2.72 Elements of Mechanical Design
Spring 2009

2.72

Elements of Mechanical Design

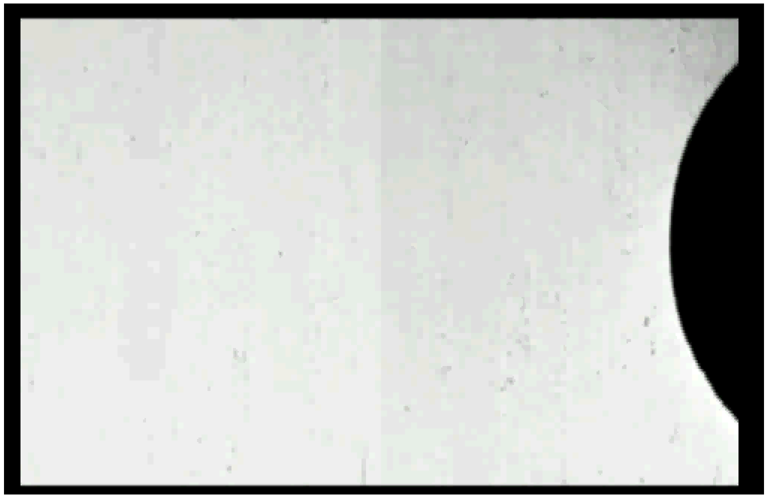
Lecture 04: Fatigue

Schedule and reading assignment

Reading quiz

Announcements

- Shaft due date
- Shaft exercise
- Goodman diagram quiz (Tuesday)
- Shear-moment qualifying quiz (Tuesday)



Please see unterhausen. "fatigue crack." March 27, 2008. YouTube. Accessed October 28, 2009. http://www.youtube.com/watch?v=iBuuVd0JlIM

Topics

- Discuss stiffness exercises
- Start fatigue

Reading

- None, for Tuesday, prep for quizzes in lab time (Given lounge, top of 35)

Reading quiz

Discuss stiffness exercises

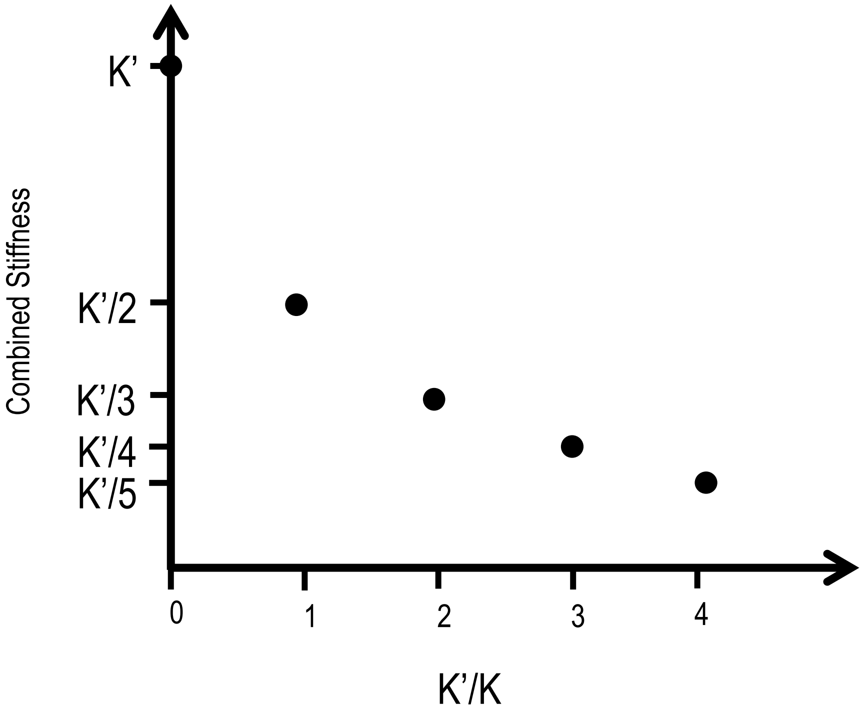
Answers

Intuition about stiffness

- Part
- Spindle
- Carriage-rail
- Etc…

Insight and perspective

Carriage bearing-rail



Shaft exercise

Are you on top of this!?

Fatigue part I

At what critical time in engineering history did fatigue became relevant?

Why does fatigue failure generate serious concern?

What type of warnings does one receive?

Fire plane wing failure

July 18, 2002 near Estes Park, Colorado

- Both crew members killed
- Delivered in July, 1945 to the U.S. Navy
- Logged 8000+ flight hours

Investigation

- NTSB found extensive fatigue
- Cracks hidden from view

Please see richmondlopez13. "Plane crash caught on tape - 6." December 5, 2007. YouTube. Accessed October 14, 2009. http://www.youtube.com/watch?v=j7kr6o1s9sI

Comet airplane failures

BOAC Flight 781 crashes on 10 January 1954

- Concluded fire was most likely cause
- Resumed on 23 March 1954

Comet G-ALYY crashes on 8 April 1954

- Pressure tests revealed fatigue
- Windows to be glued & riveted, but riveted only
- Square windows → oval
- Skin thickened
- Service in 1958

I-35 bridge failure

By Laurie Blake, Paul Mcenroe, Pat Doyle and Tony Kennedy, Star Tribune

Image from Wikimedia Commons, http://commons.wikimedia.org

MnDOT's options:

- Make repairs or find flaws & bolt on steel plating
- Fueled emotional debate

MNDOT's action:

- Thousands of bolt holes would weaken bridge
- Launched inspection, interrupted by work on bridge surface

The state's top bridge engineer:

- "We chose the inspection route….. We thought we had done all we could, but obviously something went terribly wrong."
- "Up until the late 1960s, it was thought that fatigue was not a phenomenon you would see in bridges."

How much do engineers know about fatigue?

How "exact" are fatigue models?

Experimental data

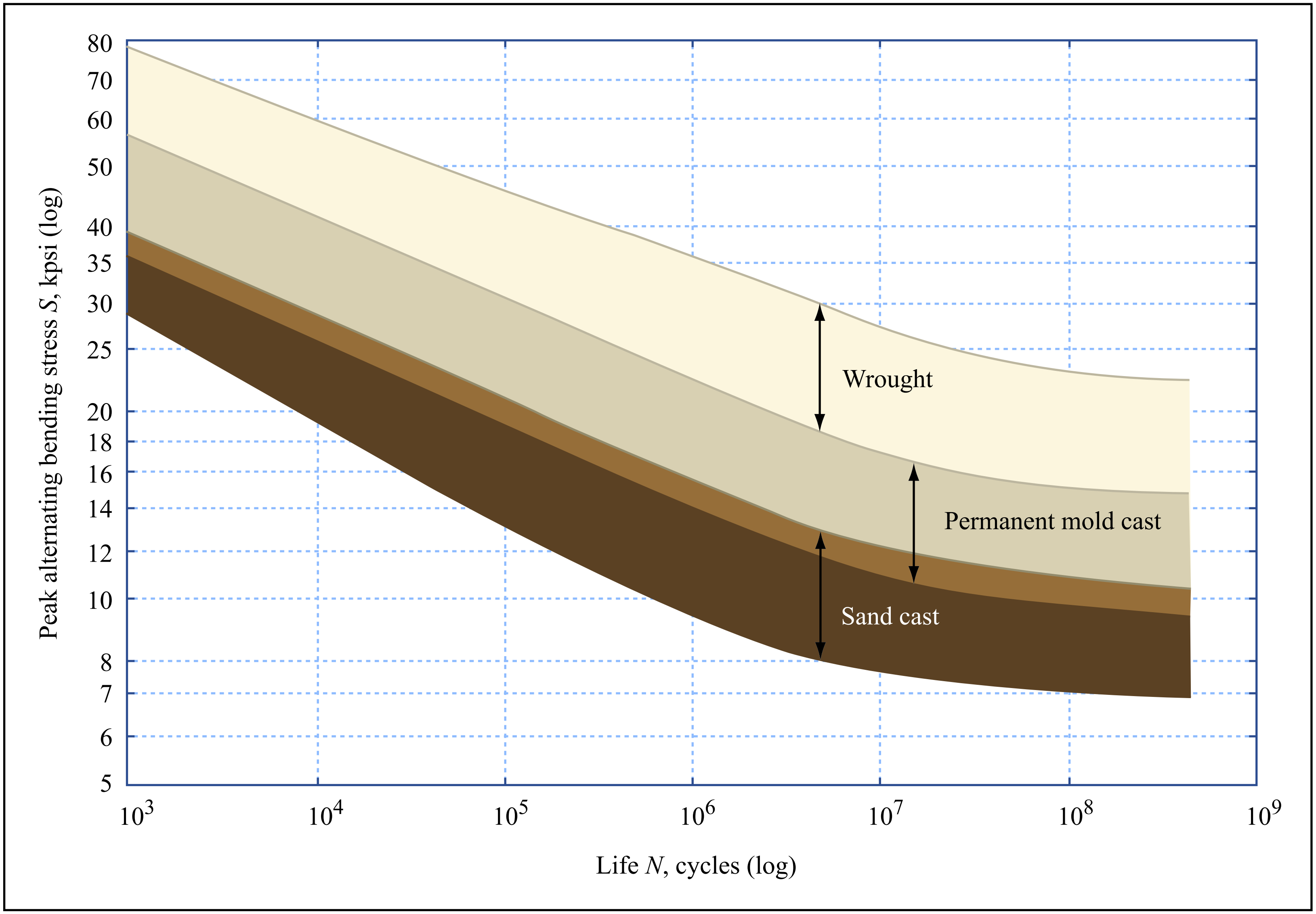


Figure by MIT OpenCourseWare. Adapted from Fig. 6-11 in Shigley & Mischke.

Experimental data

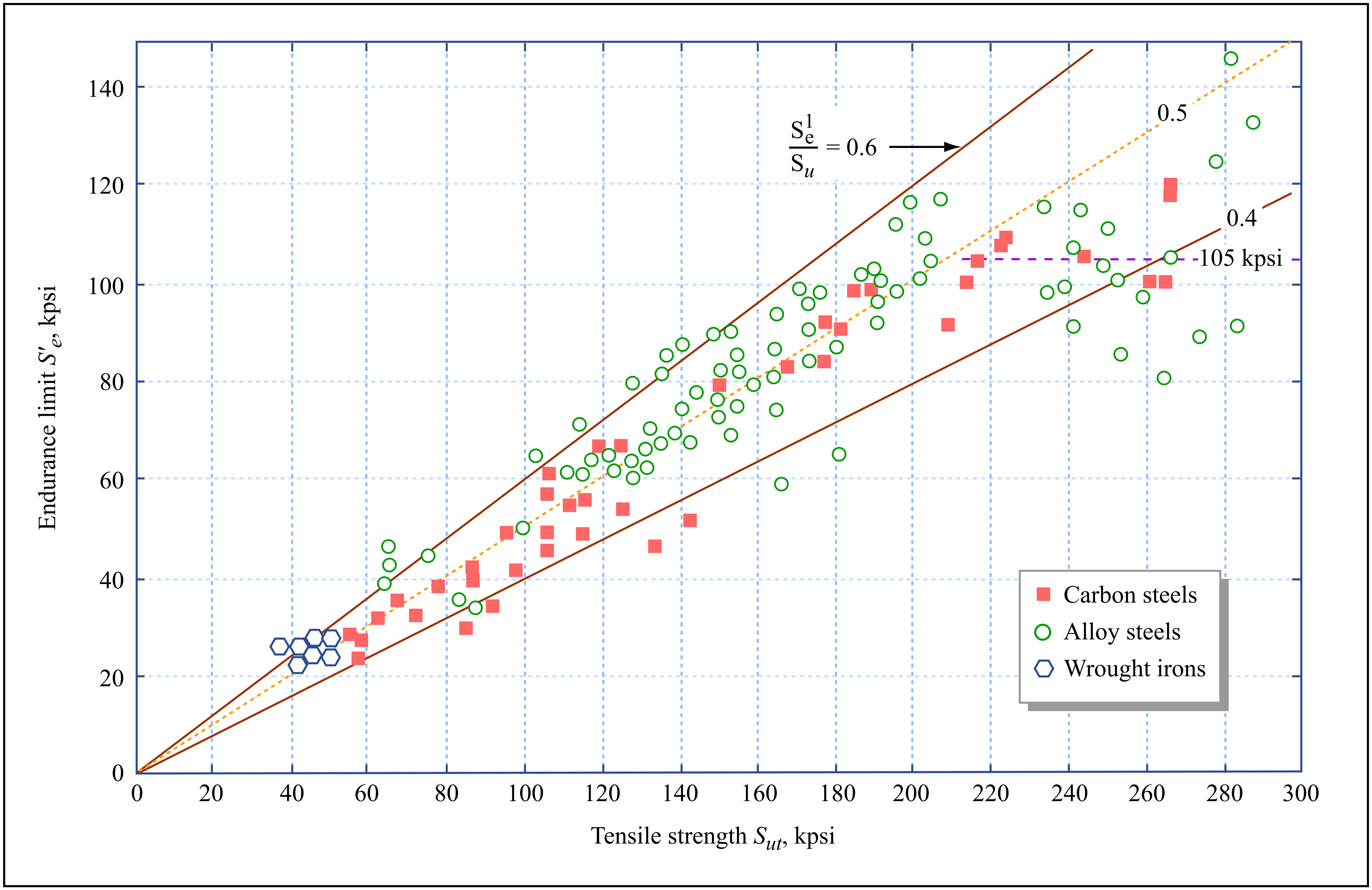


Figure by MIT OpenCourseWare. Adapted from Fig. 6-17 in Shigley & Mischke.

What actions and/or practices should be put in place as a result?

Testing and prevention

Where life-limb-$ are important

- It is your job to spec out test type and procedure
- Balance of cost vs. risk

Many people listen to REAL data

Few people listen to Eqxns

A choice: Job vs. safety

Example types

- Ultrasonic
- Liquid penetrant
- Stiffness/impulse
- Eddy-current
- Leaks
- Visual

Images removed due to copyright restrictions.
Please see http://www.labino.com/bilder/applications/00533_RT8.jpg
http://www.riverinaairmotive.com.au/img/mpi001.jpg

On foreseeable use

- Common sense
- Legal

Where do cracks come from?

Potential crack origins/causes

Inherent to material

- Imperfections, e.g. castings
- Precipitates, e.g. Al 6061 T6
- Coalescing of internal dislocations
- Grain boundaries

Image removed due to copyright restrictions. Please see http://www.emeraldinsight.com/fig/2190080205007.png

Fabrication-related

- Tool marks
- Improper assembly, e.g. forcing (car suspension-cast materials)
- Thermally induced - Weld cracks and related HAZ problems

Use-related

- High stress areas
- Scratches
- Unintended use/damage/loading (e.g. 3 finger tight and paint lid)

Fatigue: Origin of problem

Images removed due to copyright restrictions. Please see:

http://www.metallographic.com/Images/Zn-Al.jpg

http://corrosionlab.com/Failure-Analysis-Studies/Failure-Analysis-Images/20030.SCC.304H-pipeline/20030.microstructure-ditched-grain-boundaries.jpg

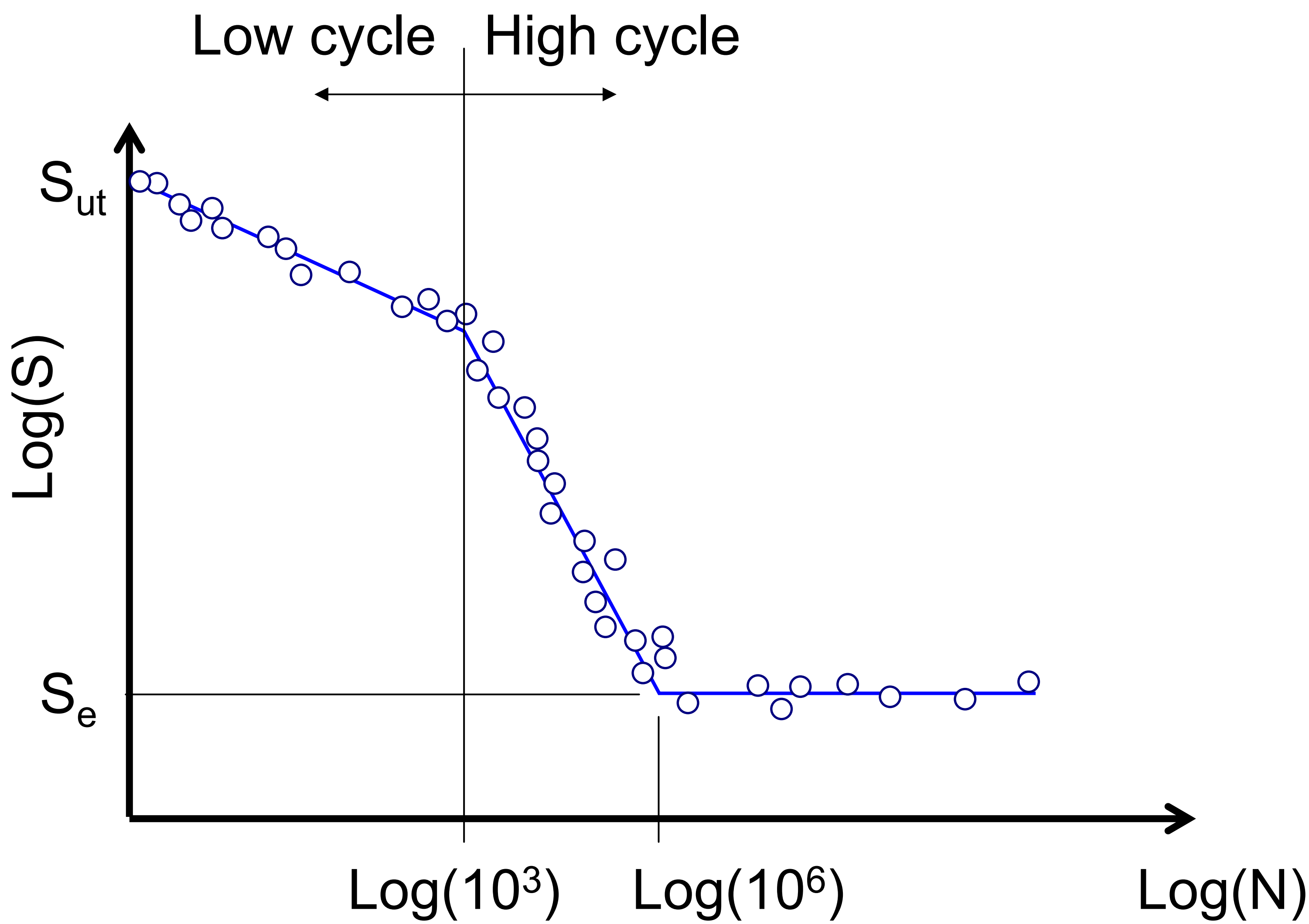
and

Fig. 4b in Henderson, Donald W., et al. "The Microstructure of Sn in Near-Eutectic Sn-Ag-Cu Alloy Solder Joints and its Role in Thermomechanical Fatigue." *Journal of Materials Research* 19 (June 2004): 1608-1612

or

Slide 36 in Kang, Sung K. "Near-Ternary Sn-Ag-Cu Solder Joints; Microstructure, Thermal Fatigue, and Failure Mechanisms." Pb-Free Workshop, TMS Annual Meeting, February 2005.

Fatigue life review



Fatigue life review

Ferrous materials/allows: Se ~ 1 000 000 – 10 000 000

- Under ideal conditions

Non-ferrous (i.e. aluminum) generally no Se…

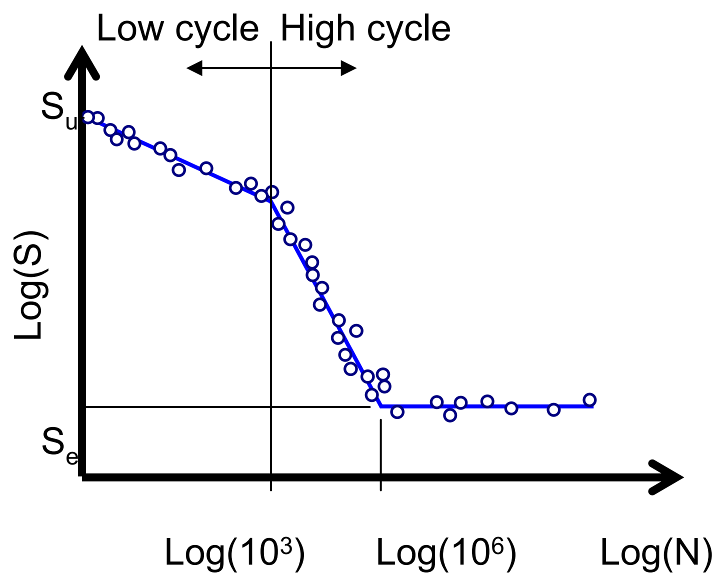
Do we use Al in places where fatigue is important?…

- Aircraft…
- History Channel Boneyard…

Science vs. engineering…

Methods

- Stress
- Strain
- Fracture mechanics



Fatigue and ethical responsibility

You will be criminally negligent if you do not augment calculations with TESTING for critical fatigue applications

Life-limb-$

Real life

Please see datsun_laurel. "R06 Front Carbon ARB Fatigue Test." Photobucket. Accessed October 14, 2009.
http://s23.photobucket.com/albums/b388/datsun_laurel/FSAE/?action=view&current=ARB_Test.flv

Real life

http://video.google.com (author?)

Please see SmithersMpls. "Carbon Frame Fatigue Test." February 9, 2007. YouTube. Accessed October 14, 2009. http://www.youtube.com/watch?v=QHO_VjVhaE8

Real life

http://yourdailymedia.com

Please see motocross. "The Nitro Circus: Channel 9 Action News." March 3, 2009. YouTube. Accessed October 14, 2009. http://www.youtube.com/watch?v=kOaYruVcvm4

What reasonable hypotheses could one hold for identifying important factors?

Fatigue life modifiers

Experimental results are used to obtain modifiers

$$S_e = \left(k_a \; k_b \; k_c \; k_d \; k_e \; k_f\right) S'_e$$

Where:

- k_a = Surface condition modification factor
- k_b = Size modification factor
- k_c = Load modification factor
- k_d = Temperature modification factor
- k_e = Reliability modification factor
- k_f = Others…

- S'_e = Rotary-beam test endurance limit
- S_e = Predicted endurance limit for your part

Endurance limit depends on many factors

For ferrous materials, the following approximations may be used for first pass design

$$S'_e = \begin{cases} 0.5\ S_{ut} & S_{ut} \le 200 kpsi \\ 100 kpsi & S_{ut} > 200 kpsi \\ 700 MPa & S_{ut} > 1400 MPa \end{cases}$$

This is for ideal conditions… but designs are never ideal

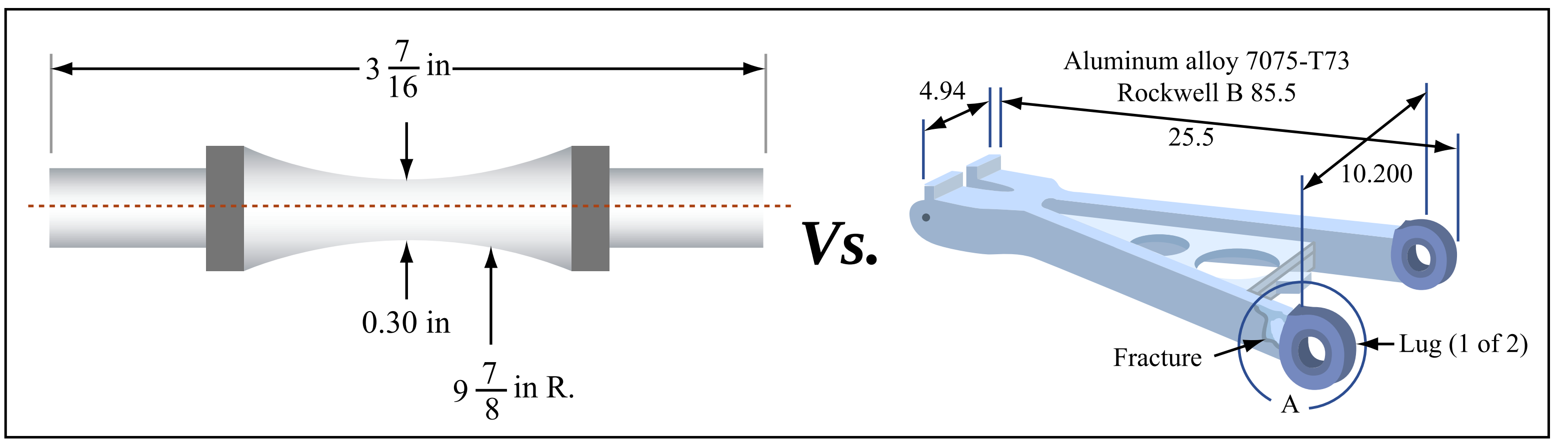


Figure by MIT OpenCourseWare. Adapted from Fig. 6-8 and 6-9 in Shigley & Mischke.

Experimental data

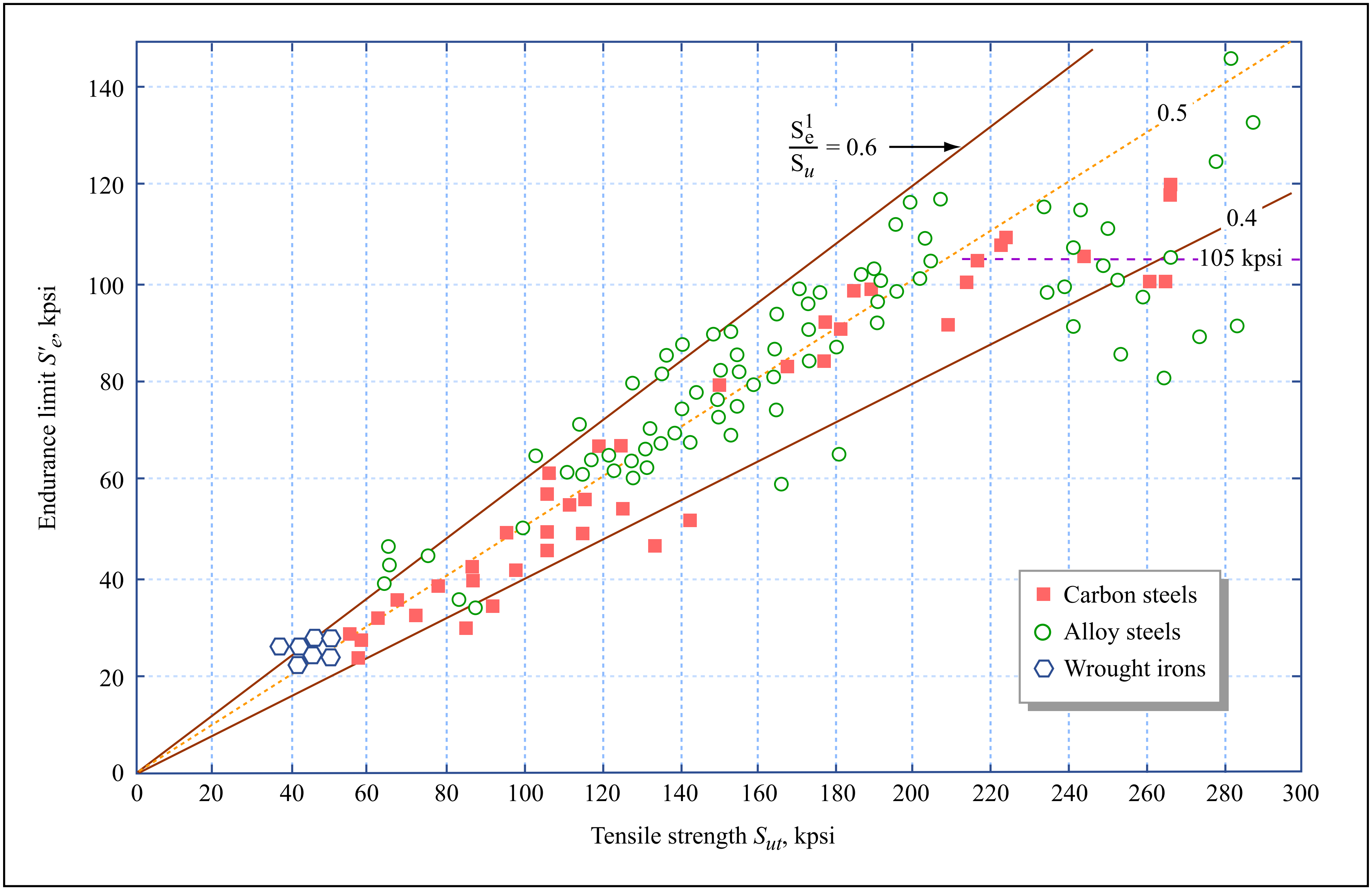


Figure by MIT OpenCourseWare. Adapted from Fig. 6-17 in Shigley & Mischke.

Fatigue life modifiers: Surface condition

Experimental results are used to obtain modifiers

$$k_a = a\ S_{ut}^{b}$$

Where:

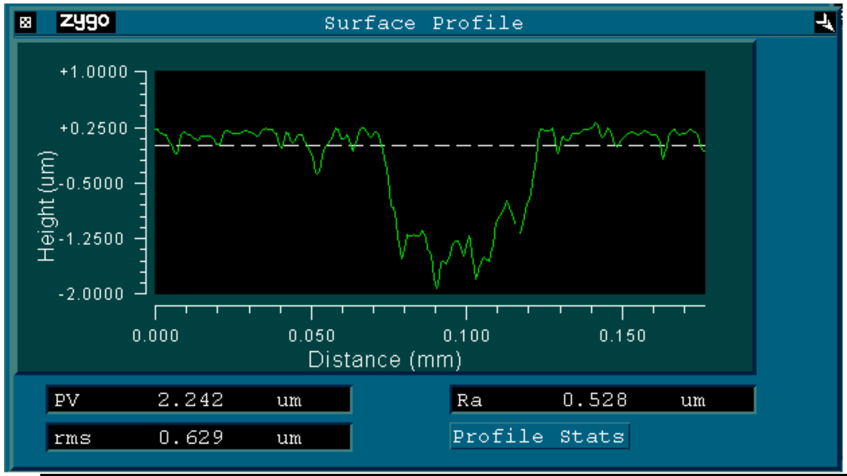
- a = function of fabrication process
- b = function of fabrication process
  - *Why does finish matter?*

Surface finish	Factor a		Exponent b
	S_{ut}, kpsi	S_{ut}, MPa	
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272.	-0.995

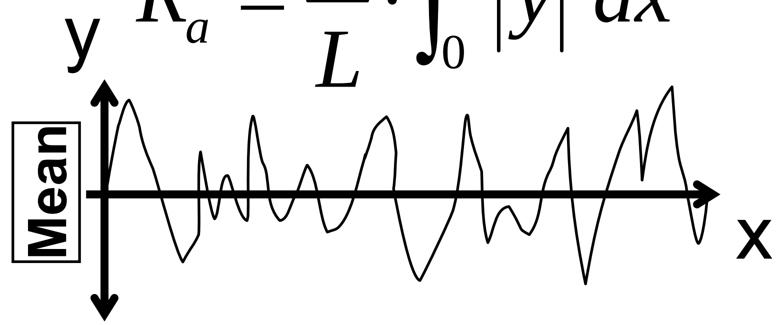
Figure by MIT OpenCourseWare. Adapted from Table 6-2 in Shigley & Mischke.

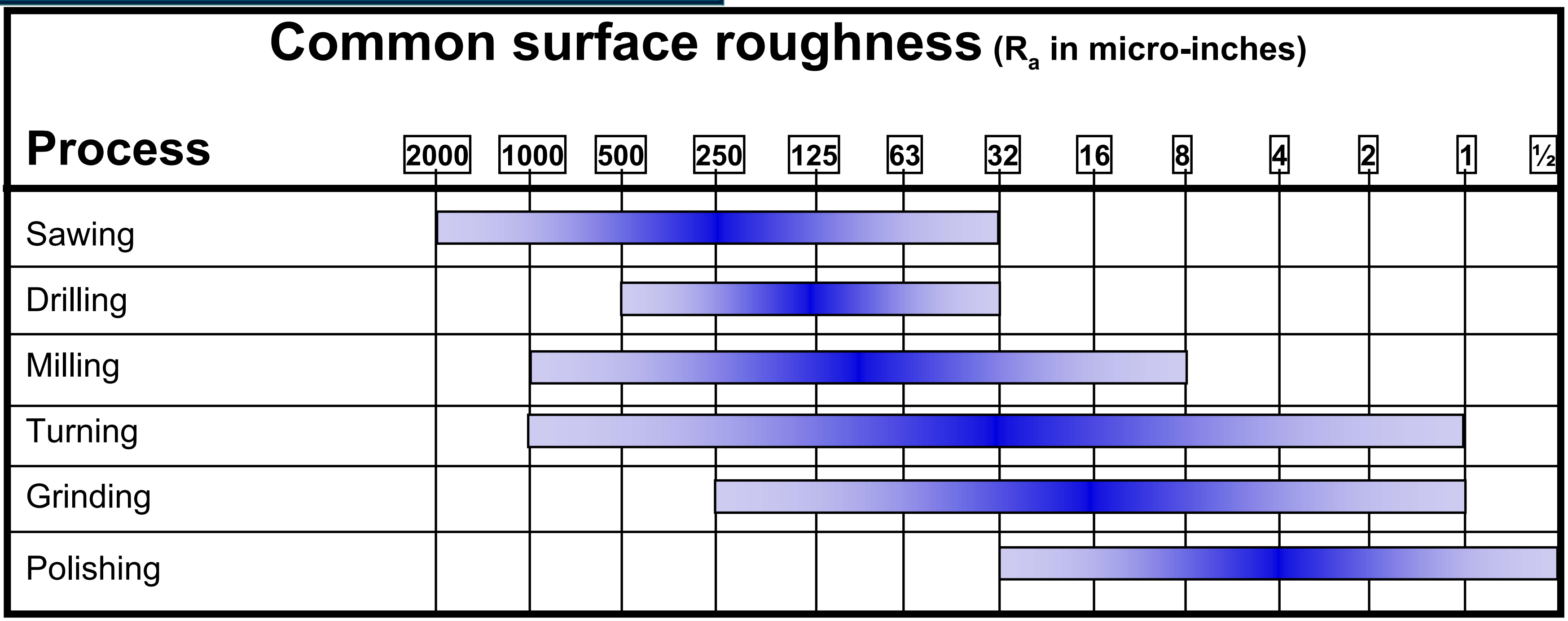
Why would surface condition matter?

Surface roughness review



$$R_a = \frac{1}{L} \cdot \int_0^L |y| \, dx$$





Only specify what you need & know your processes

Why would part size matter?

Fatigue life modifiers: Size factor

For bending and torsion of a round bar:

$$k_b = \begin{cases} 0.879\, d^{-0.107} & \text{for} \quad 0.11in < d < 2.00in \\ 0.910\, d^{-0.157} & \text{for} \quad 2.00in < d < 10.0in \end{cases}$$

For axial loading:

$$k_b = 1$$

What if the bar is not round?

- Use a 95 percent stress area
- Equate volumes, length drops out
- Relate cross sectional area of round and square bar

$$d_e = 0.808\left(h\,b\right)^{0.5}$$

Why would the type of loading matter?

Fatigue life modifiers: Loading factor

For bending and torsion of a round bar:

$$k_c = \begin{cases} 1.00 & \textit{bending} \\ 0.85 & \textit{axial} \\ 0.59 & \textit{torsion} \end{cases}$$

Why would temperature matter?

Fatigue life modifiers: Temperature factor

The effect of increasing temperature

- Yield strength typically decreases
- May be no fatigue limit for material-temperature combos

The temperature factor

- May be ESTIMATED from existing tables
- Should ALWAYS BE DETERMINED EXPERIMENTALLY FOR YOUR GIVEN MATERIAL.

Relate strength at temperature to room temp. strength

$$k_d = \frac{S_T}{S_{RT}}$$

Fatigue life modifiers: For an example steel

Temperature, °C	S_T/S_{RT}
20	1.000
50	1.010
100	1.020
150	1.025
200	1.020
250	1.000
300	0.975
350	0.943
400	0.900
450	0.843
500	0.768
550	0.672
600	0.549

Temperature, °F	S_T/S_{RT}
70	1.000
100	1.008
200	1.020
300	1.024
400	1.018
500	0.995
600	0.963
700	0.927
800	0.872
900	0.797
1000	0.698
1100	0.567

Figure by MIT OpenCourseWare. Adapted from Table 6-4 in Shigley & Mischke.

Part II

Calculations

How do statistics and probability come into play?

Standard normal distribution, mean = 0

Standard normal distribution curve generated via the probability distribution

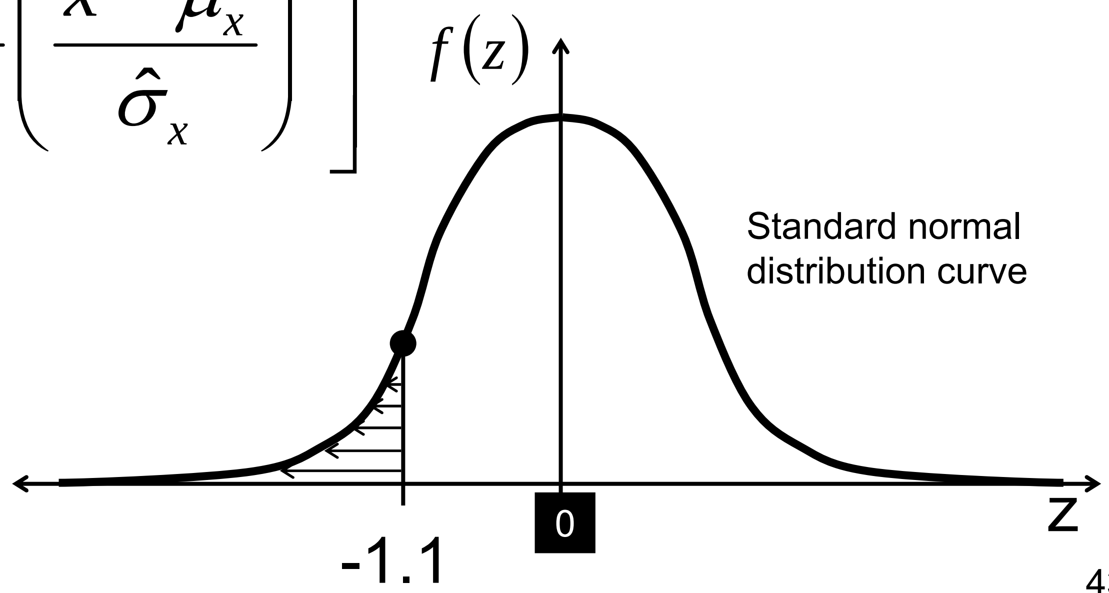
- Area under the curve = 1

$$f(z) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp\left[-\frac{1}{2}(z)^2\right]$$

This will be covered in the 2nd design lab

What if mean is not 0?

$$f(x) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\hat{\sigma}_x}\right)^2\right]$$



Non-zero means in Gaussian distributions

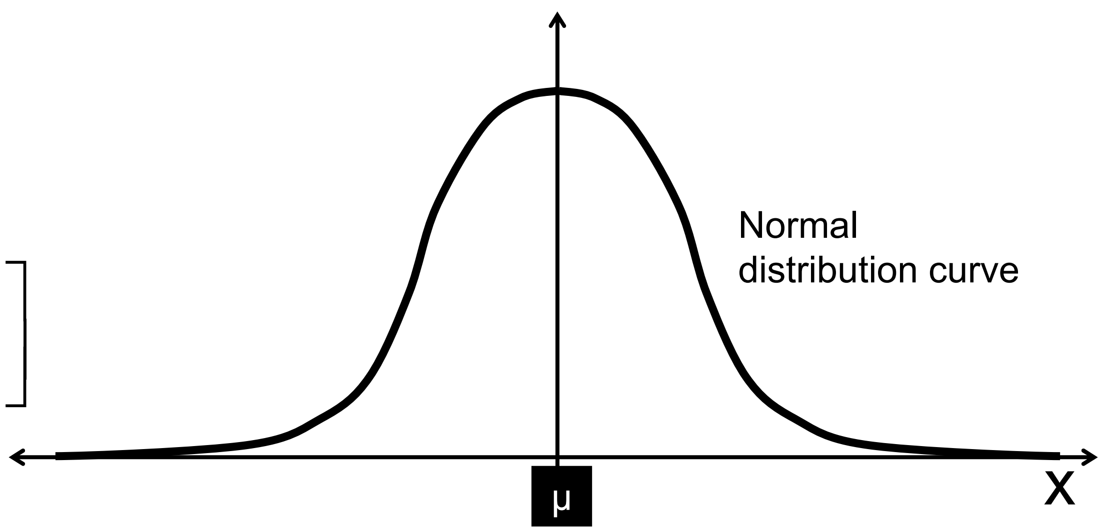
A normal Gaussian distribution is typically observed in fatigue behavior of parts

- x = variate = x   z = transformation variate   σ = standard deviation

$$f(x) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{x - \mu_x}{\hat{\sigma}_x} \right)^2 \right]$$

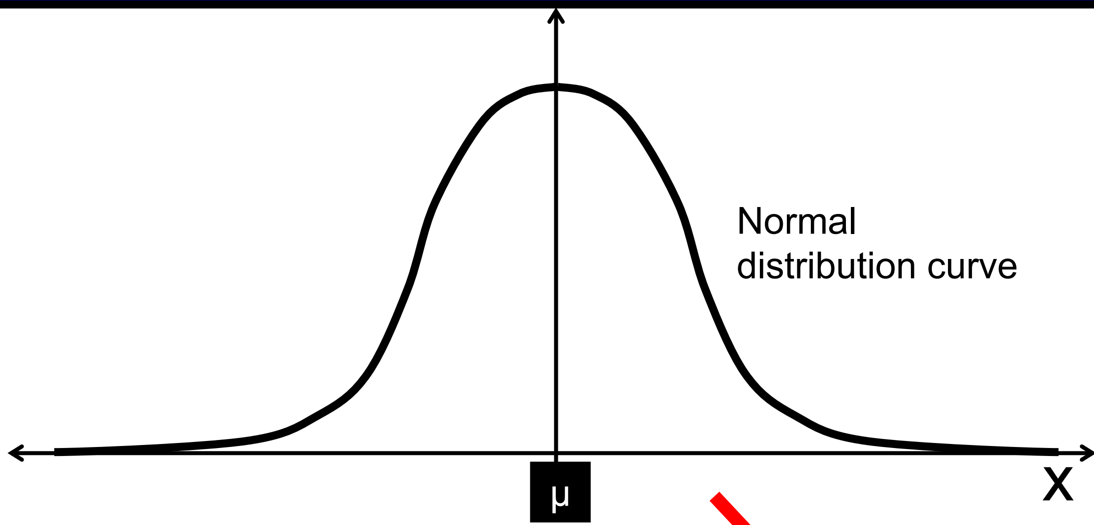
$$\mathbf{z} = \frac{\mathbf{x} - \mu_x}{\hat{\sigma}_x}$$

$$f(z) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp\left[-\frac{1}{2} (z)^2 \right]$$

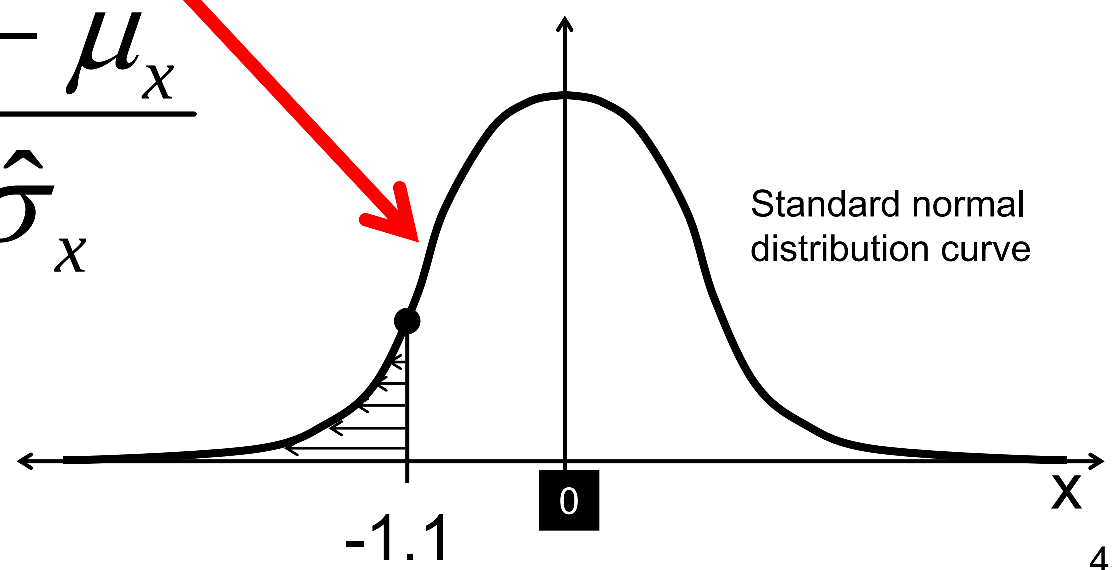


Only one table for values needed to find the area between z values…

Fatigue life modifiers: Reliability factor



$$\mathbf{z} = \frac{\mathbf{x} - \mu_x}{\hat{\sigma}_x}$$



Example use of standard normal distribution

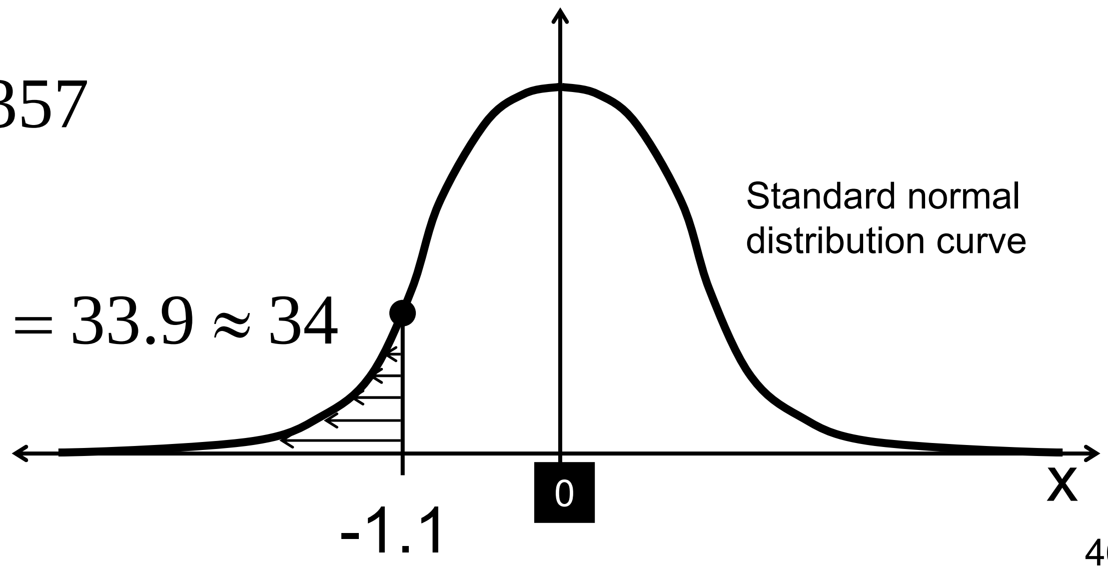
In a shipment of 250 connecting rods, the mean tensile strength is 45 kpsi and the standard deviation is 5 kpsi

- (a) Assuming a normal distribution, how many rods may be expected to have a strength less than 39.5 kpsi?
- (b) How many are expected to have a strength between 39.5kpsi and 59.5kpsi?

$$\mathbf{z}_{39.5} = \frac{\mathbf{x} - \mu_x}{\hat{\sigma}_x} = \frac{39.5 - 45.0}{5.0} = -1.10$$

$$\Phi\left(\mathbf{z}_{39.5}\right) = \Phi\left(-1.10\right) = 0.1357$$

$$N \cdot \Phi\left(\mathbf{z}_{39.5}\right) = 250 \times 0.1357 = 33.9 \approx 34$$



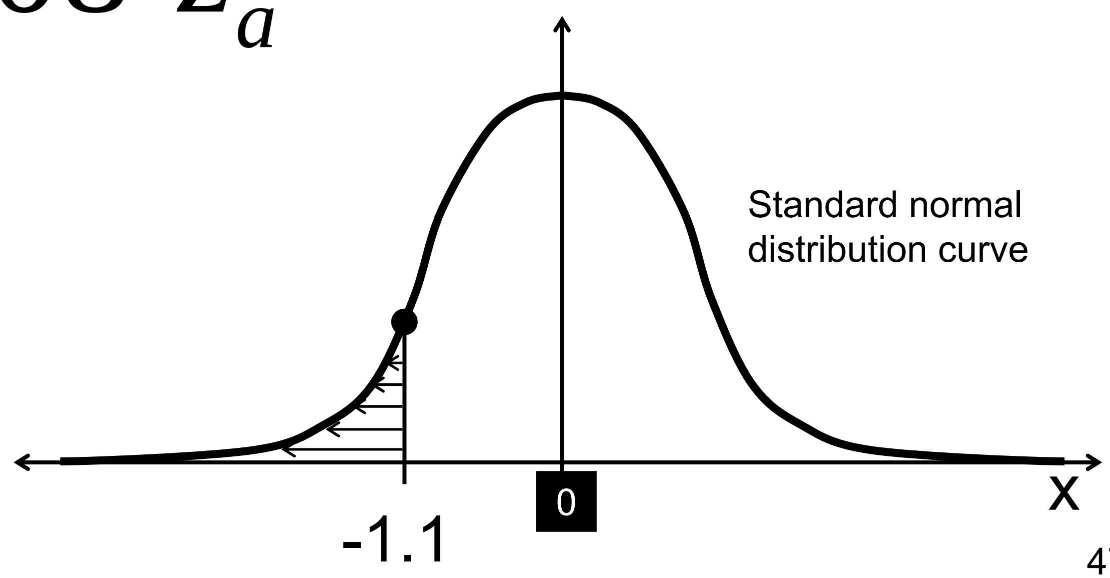
Fatigue life modifiers: Reliability factor

Most strength data is reported as mean values

- Standard deviations typically less than 8%, but you MUST KNOW what it is… run experiments…
- 68% of all measurements fall within one standard deviation
- 95% of all measurements fall within two standard deviations

- For $\sigma \sim 8\%$

$$k_e = 1 - 0.08\ z_a$$



How do we do

1st order fatigue

modeling/analysis?

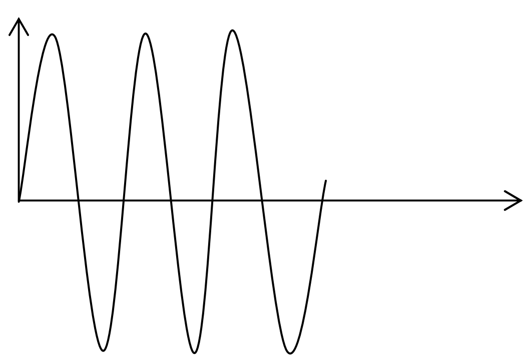
Fluctuating stresses

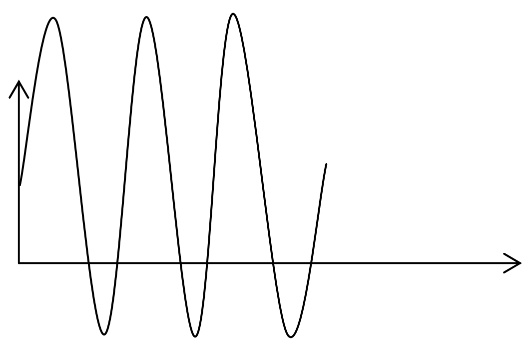
Stress values of concern

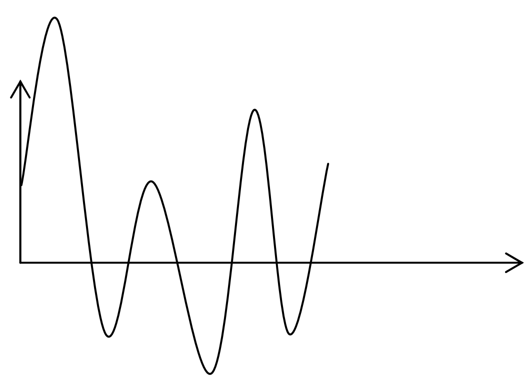
- σ_{min} Minimum stress
- σ_{max} Maximum stress
- σ_a Amplitude component = $(\sigma_{max} - \sigma_{min})/2$
- σ_m Midrange component = $(\sigma_{max} + \sigma_{min})/2$
- σ_s Steady component

- R Stress ratio = $\sigma_{min} / \sigma_{max}$
- A Amplitude ratio = σ_a / σ_m

Note the correction to σa and σm







Fluctuating stresses

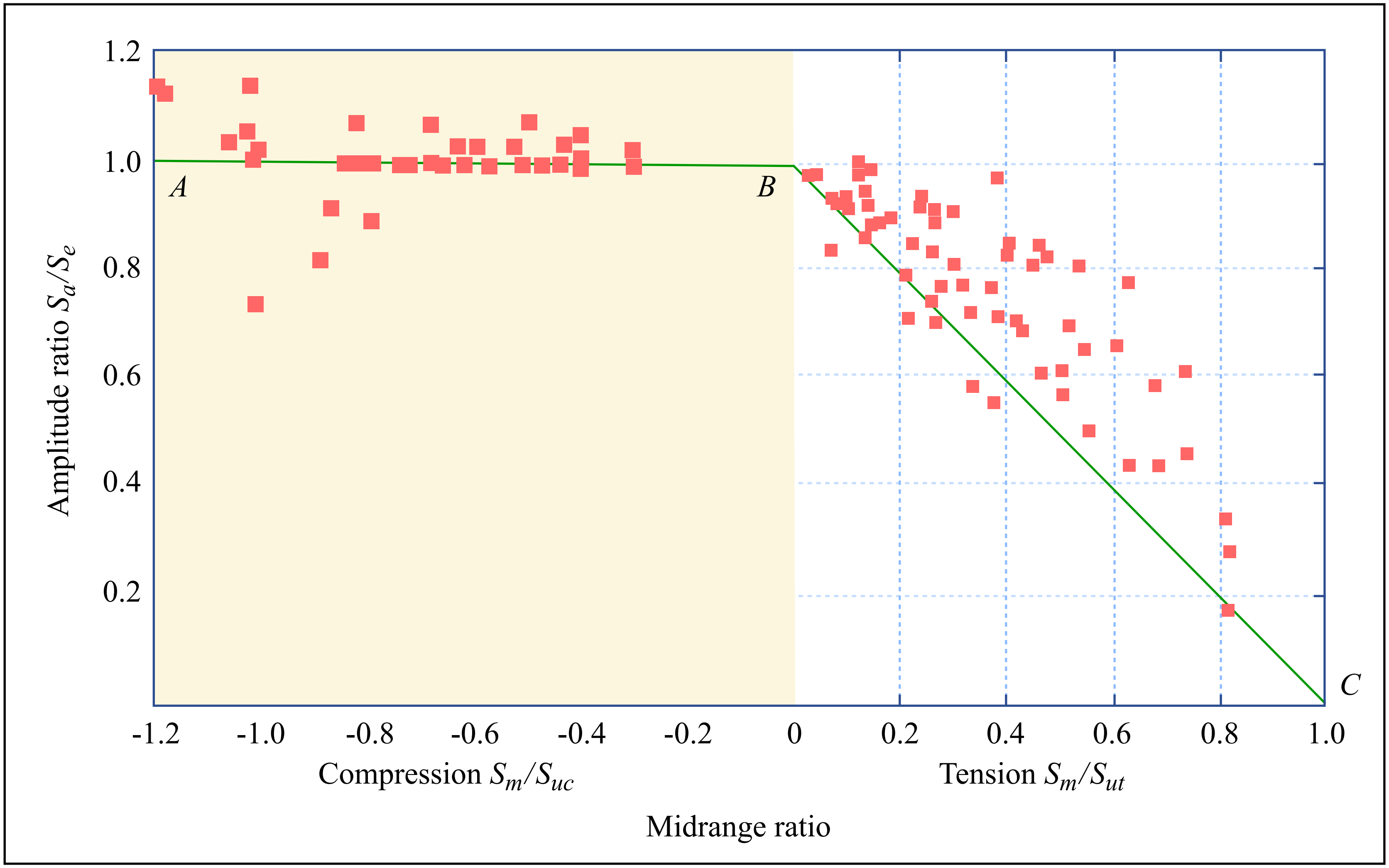


Figure by MIT OpenCourseWare. Adapted from Fig. 6-25 in Shigley & Mischke.

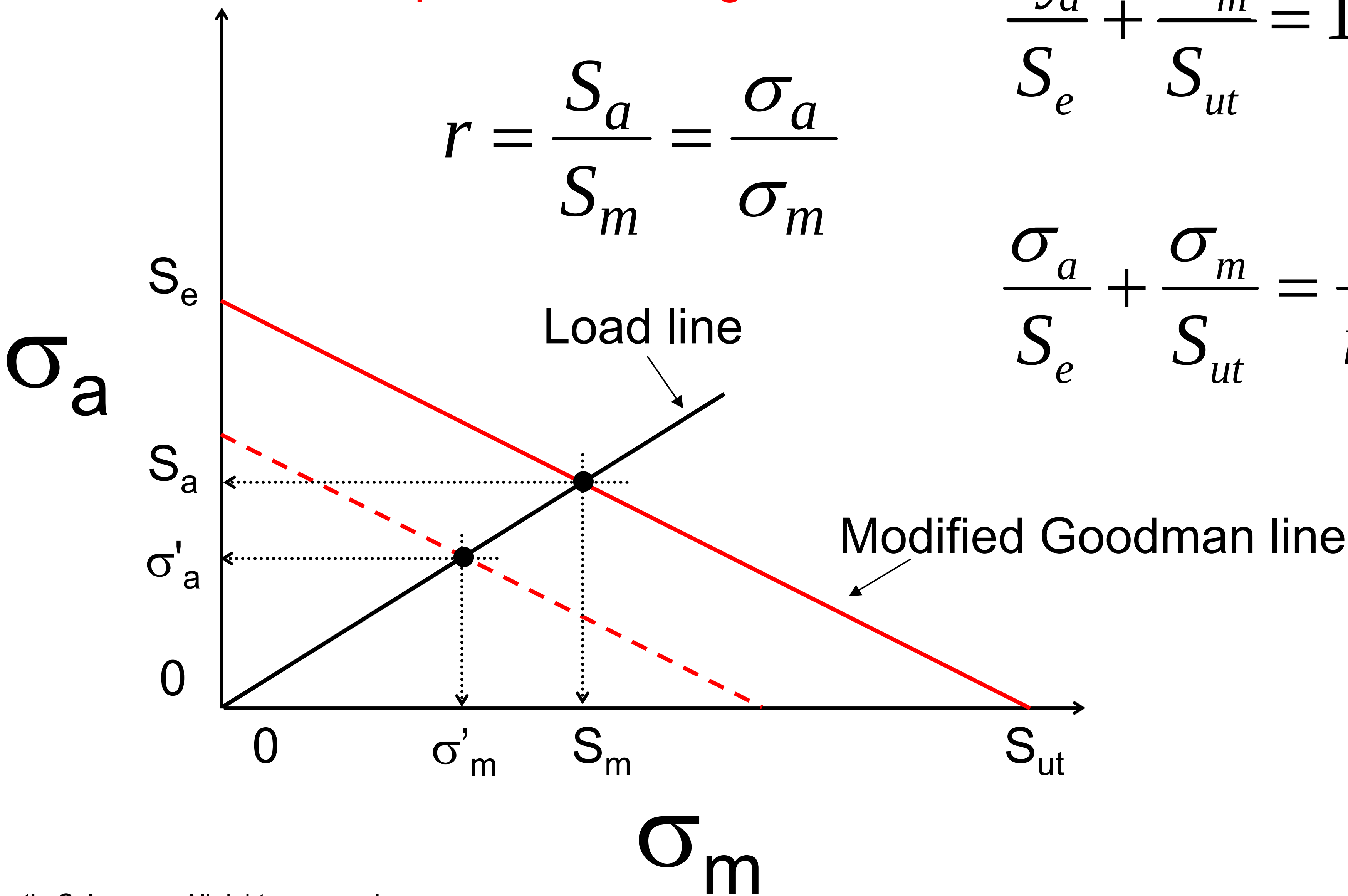
Fatigue diagram: Goodman

Capital S = strength!

$$\frac{S_a}{S_e}+\frac{S_m}{S_{ut}}=1$$

$$r=\frac{S_a}{S_m}=\frac{\sigma_a}{\sigma_m}$$

$$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_{ut}}=\frac{1}{n}$$



Stress concentration and notch sensitivity

Fatigue is due to crack propagation, hence notch sensitivity is important

- Max stress

$$\sigma_{\max} = K_f \sigma_o$$

$$\tau_{\max} = K_{fs} \tau_o$$

K_t from table A-15

- Notch sensitivity, q (usually $0 < q < 1$) accounts for material sensitivity
  - $K_f = 1 + q\,(K_t - 1)$ *Table 6-20*
  - $K_{fs} = 1 + q_{shear}(K_{ts} - 1)$ *Table 6-21*

It is always safe to use K_t

K_t rarely > 3 for good/practical designs, but check!

Example

1.5 in diameter AISI 1050 cold drawn steel (S_y = 84kpsi, S_{ut} = 100 kpsi) withstands a tensile load that ranges from 0 to 16000 lbf. K_f = 1.85, k_a = 0.797, k_b = 1, k_d = 1, k_c = 0.923. (8th edition has k_c = 0.85)

Modifications in example:

- kc = 0.85 in 8th edition

Example modified for 8th edition

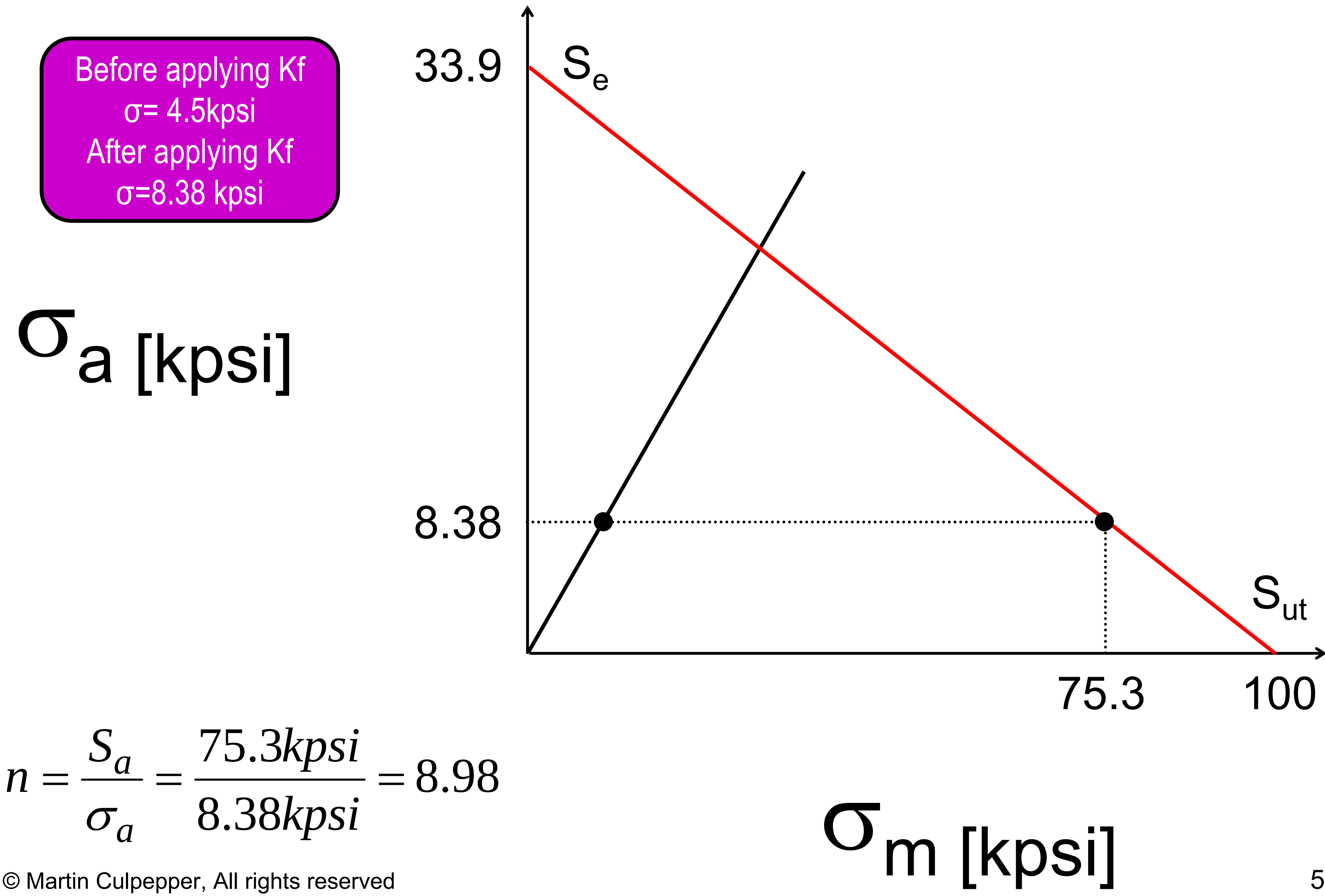
- Se = ½ Sut in 8th edition

- a. Factor of safety if σa held constant
- b. Factor of safety if σm held constant
- c. Factor of safety if σa/σm = constant

- $S_e = k_a k_b k_c k_d S'_e = k_a k_b k_c k_d\,(0.5\ S_{ut})$ = 33.9kpsi

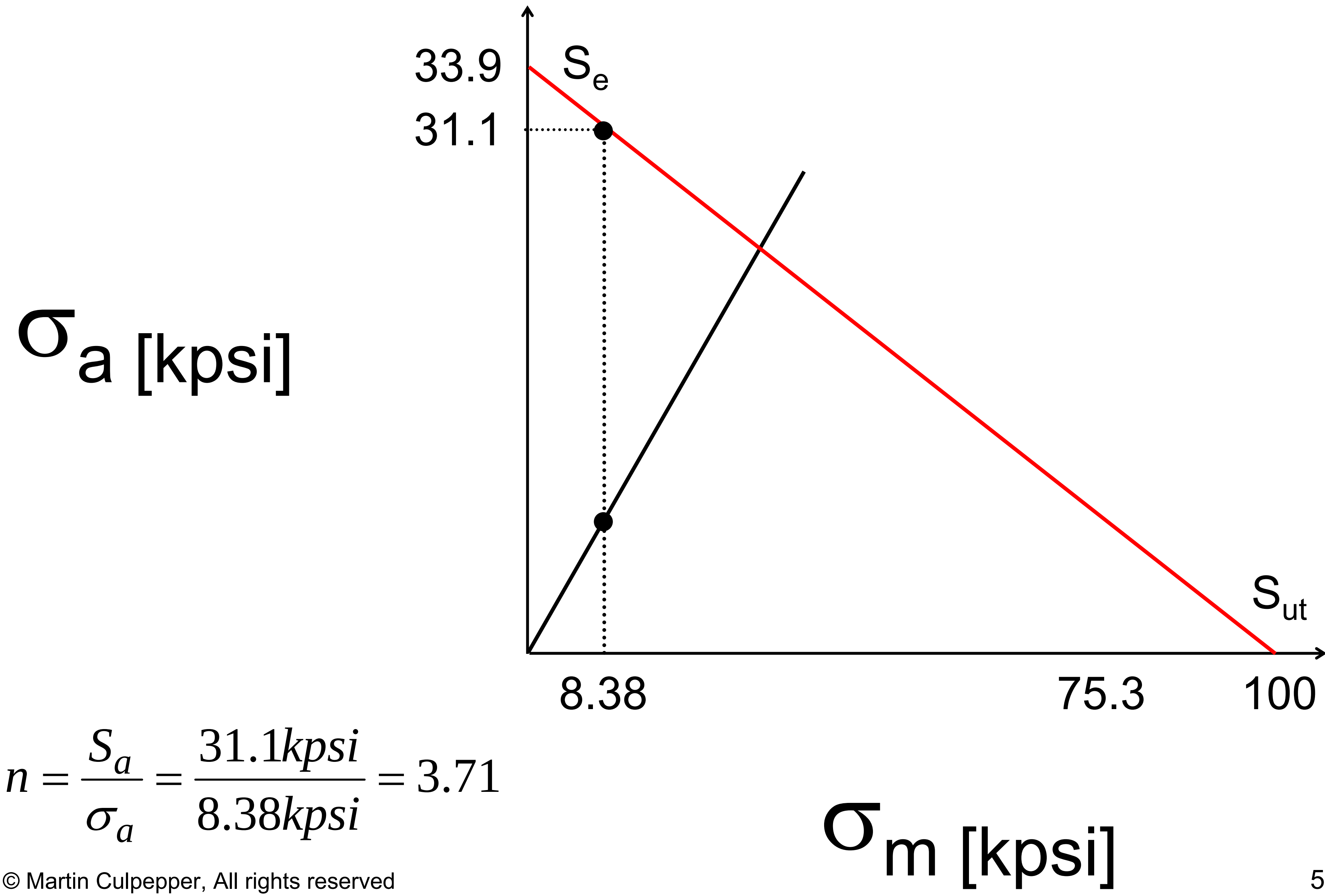
Part b: σ_a held constant

Before applying Kf
σ= 4.5kpsi
After applying Kf
σ=8.38 kpsi



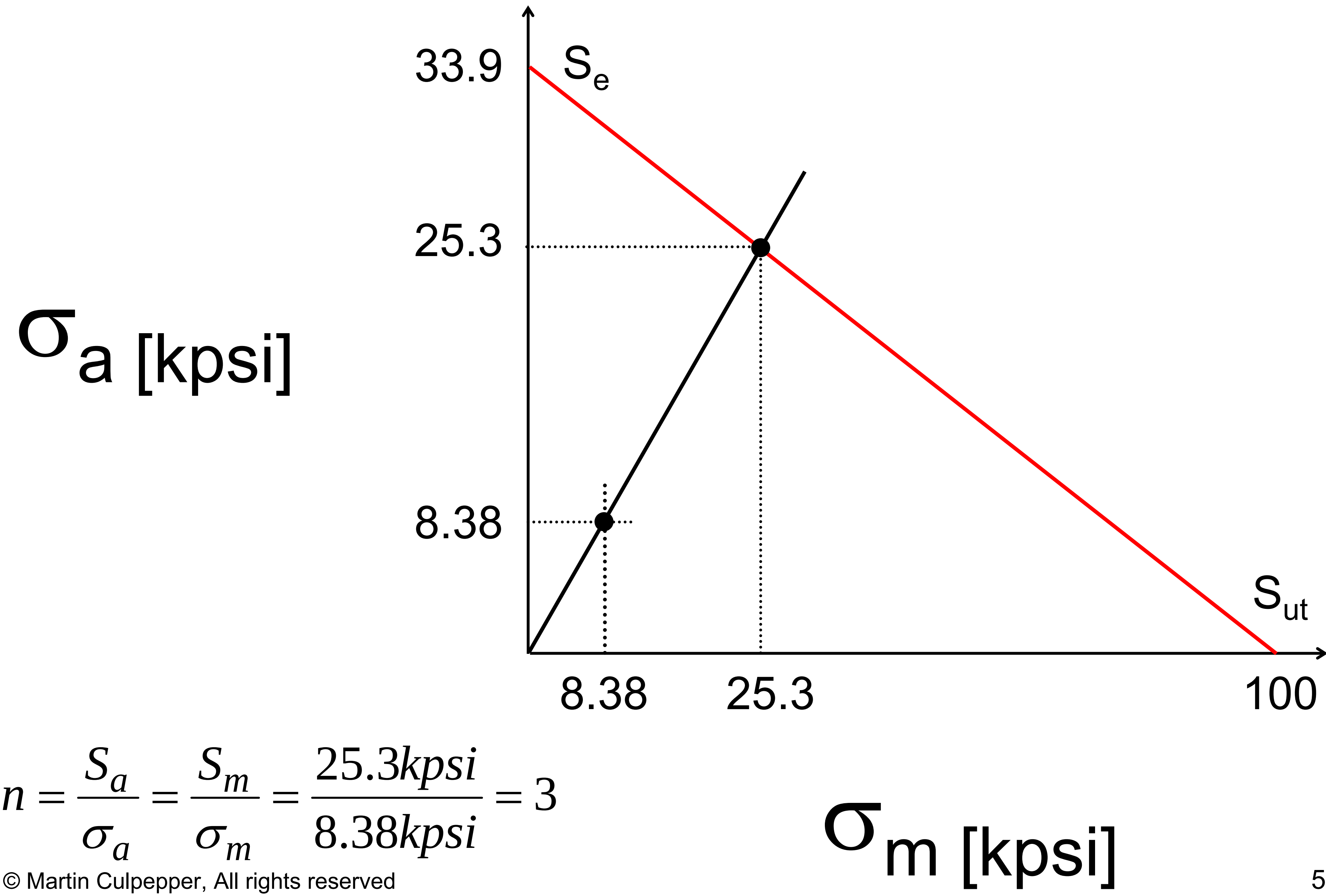
$$n = \frac{S_a}{\sigma_a} = \frac{75.3kpsi}{8.38kpsi} = 8.98$$

Part a: σ_m held constant



$$n = \frac{S_a}{\sigma_a} = \frac{31.1kpsi}{8.38kpsi} = 3.71$$

Part c: σ_a / σ_m held constant



$$n = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{25.3kpsi}{8.38kpsi} = 3$$

What about your shaft?

Step 1: Free body diagram

- Cutting forces (2.008 person and/or next week)
- Driving loads
- Reaction loads
- Preloads
- Others… OS! loads?

I can be here Saturday to help, if people ask!!!

Step 2: Parametric geometry & load variables

Step 3: Material properties

Step 4: Force magnitudes estimates/calculations

Step 5: Stress & fatigue

- σ_x, σ_y, τ_{xy}, K_t, q, K_f, σ_a, σ_m, σ_s, σ_x,

In the end, SH… so you should program this into excel:

- In case you need to change variables… there are always changes!
- Optimization in excel.