2.72 Elements of Mechanical Design

Spring 2009

2.72
Elements of Mechanical Design

Lecture 05: Structures

Schedule and reading assignment

Quizzes

- Quiz – None

Topics

- Finish fatigue
- Finish HTMs in structures

Reading assignment

- None
- Quiz next time on HTMs

Matrix Review

What is a Matrix?

A matrix is an easy way to represent a system of linear equations

Linear algebra is the set of rules that governs matrix and vector operations

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

"Vector"

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$$

"Matrix"

Matrix Addition/Subtraction

You can only add or subtract matrices of the same dimension

Operations are carried out entry by entry

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$$

(2 x 2) (2 x 2) (2 x 2)

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} - \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 - b_1 & a_2 - b_2 \\ a_3 - b_3 & a_4 - b_4 \end{bmatrix}$$

(2 x 2) (2 x 2) (2 x 2)

Matrix Multiplication

An *m* x *n* matrix times an *n* x *p* matrix produces an *m* x *p* matrix

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{bmatrix}$$

(2 x 2) (2 x 2) (2 x 2)

Matrix Properties

Notation: A, B, C **= matrix ,** c **= scalar**

Cumulative Law: $A + B = B + A$

Distributive Law: $c(A + B) = cA + cB$

$C(A + B) = CA + CB$

Associative Law: $A + (B - C) = (A + B) - C$

$A(BC) = (AB)C$

NOTE that AB does not equal BA !!!!!!!

Matrix Division

To divide in linear algebra we multiply each side by an inverse matrix:

$$AB = C$$

$$A^{-1}AB = A^{-1}C$$

$$B = A^{-1}C$$

Inverse matrix properties:

$$A^{-1}A = AA^{-1} = I$$ (The identity matrix)

$$(AB)^{-1} = B^{-1}A^{-1}$$

Structures

Machines structures

Structure = backbone = affects everything

Satisfies a multiplicity of needs

- Enforcing geometric relationships (position/orientation)
- Material flow and access
- Reference frame

Requires first consideration and serves to link modules:

- Joints (bolted/welded/etc…)
- Bearings
- Shafts
- Parts
- Tools
- Sensors
- Actuators

Image removed due to copyright restrictions. Please see

http://www.clarkmachinetools.com/2003_1.jpg

Key issues with structural design

Machine concepts

- Topology
- Material properties

Image removed due to copyright restrictions. Please see
http://www.fortune-cnc.com/uploads/images/1600ge_series.jpg

Principles

- Thermomechanical
- Elastomechanics
- Kinematics
- Vibration

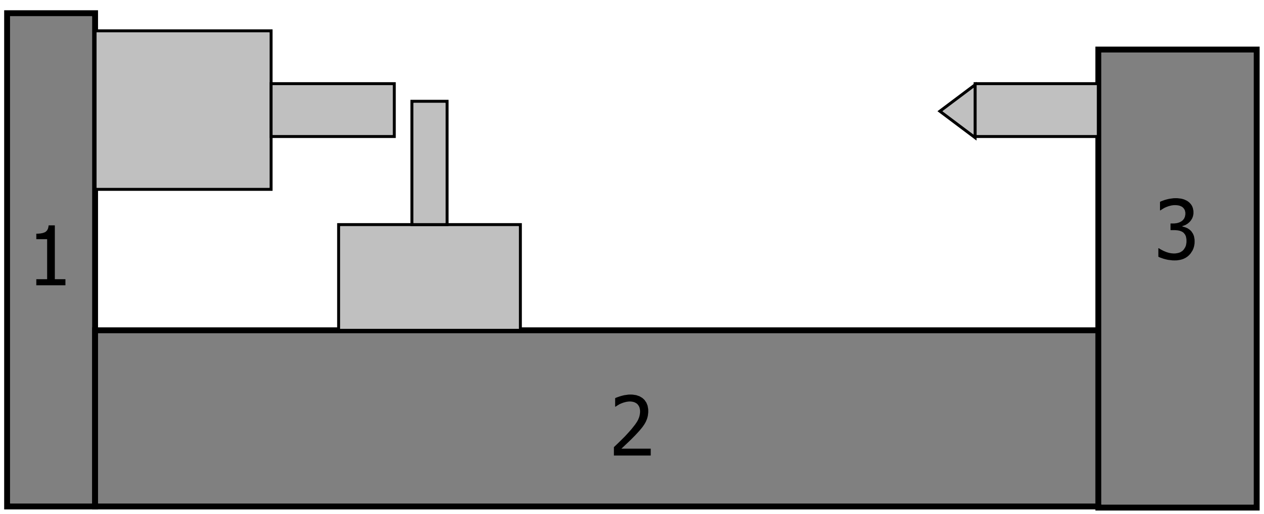
Key tools that help

- Stick figures
- Parametric system/part error model

Visualization of the:
Load path
Vibration modes
Thermal growth

Modeling: stick figures

Image removed due to copyright restrictions. Please see
http://americanmachinetools.com/images/diagram-lathe.jpg

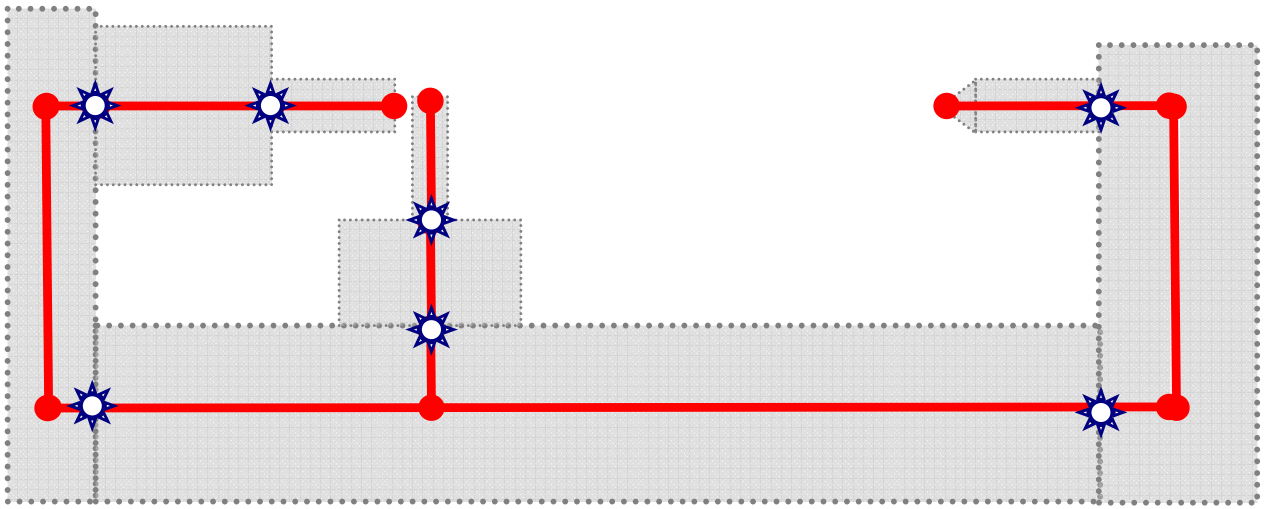


Modeling: stick figures

1. Stick figures

2. Beam bending

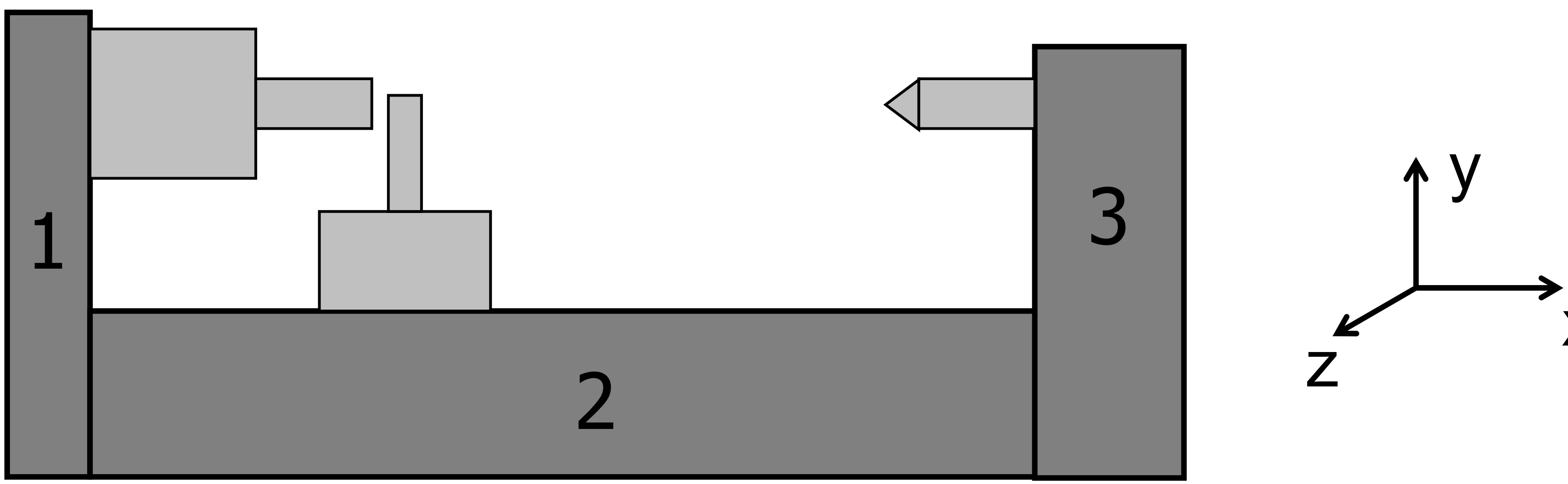
3. System bend.



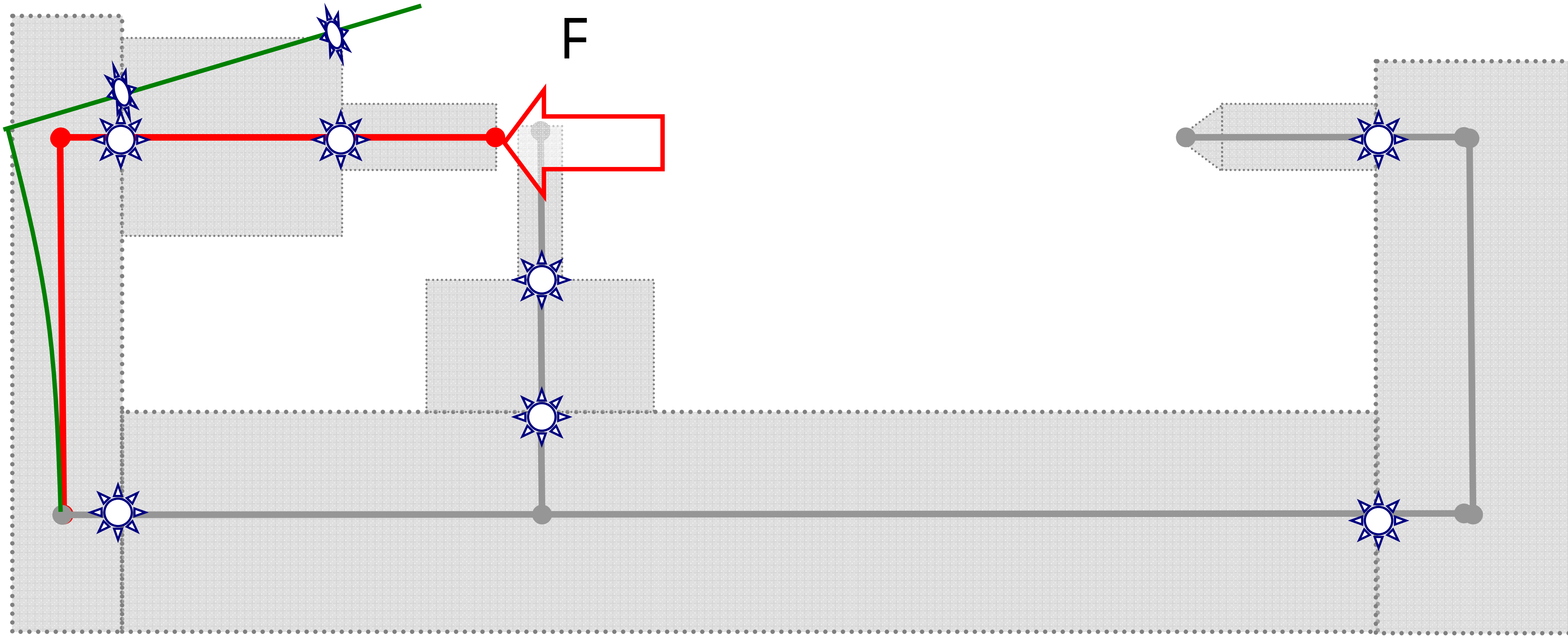
These types of models are idealizations of the physical behavior. The designer must KNOW:

(a) if beam bending assumptions are valid

(b) how to interpret and use the results o this type of these models

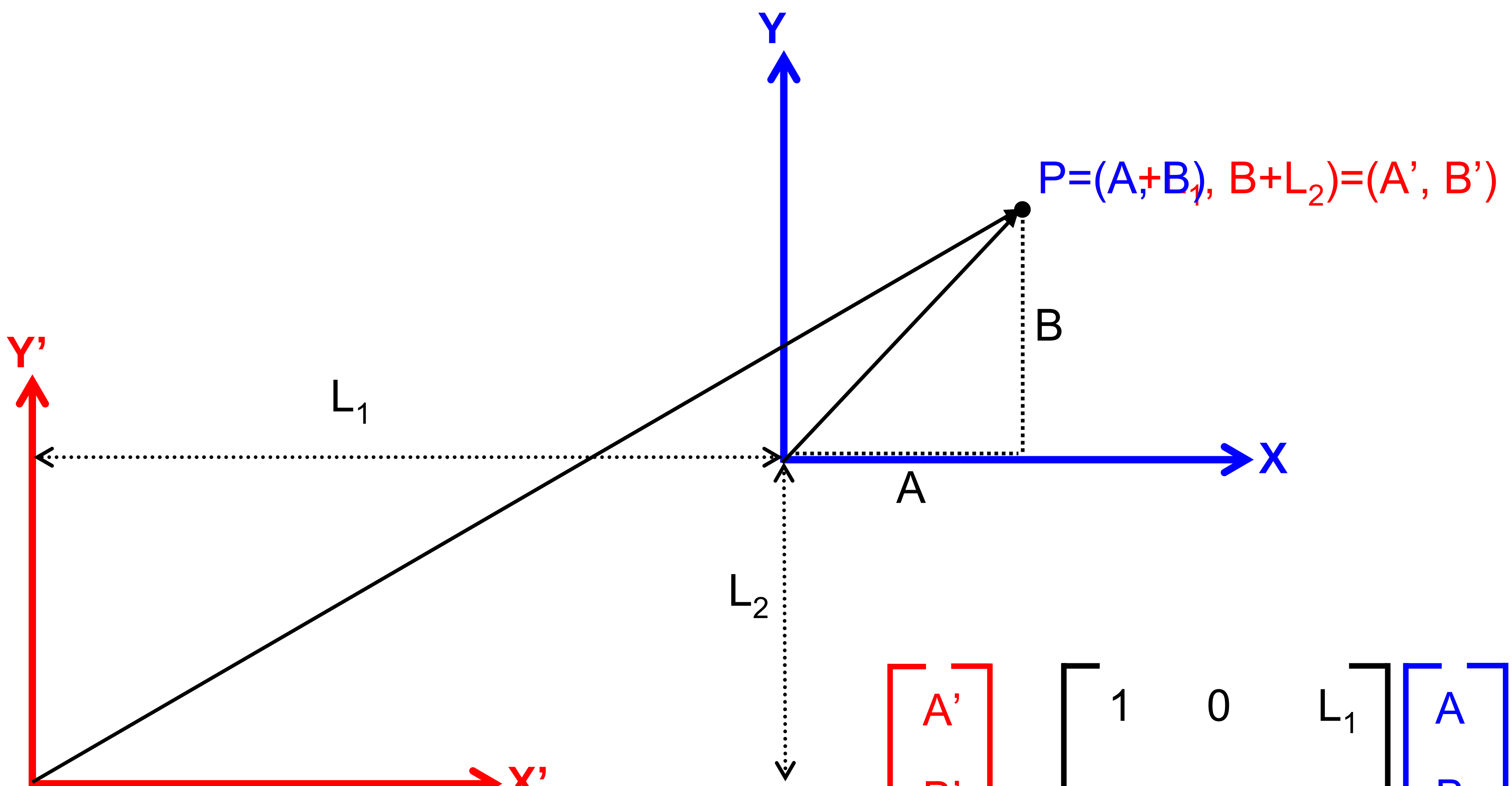


Modeling: stick figures



Transformation Matrices

Translational Transformation Matrix



$$\begin{bmatrix} A' \\ B' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1 \\ 0 & 1 & L_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ 1 \end{bmatrix}$$

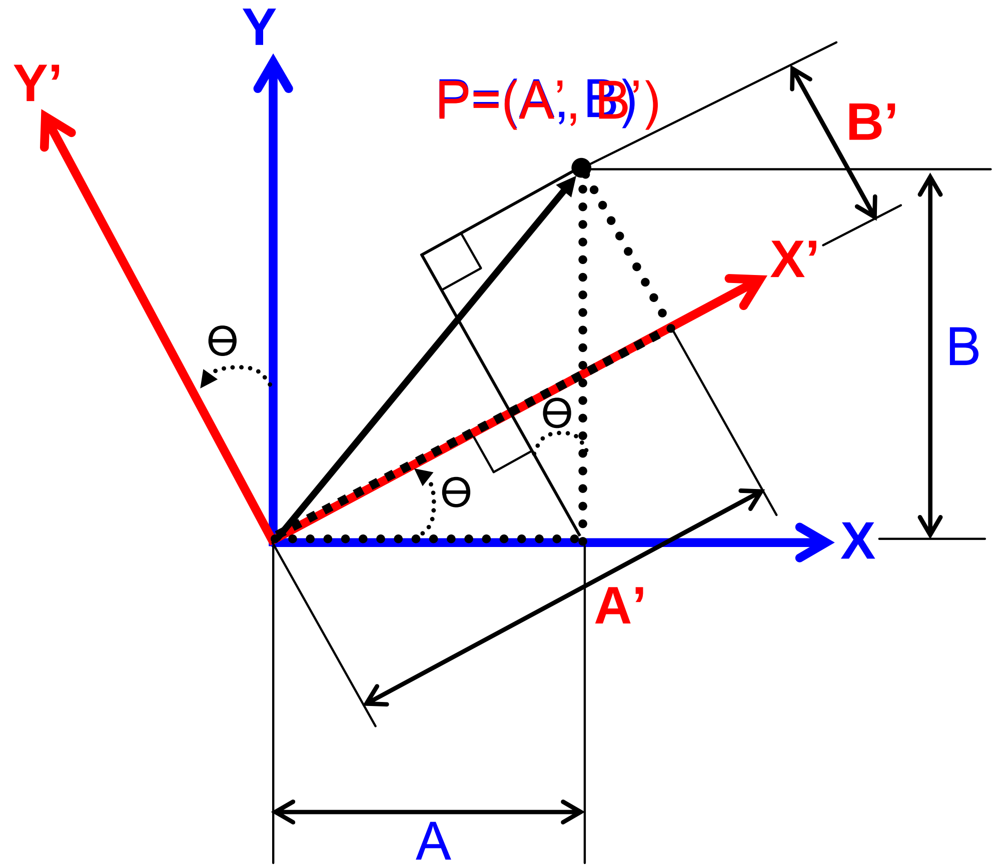
Translational Transformation Matrix

General 2D transformation matrix

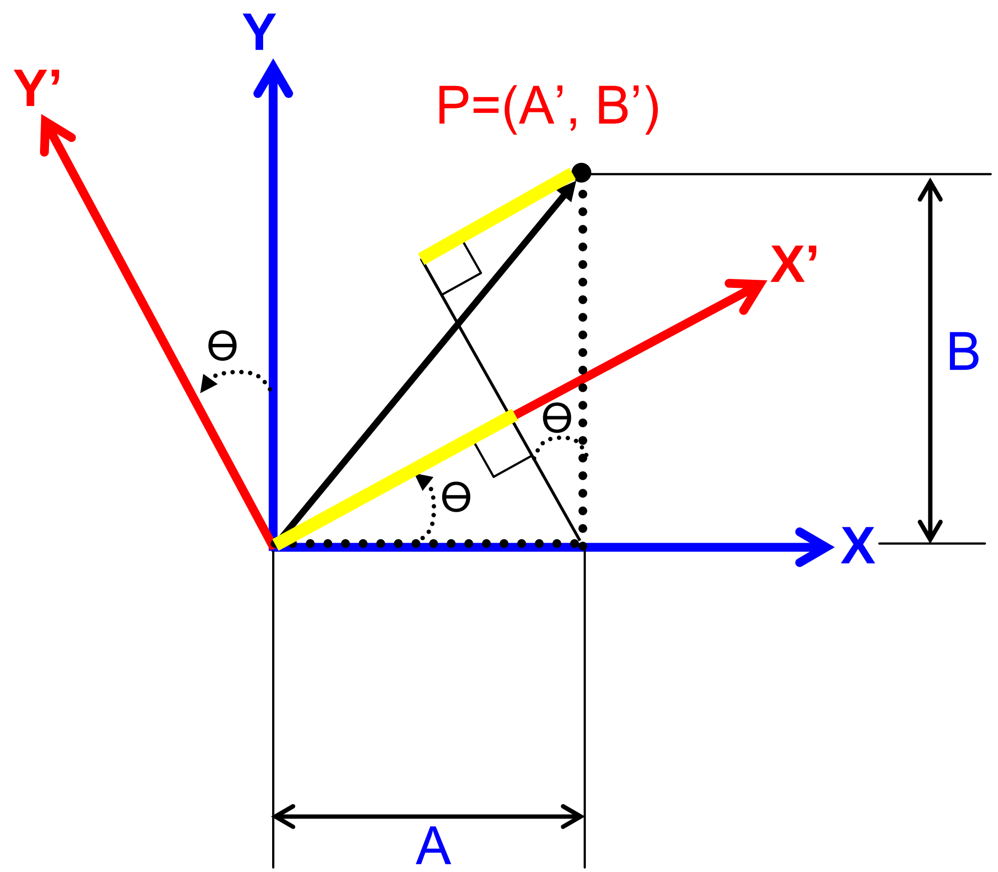
$$\begin{bmatrix} 1 & 0 & \Delta X \\ 0 & 1 & \Delta Y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & L_1 \\ 0 & 1 & L_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotational Transformation Matrix

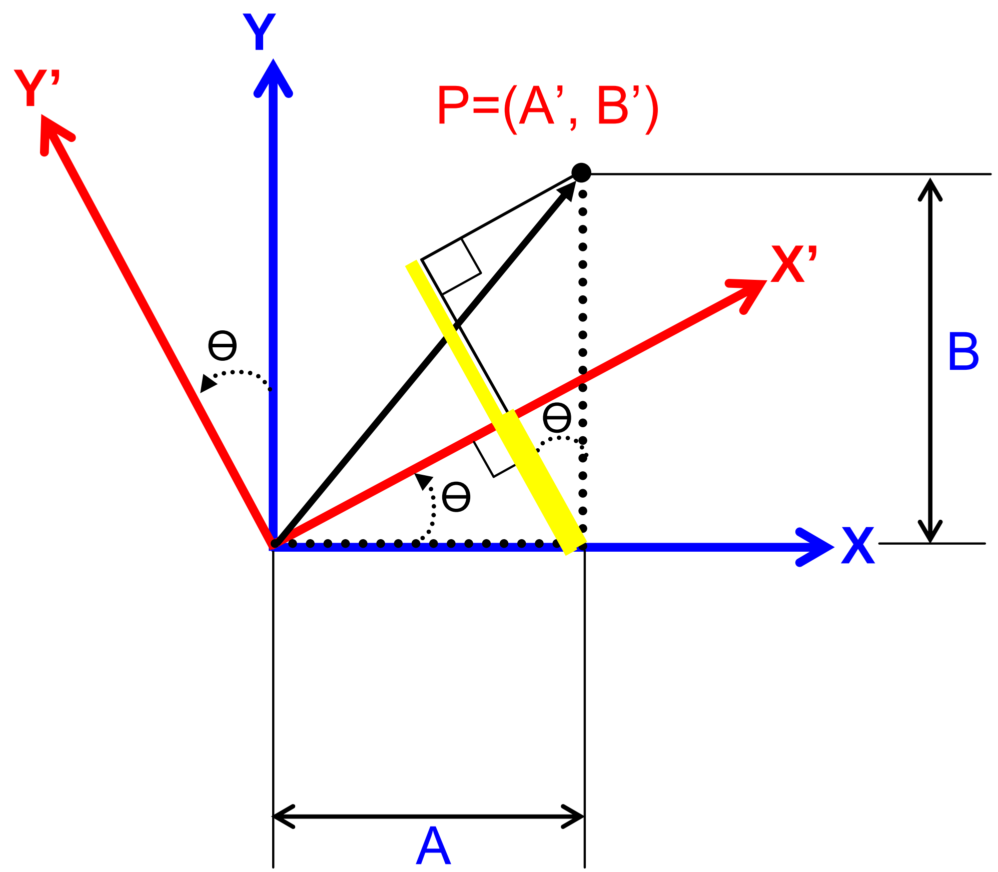


Rotational Transformation Matrix



A' = A cosΘ + B sin Θ

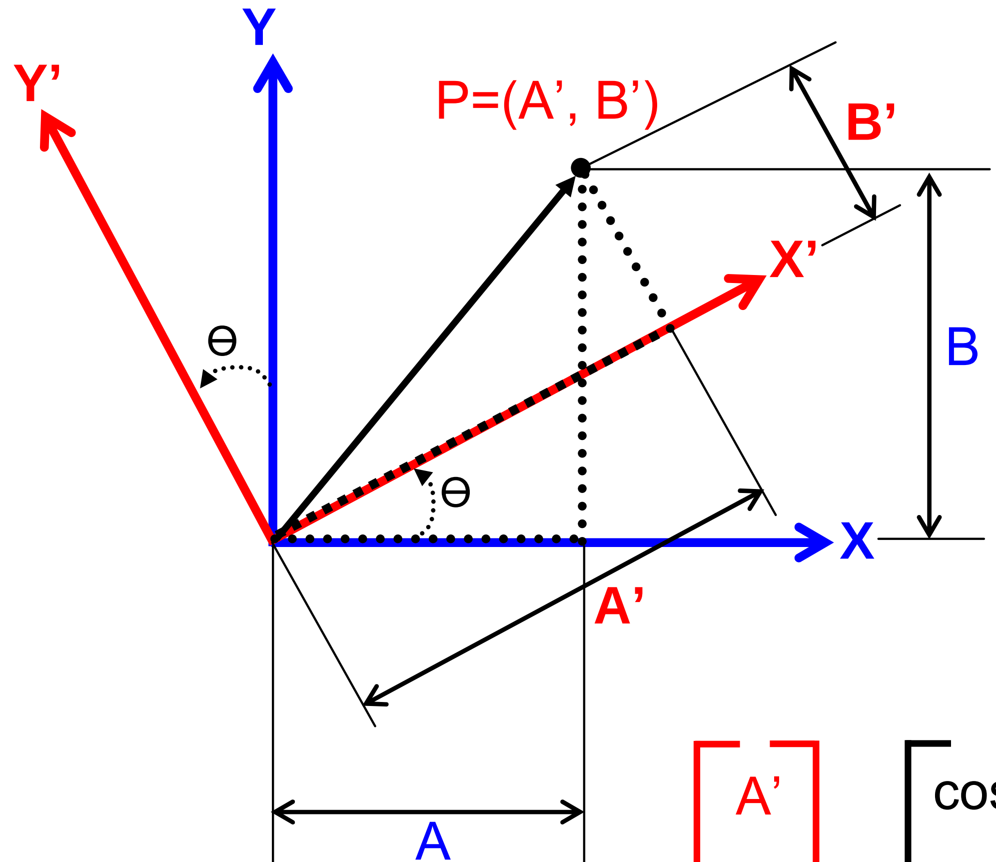
Rotational Transformation Matrix



A' = A cosϴ + B sin ϴ

B' = -A sinϴ + B cos ϴ

Rotational Transformation Matrix



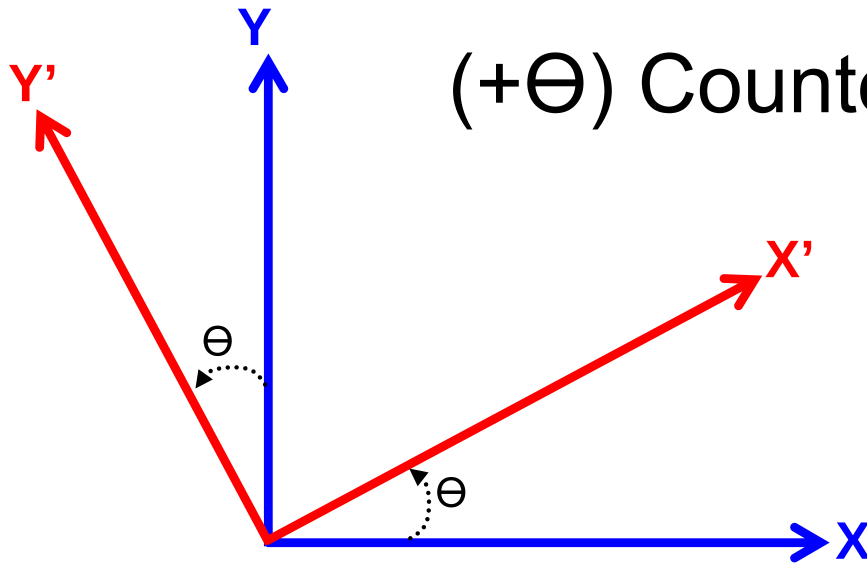
$$A' = A\cos\theta + B\sin\theta$$

$$B' = -A\sin\theta + B\cos\theta$$

$$\begin{bmatrix} A' \\ B' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ 1 \end{bmatrix}$$

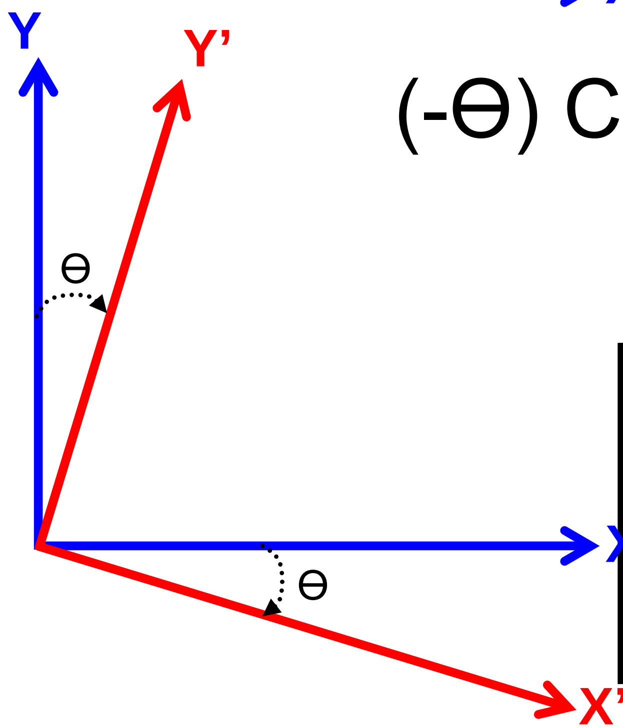
Rotational Transformation Matrix

General 2D rotational matrix:



(+Θ) Counter Clockwise

$$\begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(-Θ) Clockwise

$$\begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

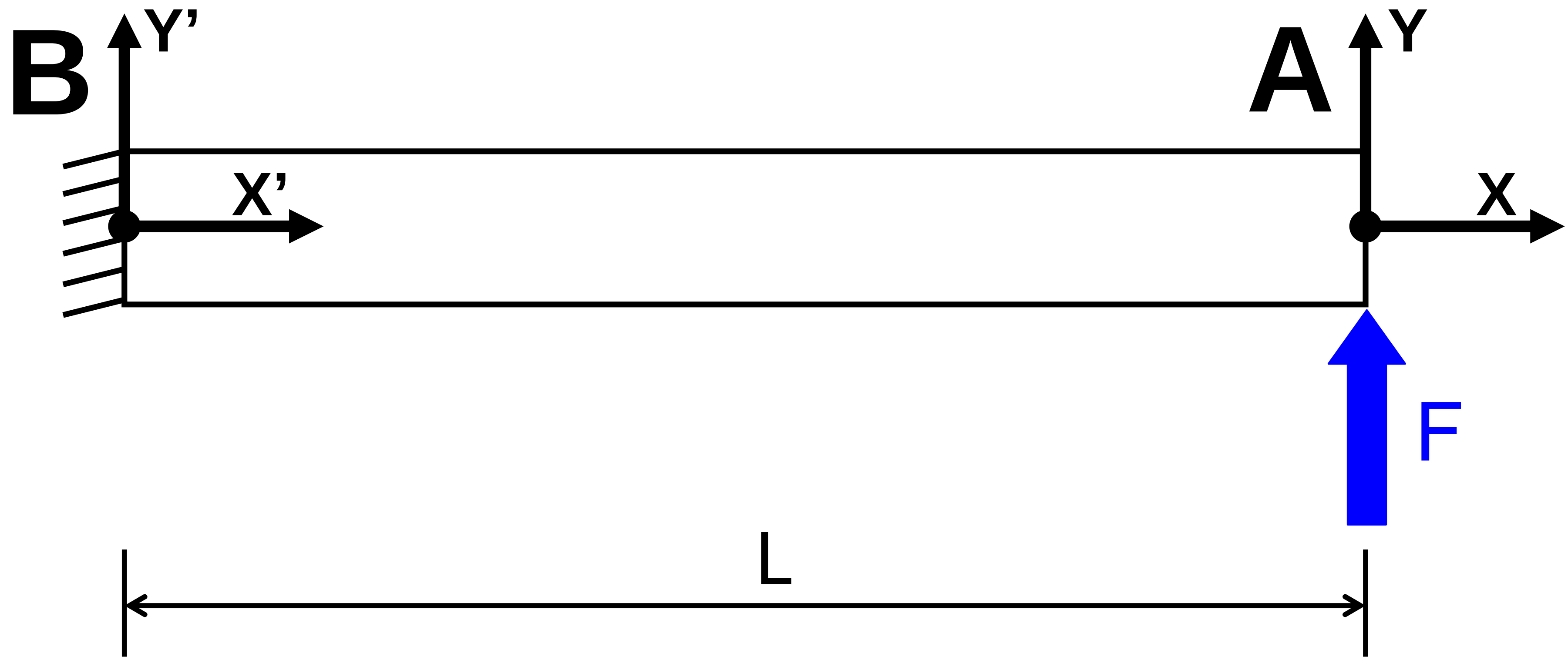
Homogeneous Transformation Matrix

General 2D HTM translational and rotational matrix:

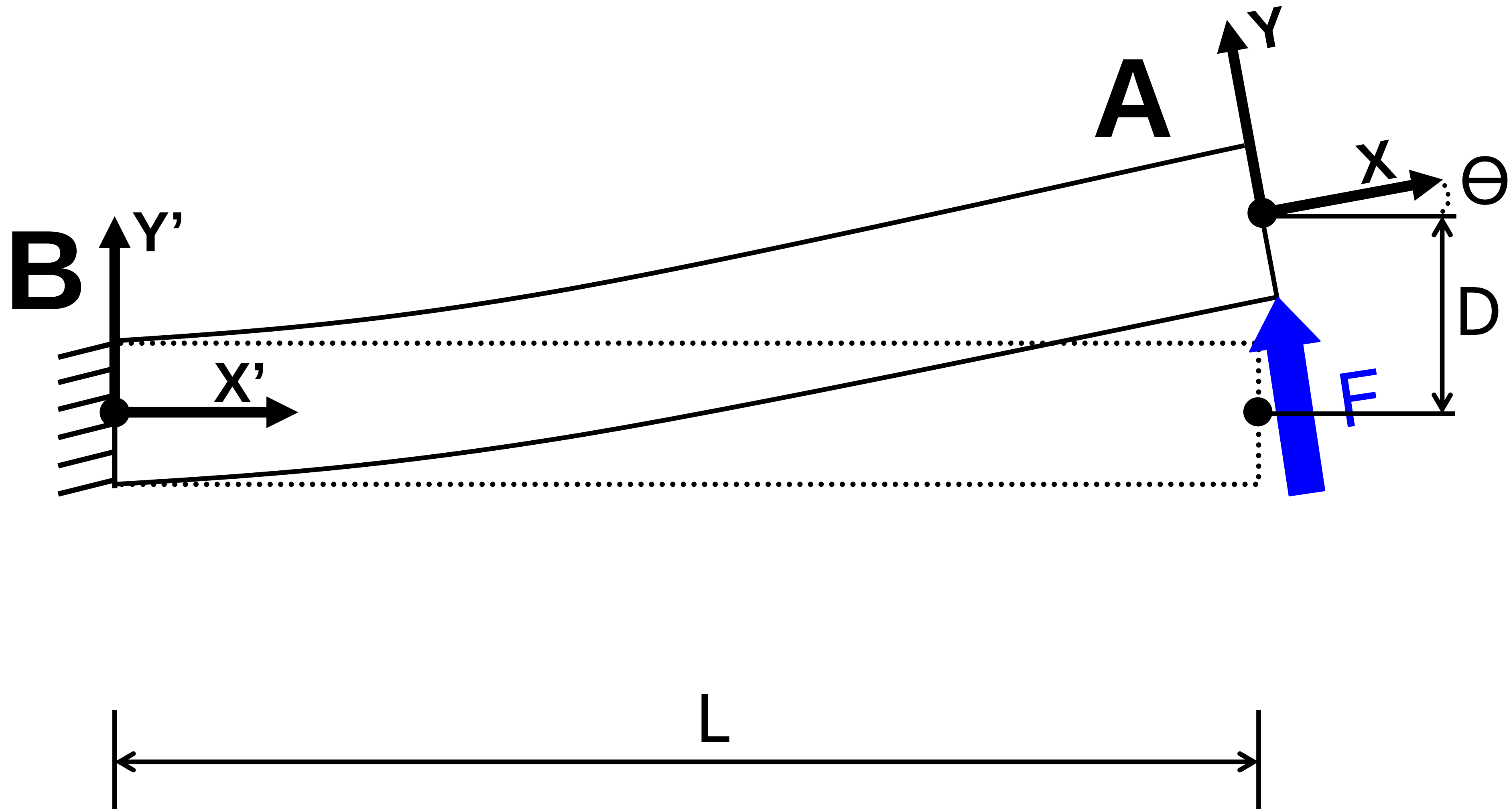
$$\begin{bmatrix} \cos\Theta & \sin\Theta & \Delta x \\ -\sin\Theta & \cos\Theta & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

HTM Applications

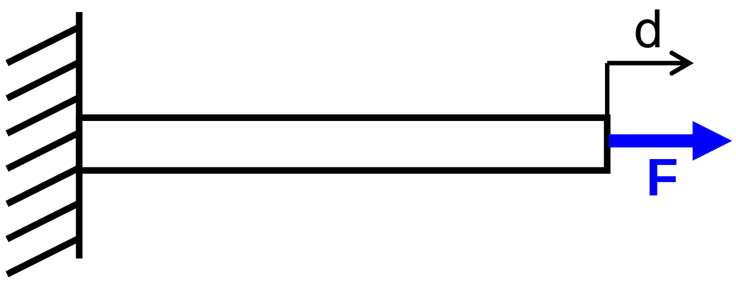
Simple Beam Example



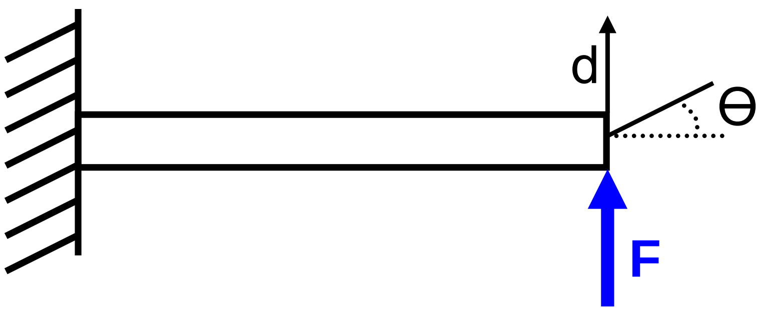
Simple Beam Example



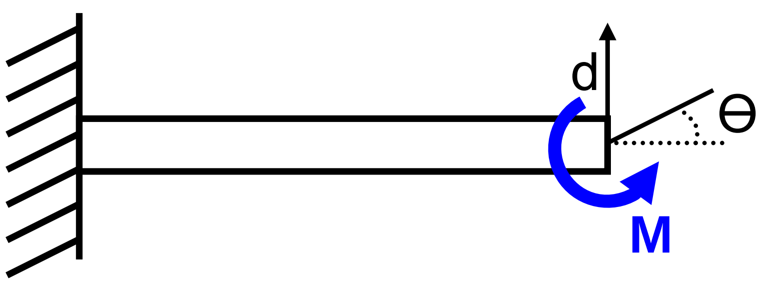
Useful Force-deflection Equations



$$d = \frac{FL}{EA}$$

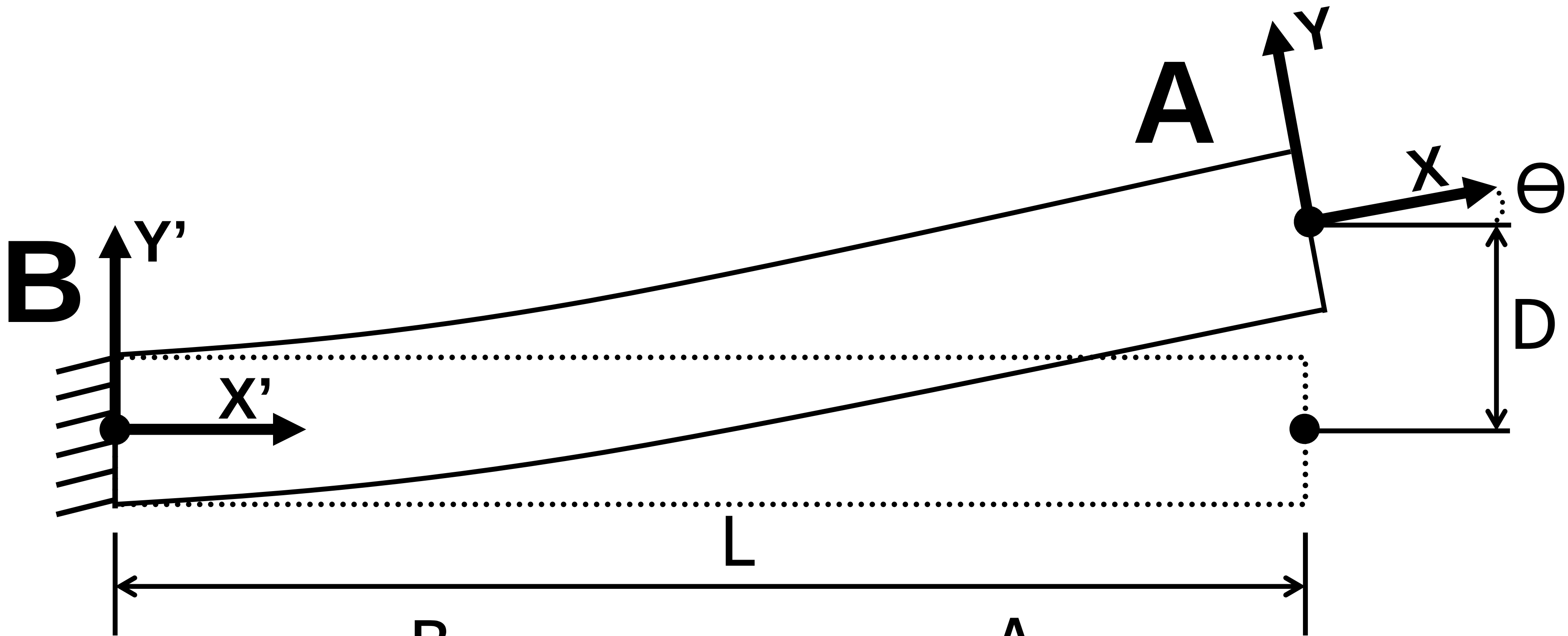


$$d = \frac{FL^3}{3EI} \qquad \Theta = \frac{FL^2}{2EI}$$



$$d = \frac{ML^2}{2EI} \qquad \Theta = \frac{ML}{EI}$$

Simple Beam Example

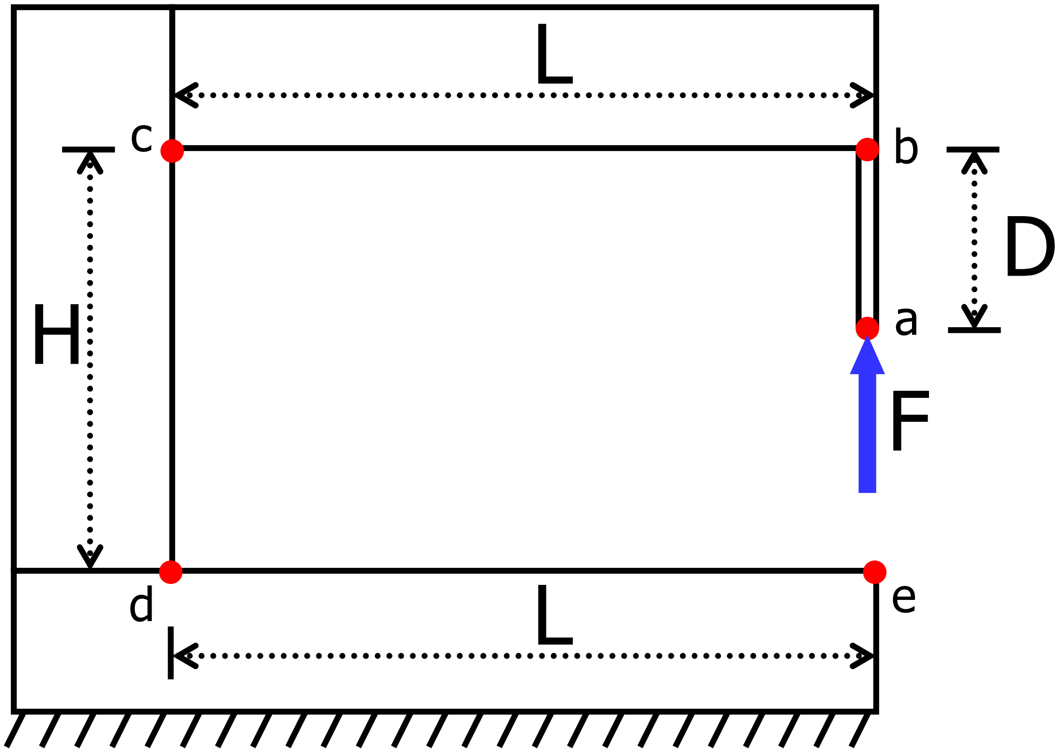


$$^{B}\begin{bmatrix} L \\ \\ 1 \end{bmatrix} \quad {}^{B}H_{A} = \begin{bmatrix} \cos\Theta & -\sin\Theta & L \\ \sin\Theta & \cos\Theta & D \\ 0 & 0 & 1 \end{bmatrix} \, ^{A}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$D = \frac{FL^3}{3EI}$$

$$\Theta = \frac{FL^2}{2EI}$$

Drill Press Example

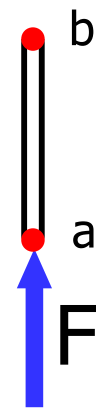


Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

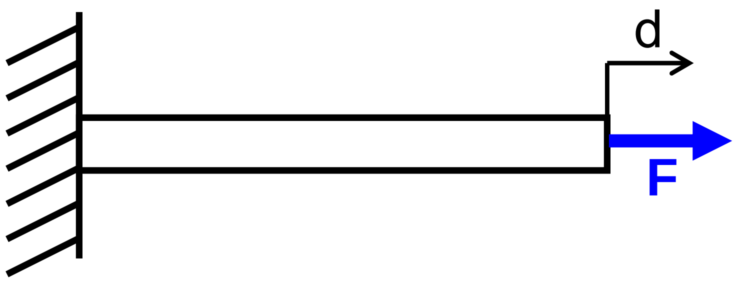
Young's Modulus of Material = E

Find the HTM from a to b:

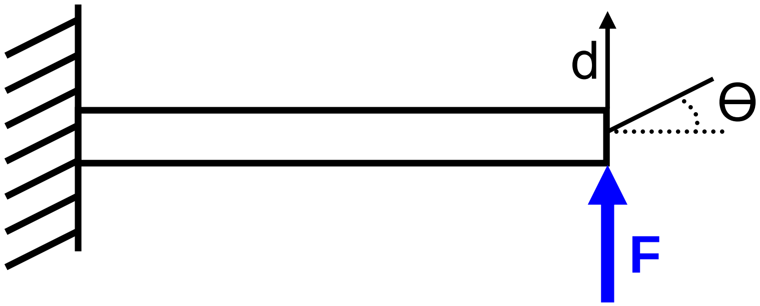


$$
{}^{b}\mathbf{H}_{a} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -(D-\delta) \\ 0 & 0 & 1 \end{bmatrix}
$$

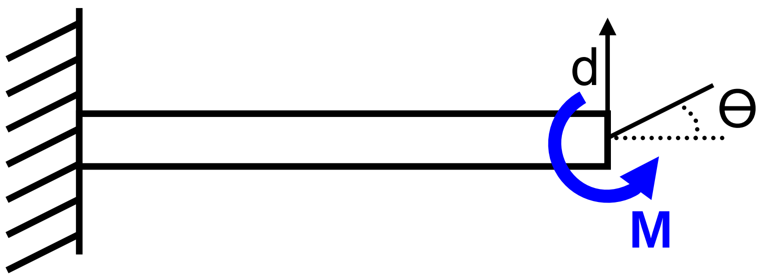
Useful Force-deflection Equations



$$d = \frac{FL}{EA}$$

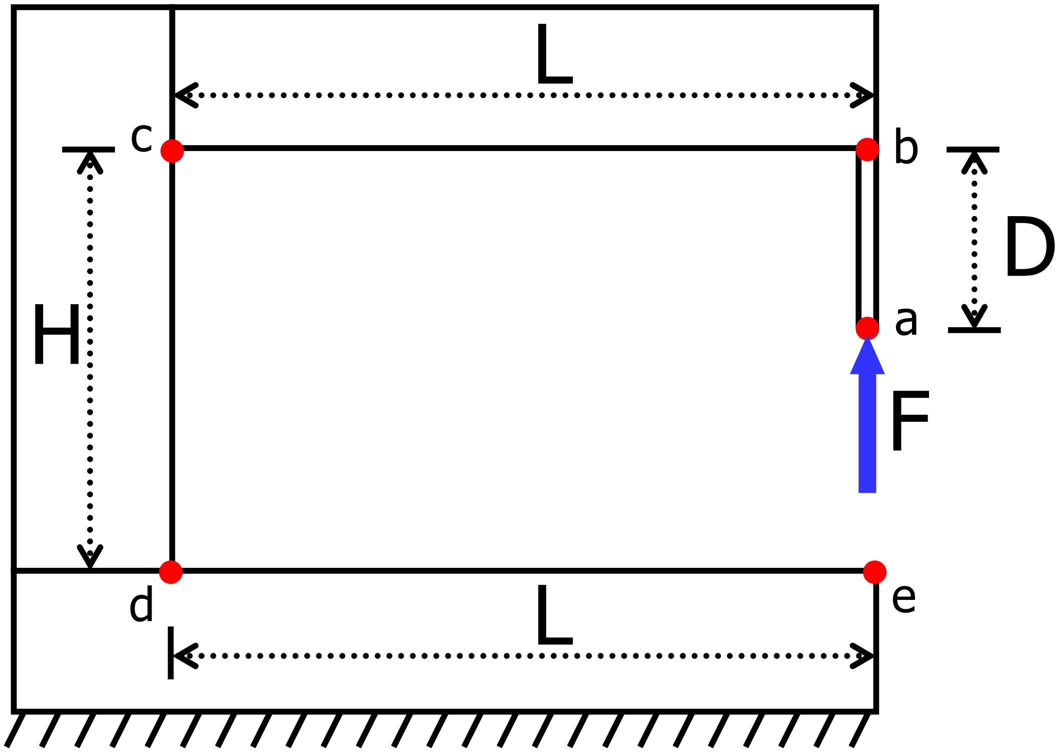


$$d = \frac{FL^3}{3EI} \qquad \theta = \frac{FL^2}{2EI}$$



$$d = \frac{ML^2}{2EI} \qquad \theta = \frac{ML}{EI}$$

Drill Press Example

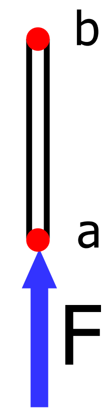


Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

Young's Modulus of Material = E

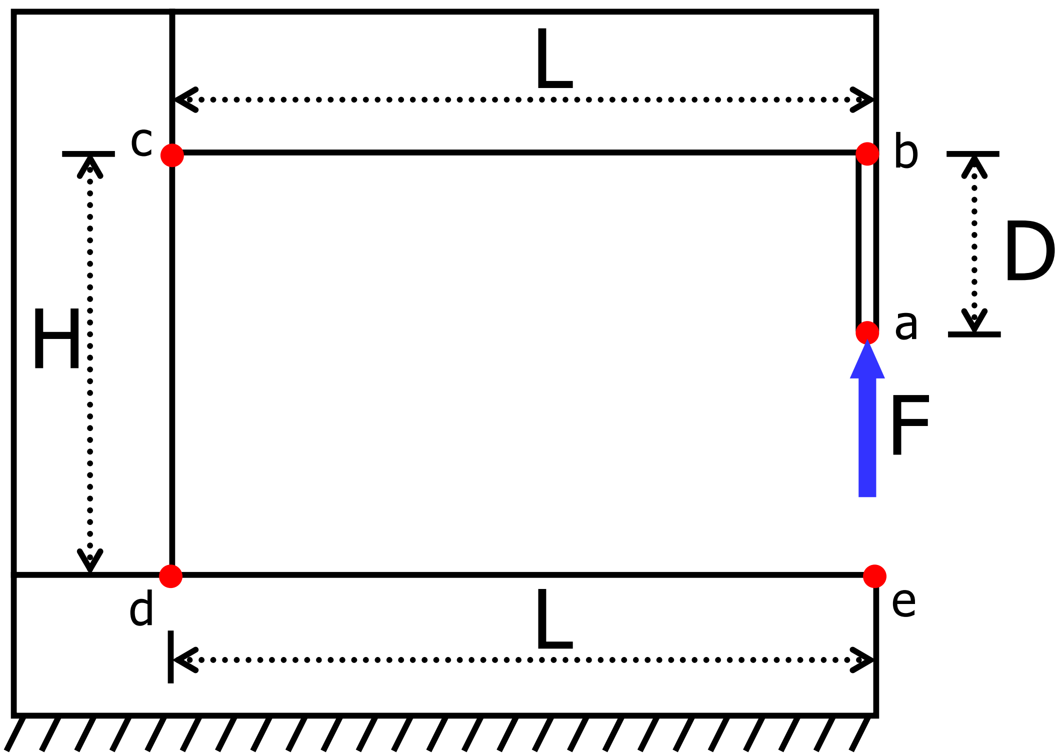
Find the HTM from a to b:



$$
{}^{b}\mathbf{H}_{a} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -(D-\delta) \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
\delta = \frac{FD}{EA_d}
$$

Drill Press Example

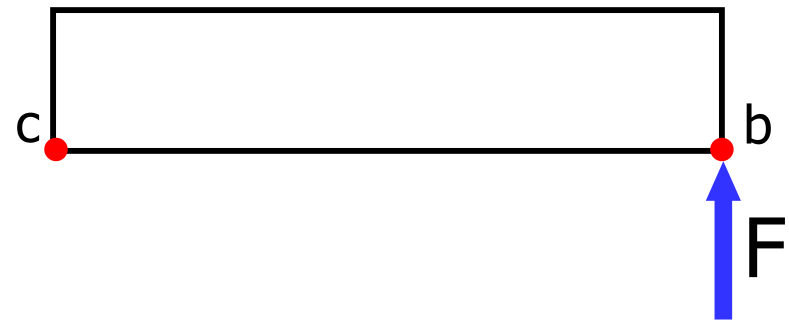


Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

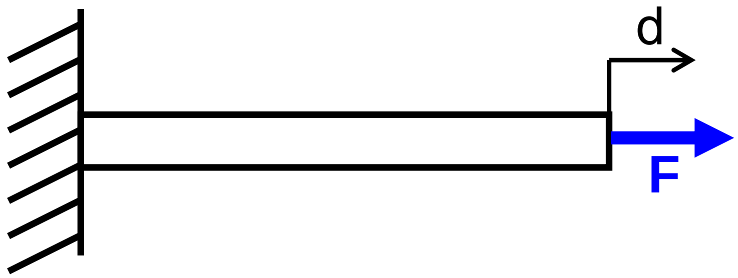
Young's Modulus of Material = E

Find the HTM from b to c:

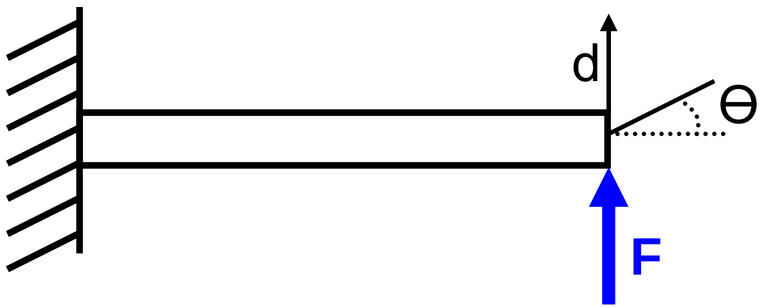


$$ {}^{c}\mathbf{H}_{b} = \begin{bmatrix} \cos\theta & -\sin\theta & L \\ \sin\theta & \cos\theta & \delta \\ 0 & 0 & 1 \end{bmatrix} $$

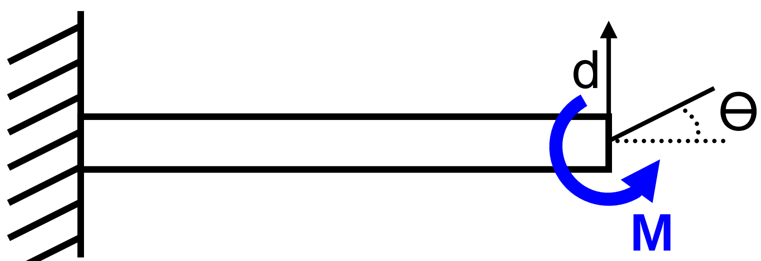
Useful Force-deflection Equations



$$d = \frac{FL}{EA}$$

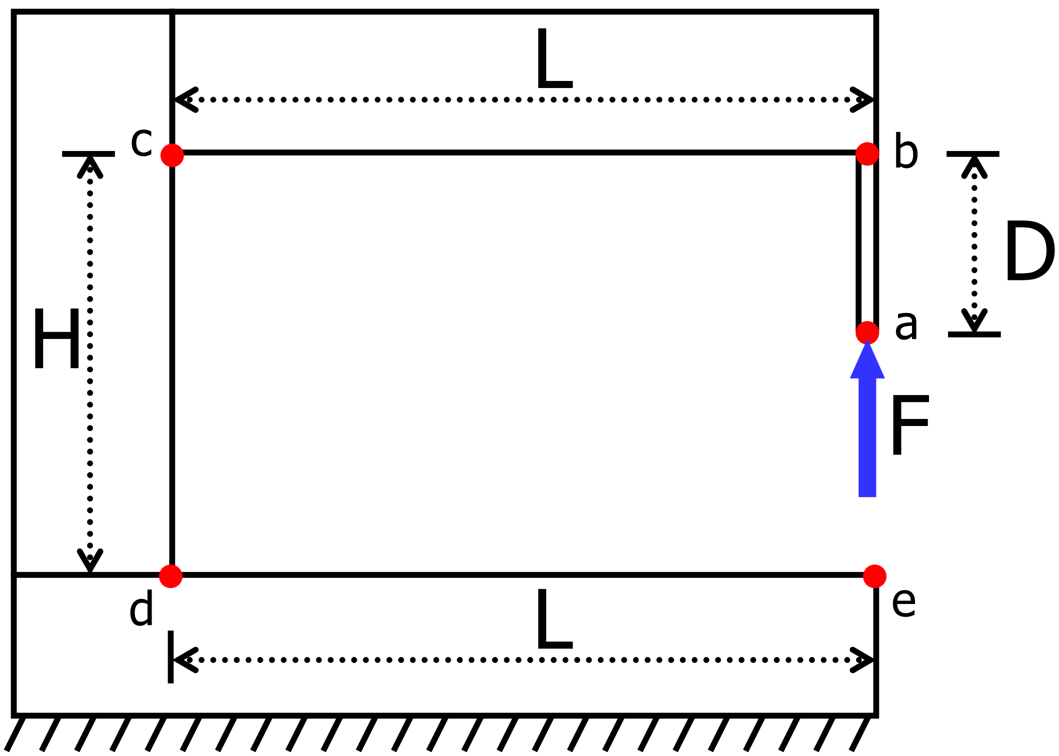


$$d = \frac{FL^3}{3EI} \qquad \Theta = \frac{FL^2}{2EI}$$



$$d = \frac{ML^2}{2EI} \qquad \Theta = \frac{ML}{EI}$$

Drill Press Example

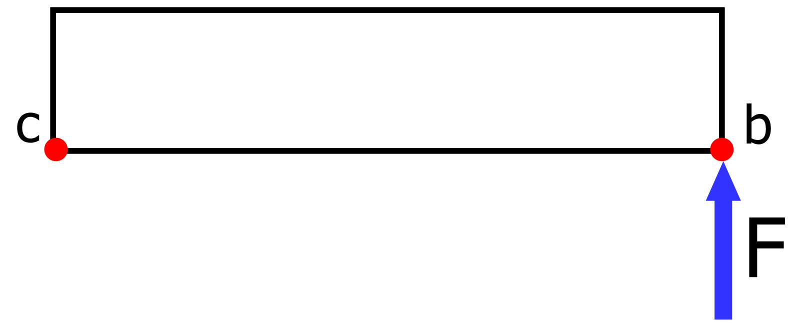


Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

Young's Modulus of Material = E

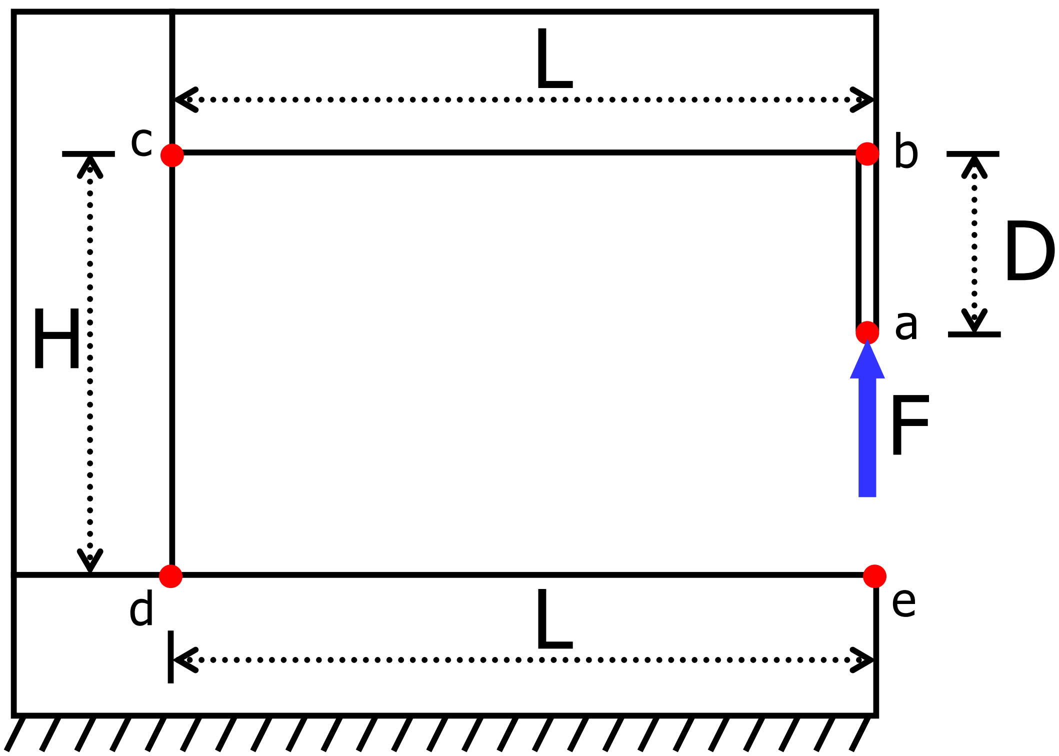
Find the HTM from b to c:



$$ {}^{c}\mathbf{H}_{b} = \begin{bmatrix} \cos\theta & -\sin\theta & L \\ \sin\theta & \cos\theta & \delta \\ 0 & 0 & 1 \end{bmatrix} $$

$$ \delta = \frac{FL^3}{3EI} \qquad \theta = \frac{FL^2}{2EI} $$

Drill Press Example

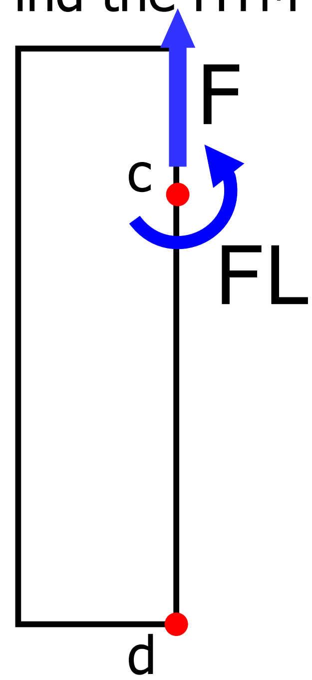


Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

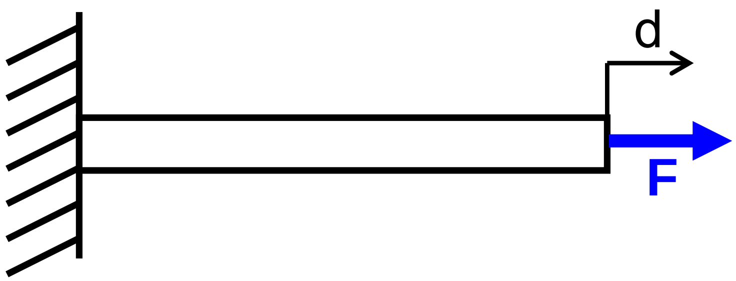
Young's Modulus of Material = E

Find the HTM from c to d:

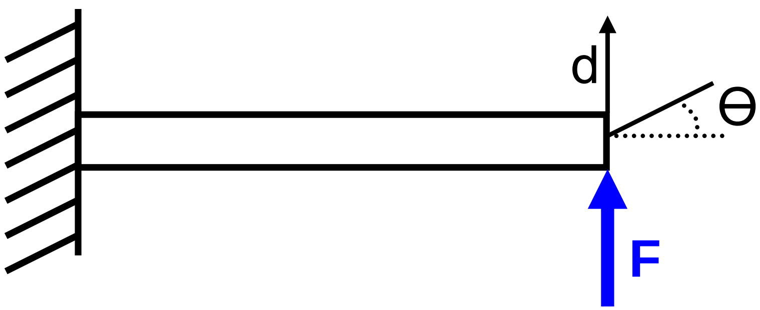


$$ {}^{d}\mathbf{H}_{c} = \begin{bmatrix} \cos\Theta & -\sin\Theta & -\delta_1 \\ \sin\Theta & \cos\Theta & H+\delta_2 \\ 0 & 0 & 1 \end{bmatrix} $$

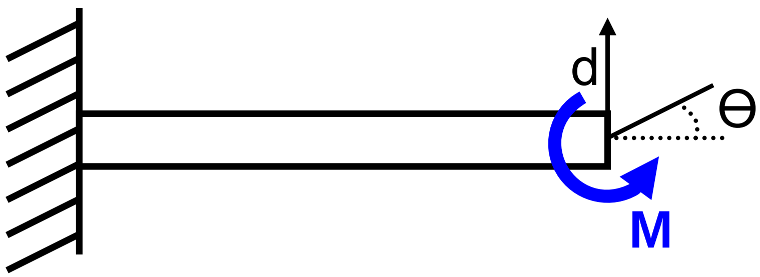
Useful Force-deflection Equations



$$d = \frac{FL}{EA}$$

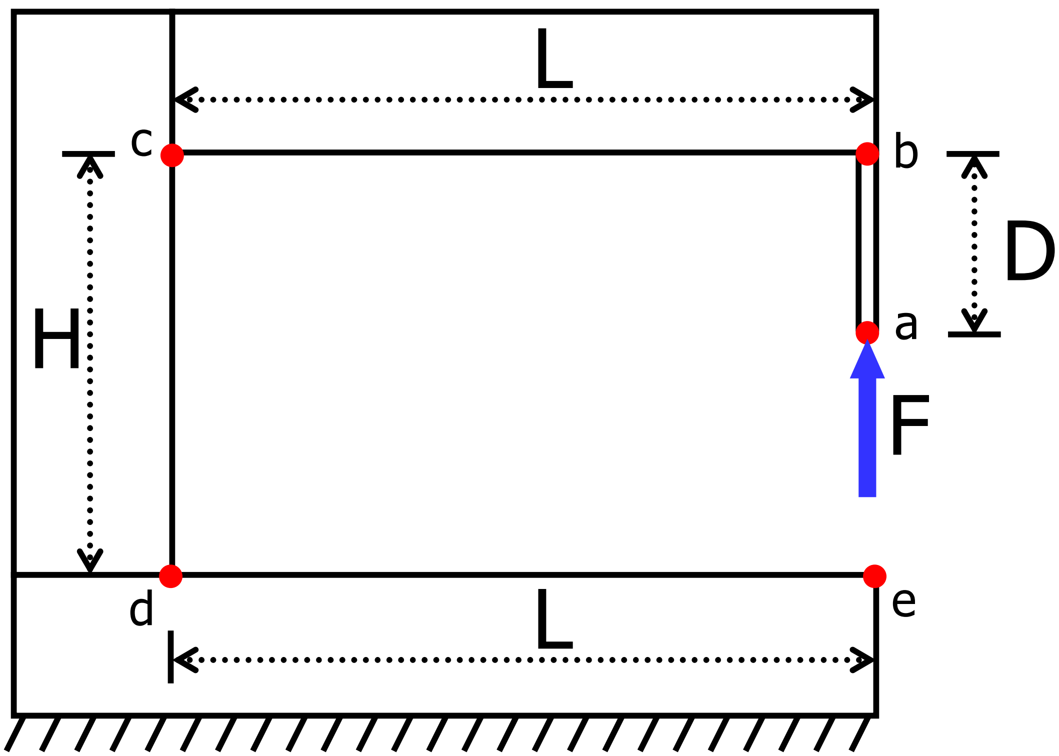


$$d = \frac{FL^3}{3EI} \qquad \theta = \frac{FL^2}{2EI}$$



$$d = \frac{ML^2}{2EI} \qquad \theta = \frac{ML}{EI}$$

Drill Press Example

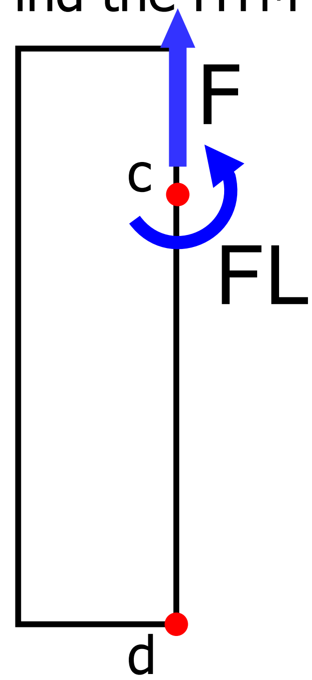


Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

Young's Modulus of Material = E

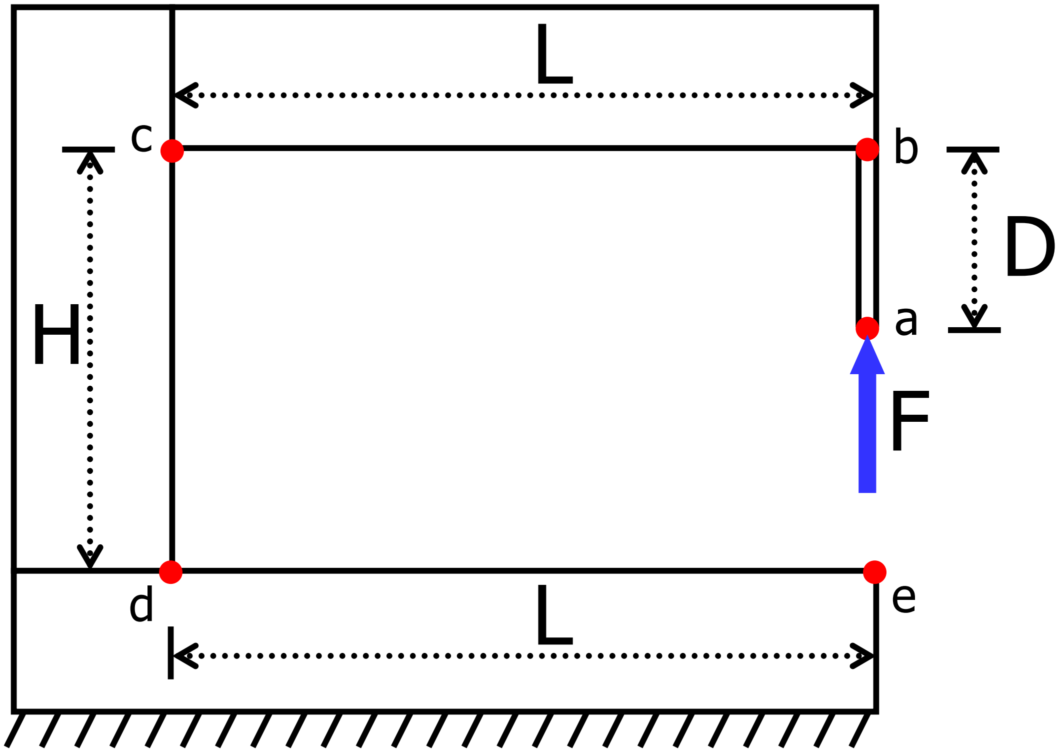
Find the HTM from c to d:



$$ {}^{d}\mathbf{H}_{c} = \begin{bmatrix} \cos\Theta & -\sin\Theta & -\delta_1 \\ \sin\Theta & \cos\Theta & H+\delta_2 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ \Theta = \frac{FLH}{EI} \qquad \delta_1 = \frac{FLH^2}{2EI} \qquad \delta_2 = \frac{FH}{EA} $$

Drill Press Example

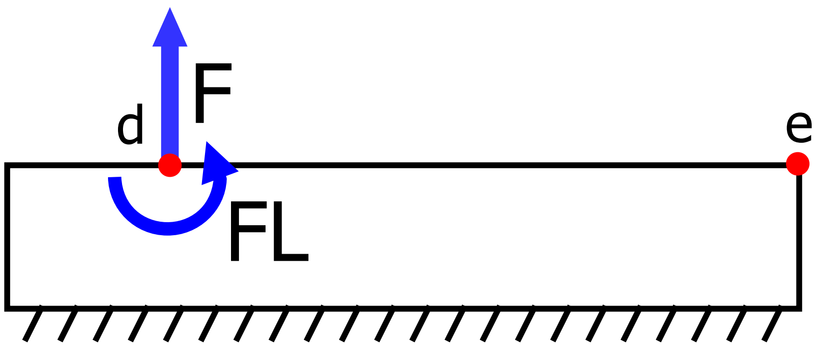


Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

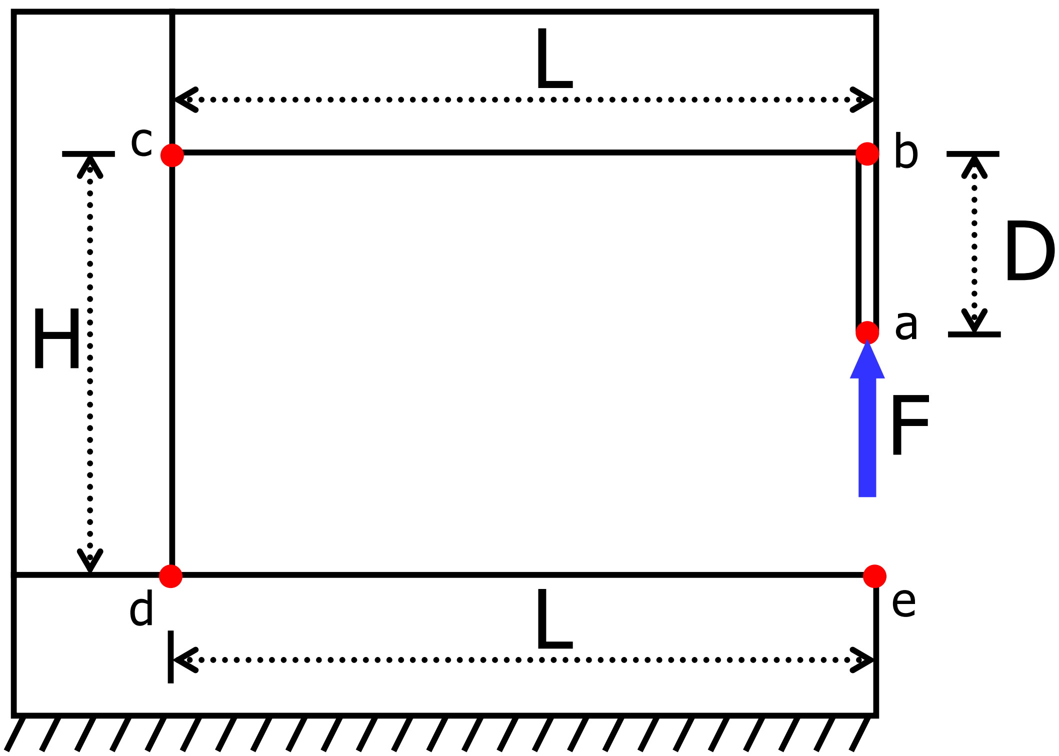
Young's Modulus of Material = E

Find the HTM from d to e:



$$ {}^{e}\mathbf{H}_{d} = \begin{bmatrix} 1 & 0 & -L \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

Drill Press Example



Cross-Sectional Area of large sections = A

Cross-Sectional Area of Drill Bit = A_d

Young's Modulus of Material = E

Find the HTM from a to e:

$$^{e}\mathbf{H}_a = {}^{e}\mathbf{H}_d \, {}^{d}\mathbf{H}_c \, {}^{c}\mathbf{H}_b \, {}^{b}\mathbf{H}_a$$

Find the vector ${}^{a}\vec{\mathbf{V}}_e$ from e to a:

$${}^{a}\vec{\mathbf{V}}_e = {}^{e}\mathbf{H}_a \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Method for building system's HTM

- Identify key nodes around the system's structural loop
- Create HTMs for each member between each node
- Multiply the member's HTMs in the correct order

More on HTMs

3D HTMs

For x-axis rotation

$$\begin{bmatrix} 1 & 0 & 0 & X \\ 0 & \cos\theta_x & \sin\theta_x & Y \\ 0 & -\sin\theta_x & \cos\theta_x & Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For y-axis rotation

$$\begin{bmatrix} \cos\theta_y & 0 & -\sin\theta_y & X \\ 0 & 1 & 0 & Y \\ \sin\theta_y & 0 & \cos\theta_y & Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For z-axis rotation

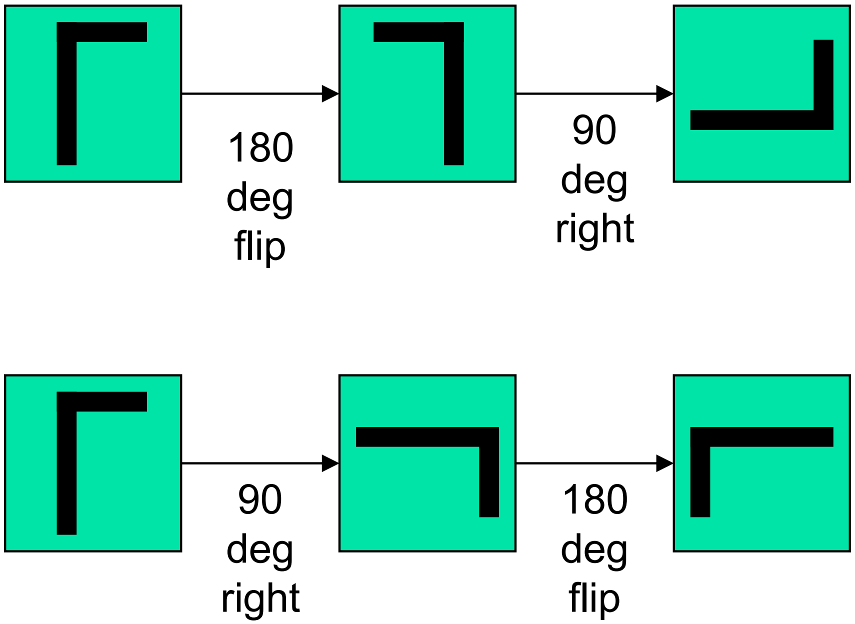
$$\begin{bmatrix} \cos\theta_z & \sin\theta_z & 0 & X \\ -\sin\theta_z & \cos\theta_z & 0 & Y \\ 0 & 0 & 1 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For small Θ:

cos(Θ)~1 & sin(Θ)~ Θ

HTM Rotation

•Remember order of multiplication matters:



•To combine a translation and rotation, again multiply the HTM matrices together

•Note that the order of the rotation and translation matrices does matter, so makes sure the answer makes sense!!!