$k = \frac{dF}{d\delta} = \frac{d}{d\delta} \left\{ \frac{E \cdot b}{4} \cdot \left[\frac{h}{L} \right]^3 \cdot \delta \right\} \rightarrow \frac{E \cdot b}{4} \cdot \left[\frac{h}{L} \right]^3$$

Superposition

You must be careful, following assumptions are needed

- Cause and effect are linearly related
- No coupling between loads, they are independent
- Geometry of beam does not change too much during loading
- Orientation of loads does not change too much during loading

Use your head, when M = 0, what is going on

Superposition is not plug and chug

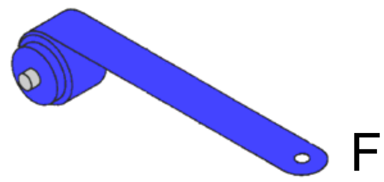
- You must visualize
- You must think

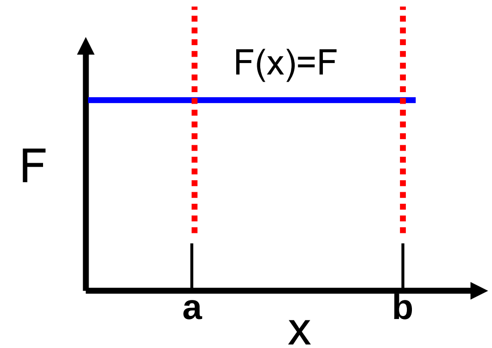
Types of springs and behaviors

Springs and stiffness

- $k_F = {}^{dF(x)}/_{dx}$

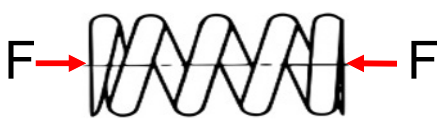
Constant force spring

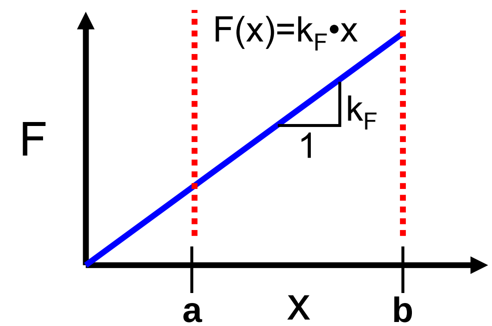




- $k_F = {}^{dF(x)}/_{dx} = 0$
- $\Delta E_{b\text{-}a} = F \bullet (x_b - x_a)$

Constant stiffness spring

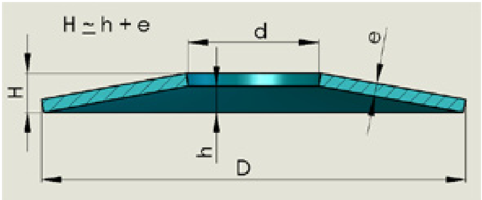


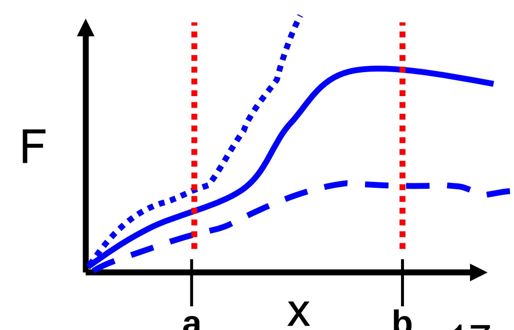


- k_F = Constant
- $\Delta E_{b\text{-}a} = 0.5 \bullet k_F \bullet (x_b^2 - x_a^2)$

Non-linear force spring

- k_F = function of x
- $\Delta E_{b\text{-}a} = \int F(x) \bullet dx$





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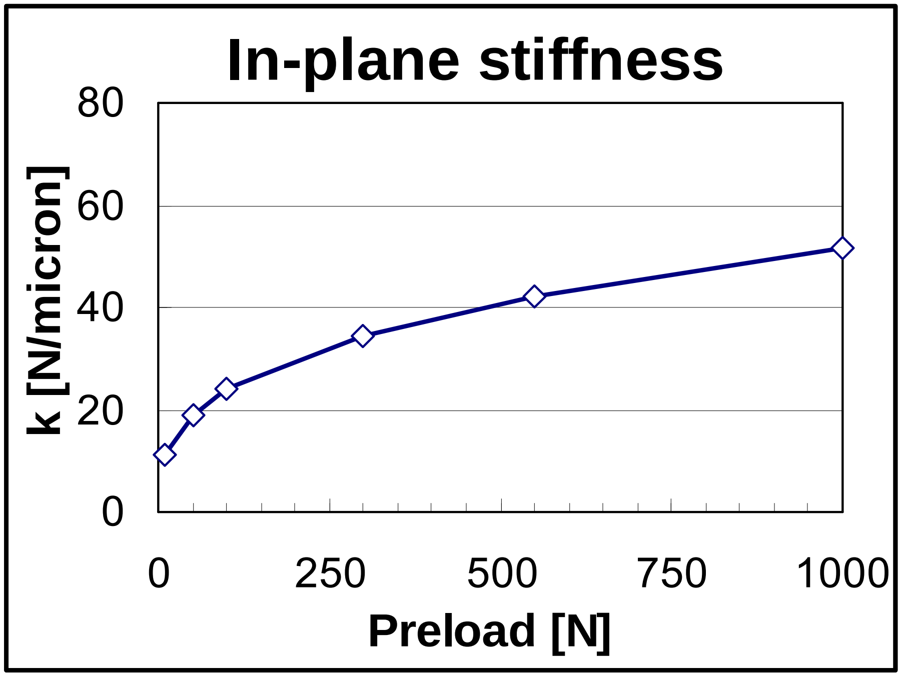
Non-conformal contact – ball on flat

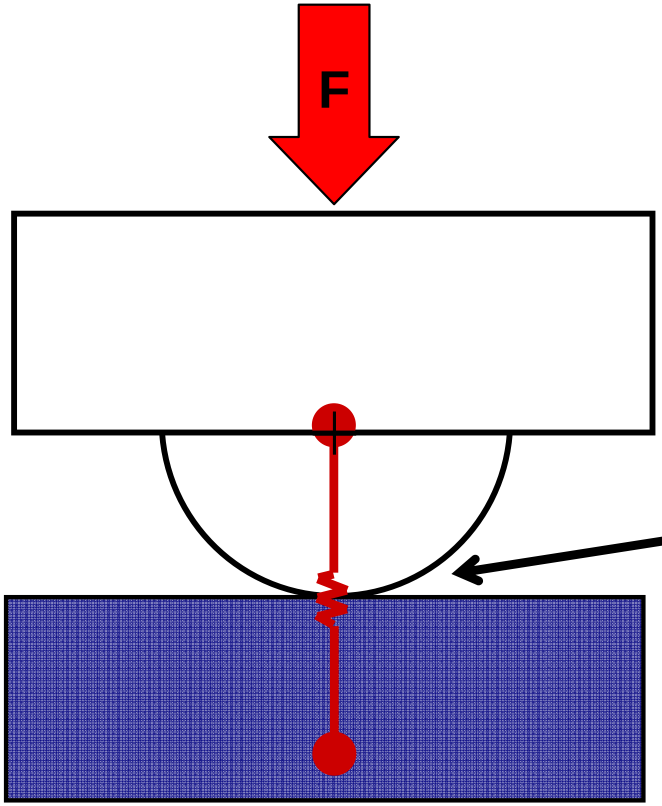
Non-conformal contacts often non-linear

- Example: bearings, belleville washers, structural connections

- Is anything ever perfectly conformal?

- Specific case: Hertzian contact





$$k_n(F) = \text{Constant} \cdot \left(R^{1/3} \cdot E^{2/3} \right) \cdot F^{1/3}$$

Linearization of non-linear springs

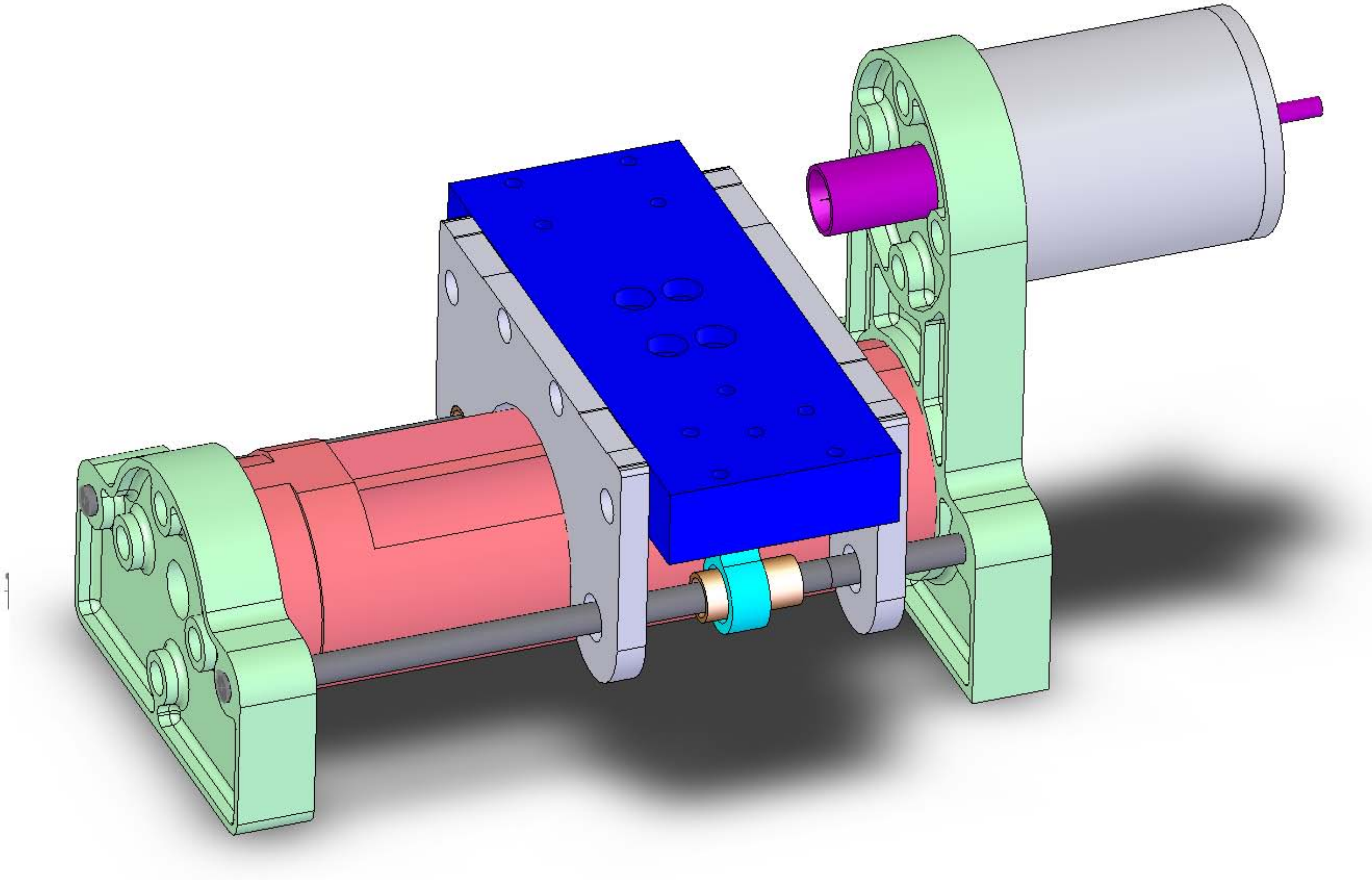
If you can linearize over the appropriate range… then you can use superposition

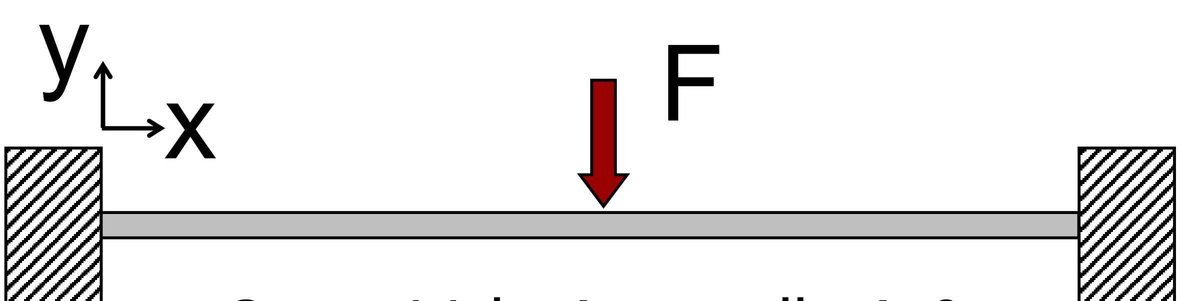
So how would, and when could, you do this?

- R = ball radius
- E = modulus of both materials (both steel)
- F = contact load

$$\frac{dF}{d\delta} = k_n(F) = \text{Constant} \cdot \left(R^{1/3} \cdot E^{2/3} \right) \cdot F^{1/3}$$

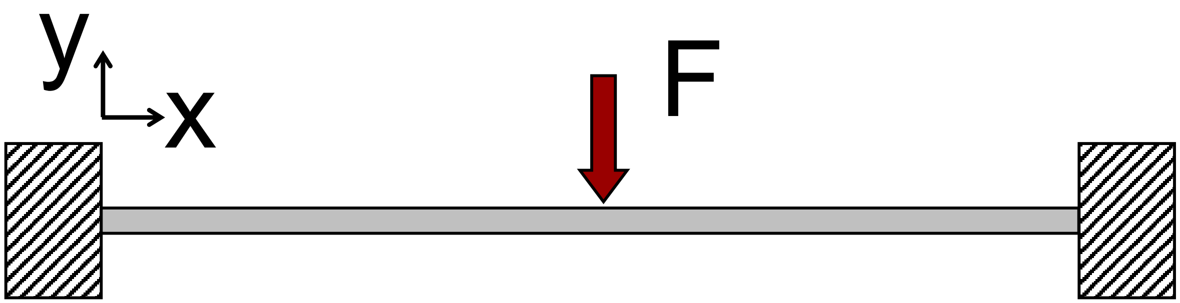
Practical application to the lathe problem





Case 11 in Appendix A-9

Practical application to the lathe problem



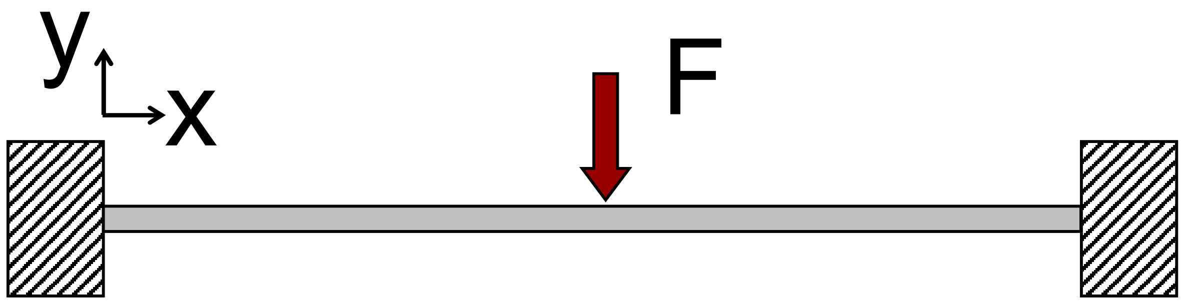
$$y(x)\Big|_{A\rightarrow B} = \frac{1}{96EI}\cdot F\cdot x^2\left(11x-9l\right)$$

$$y\left(l/2\right) = \frac{1}{96EI}\cdot F\cdot \frac{7}{8}l^3$$

$$k\Big|_{Beam} = \frac{768}{7}\cdot\frac{E\ I}{l^3}$$

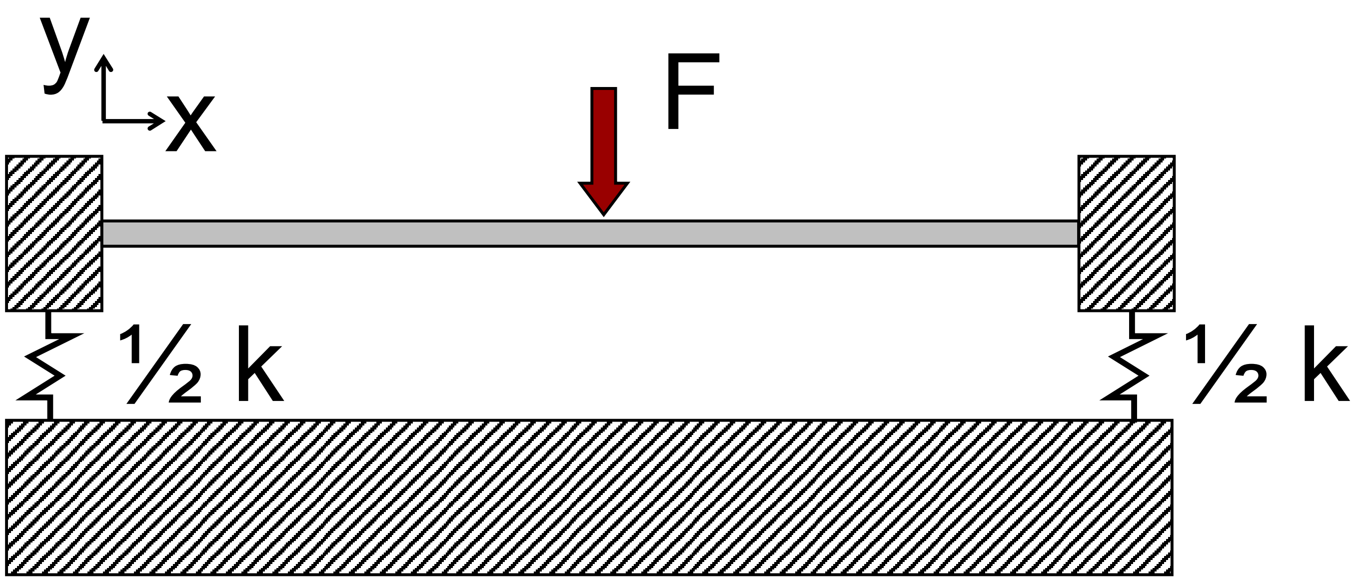
But, is this really what is going on?

Practical application to the lathe problem

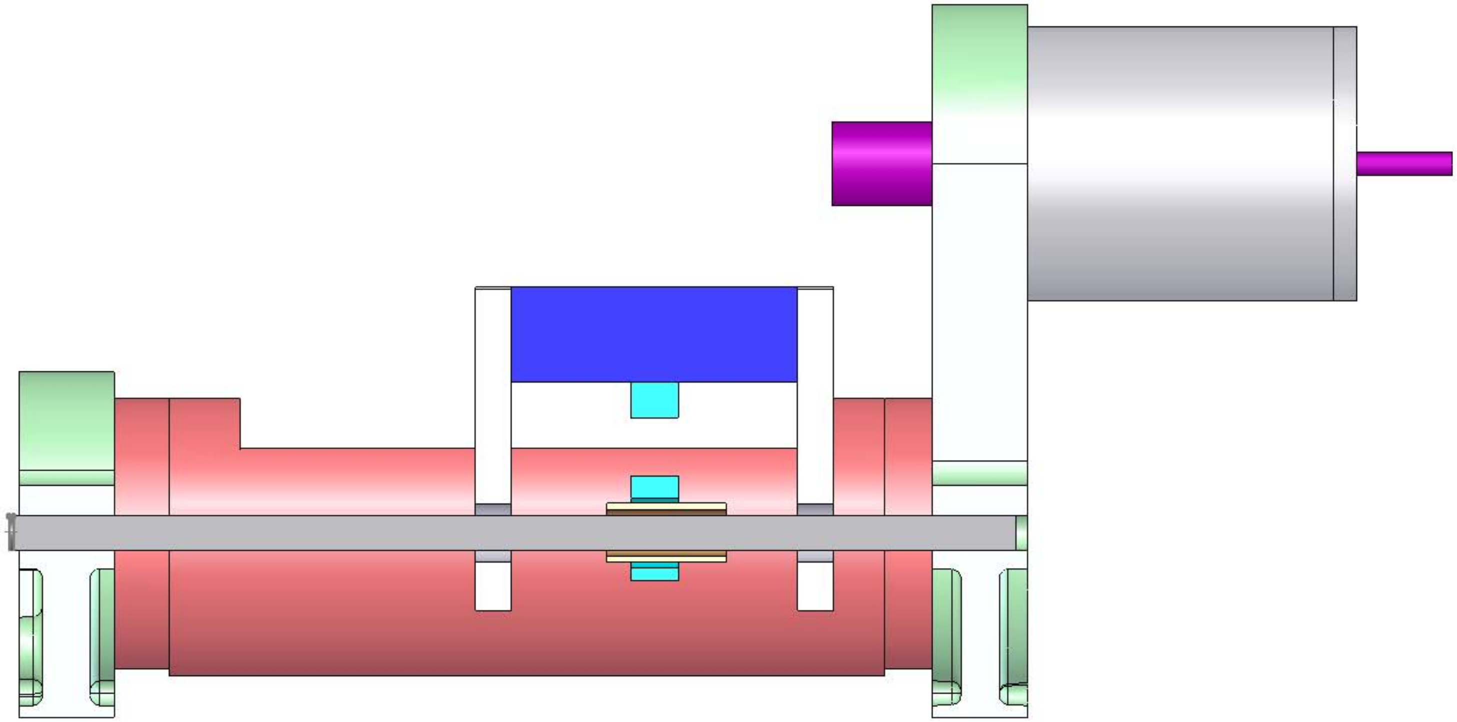


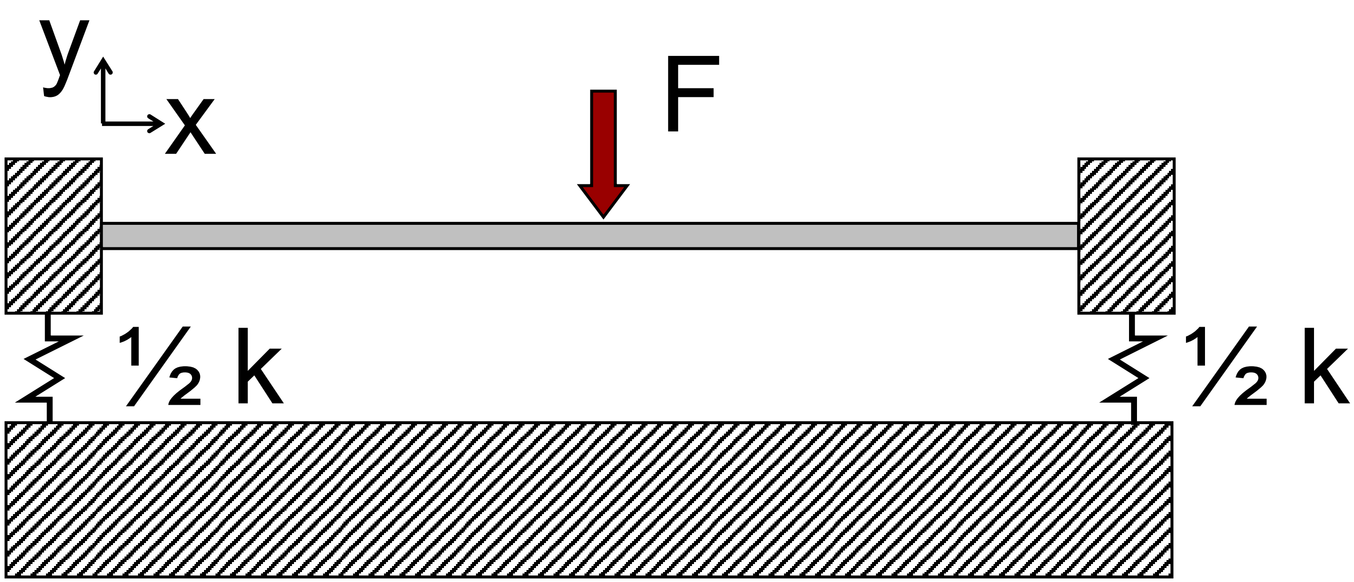
$$k\Big|_{Beam} = \frac{768}{7}\cdot\frac{E\ I}{l^3}$$

Vs. ?



Practical application to the lathe problem

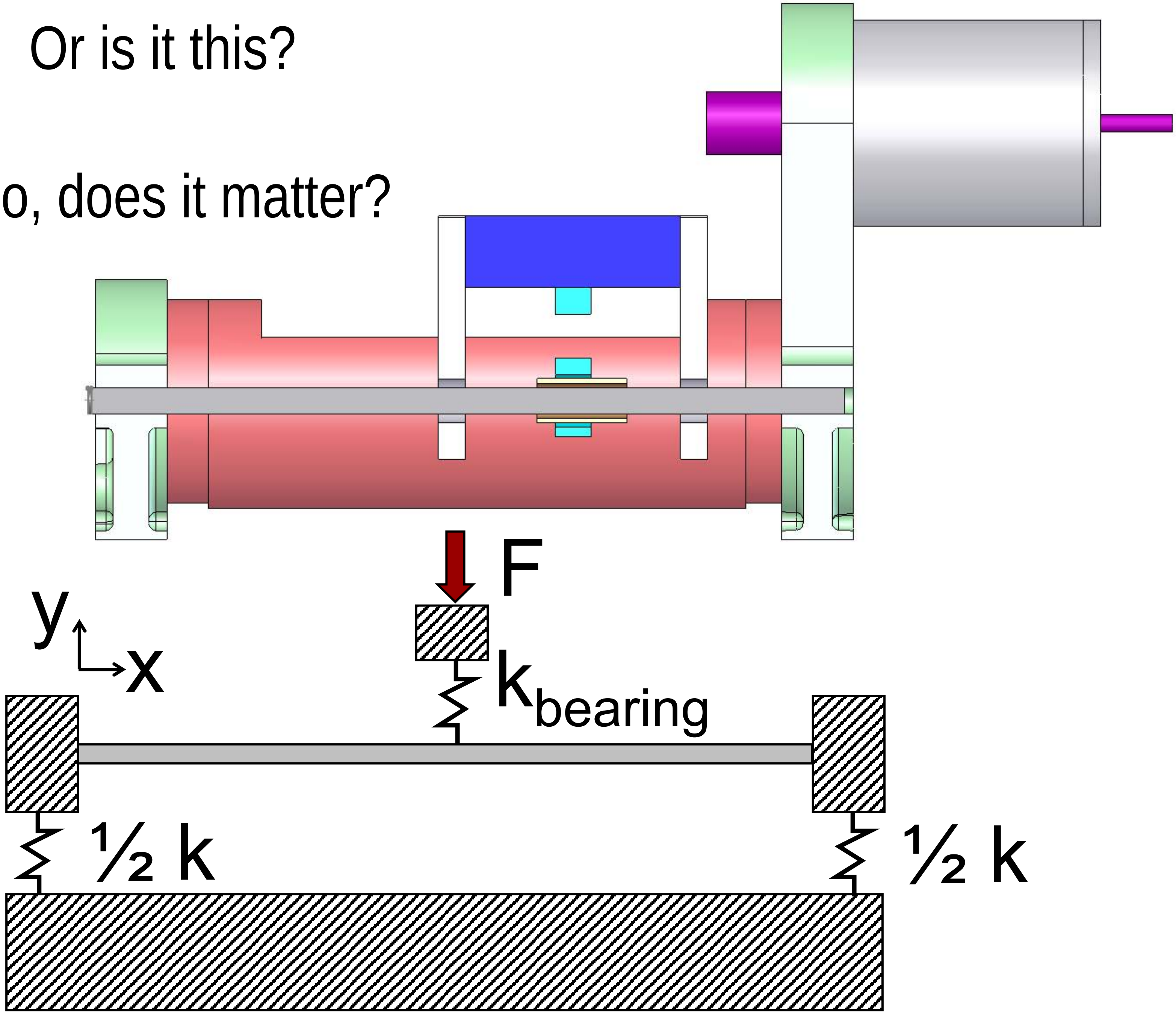




Practical application to the lathe problem

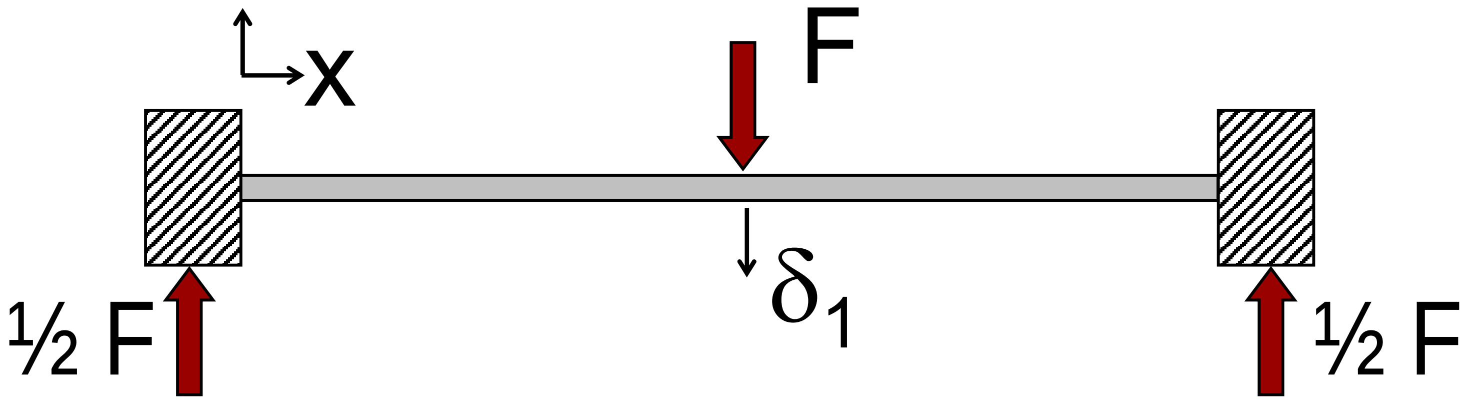
Or is it this?

If so, does it matter?



Practical application to the lathe problem

δ_1



δ_2

Group work

Obtain an equation for δ_{total} in terms of F, k and *l*

Estimate when k is important / should be considered?

What issue/scenario would cause k not to be infinite?

Look at these causes, if a stiffness is involved, would linearity, and therefore superposition apply?