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2.72 Elements of Mechanical Design  
Spring 2009

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*2.72*

*Elements of  
Mechanical Design*

*Lecture 06: Constraints*

# Schedule and reading assignment

## Quiz

- ❑ Quiz – HTMs

## Topics

- ❑ Principles of exact constraint
- ❑ Bearing layout exercises
- ❑ Spindle shaft constraint/bearing layout

## Reading assignment

- ❑ Chapter 11 in Shigley and Mischke
  - *Read sections 11.1 – 11.6, 11.9*
  - *Skim sections 11.7 – 11.8, 11.10 – 11.12*
- ❑ This is 40ish pages, but most of it is pictures/graphs/examples

*Principles of  
exact constraint*

# Under, exact and over constraint

## Constraints are fundamental to mechanical design

- ❑ A mechanical designers goal is to control, i.e. to constrain, parts so that they are where they are supposed to be.

## Exact constraint:

- ❑ There should be one constraint for each degree-of-freedom that is constrained.

## Under constraint

- ❑ Too few constraints, part is not held in all the directions it needs to be

## Over constraint

- ❑ Too many constraints, some constraints may fight each other when trying to do the same job.

# Mechanical constraints

**We want to learn how to model and design each**

**We first need to know:**

- ❑ How they should be used
- ❑ What their functional requirements are

**How they should be used and their FRs depend upon:**

- ❑ How they are laid out
- ❑ Dos and don'ts

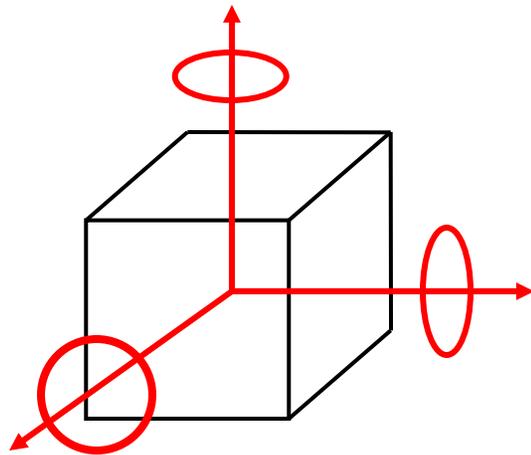
**Learn to lay them out right**

- ❑ Use this to obtain their Functional Requirements
- ❑ Then do the detailed design of each

# Constraints and Degrees-of-freedom

Rigid components have 6 degrees-of-freedom

We will represent an ideal constraint as a line



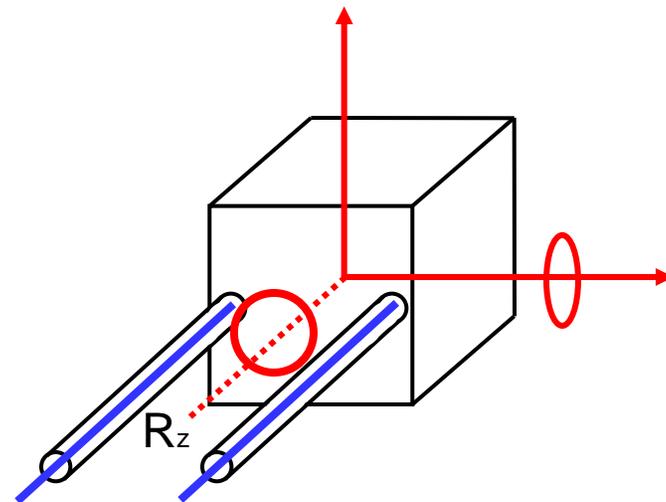
$$6 - C = R$$

$C$  = # of Non-Redundant Constraints

$R$  = # of Independent Degrees of Freedom

Example:

$$6 - 2 \text{constraints} = 4 \text{DoFs}$$



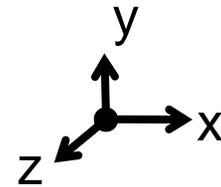
Courtesy John Hopkins, MIT

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# Constraints, compliance and motion

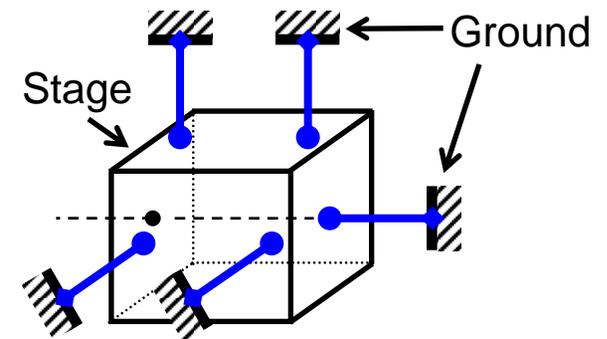
## Exact constraint: Achieve desired motion

- ❑ By applying minimum number of constraints
- ❑ Arranging constraints in correct **constraint topology**
- ❑ Adding constraints only when necessary



## For now:

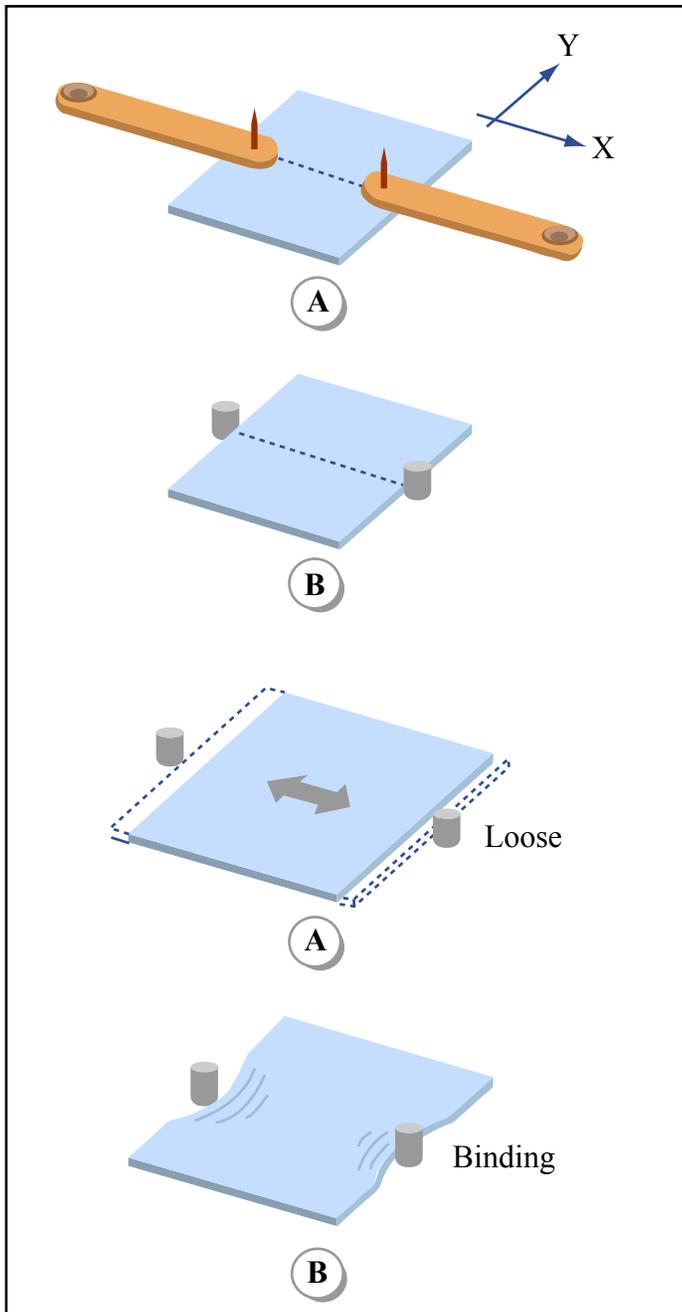
- ❑ Start with ideal constraints
- ❑ Considering small motions
- ❑ Constraints = lines



## Focus on rigid stage attached to ground

- ❑ What do we mean by rigid?
- ❑ What do we mean by constraint? - Stiffness ratios

# The benefits of exact constraint



**Parts of machines are now always the same strength and stiffness.**

**Large, stiff components have a tendency to “kill” their smaller counterparts when they are connected so that they are forced to fight.**

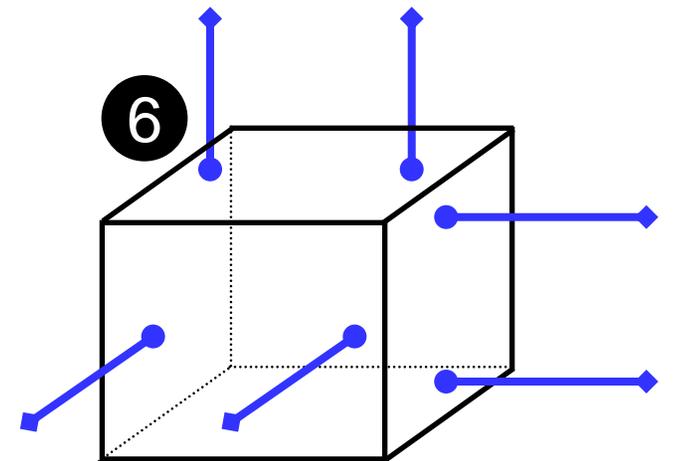
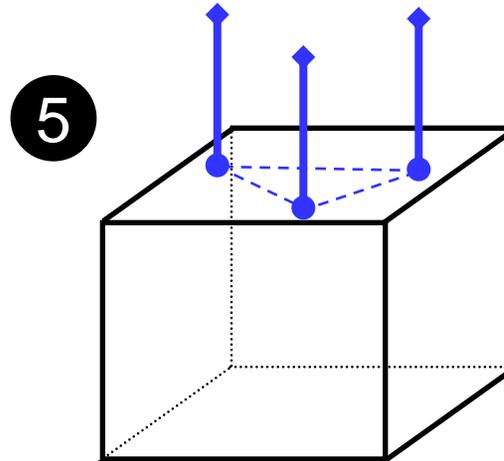
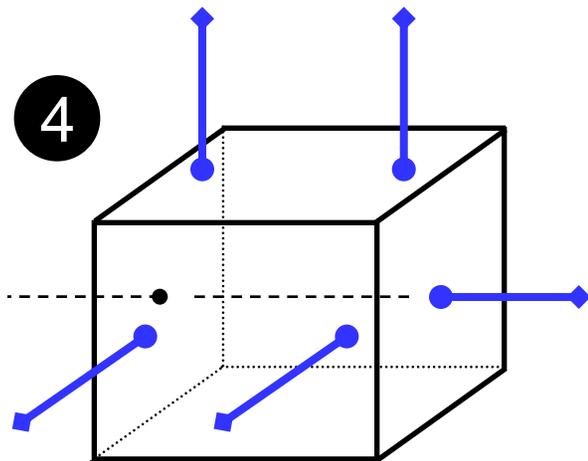
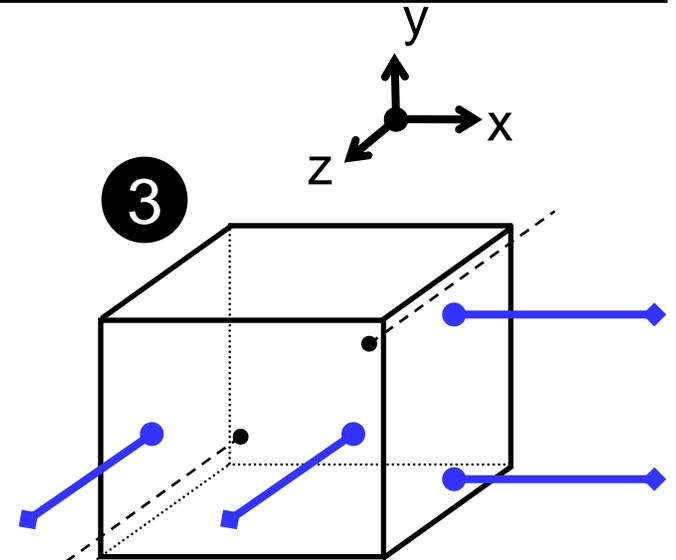
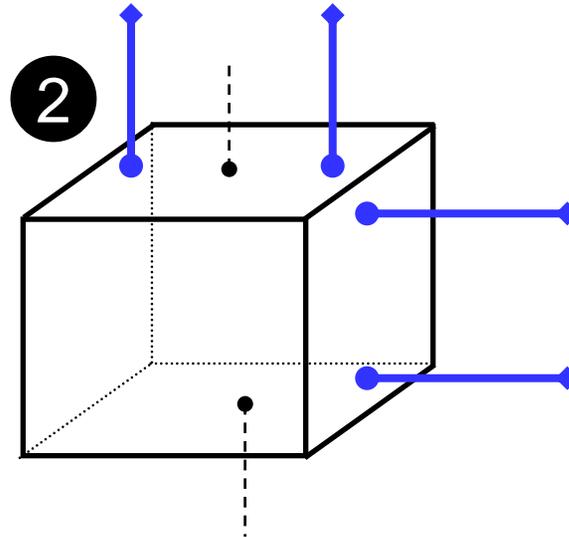
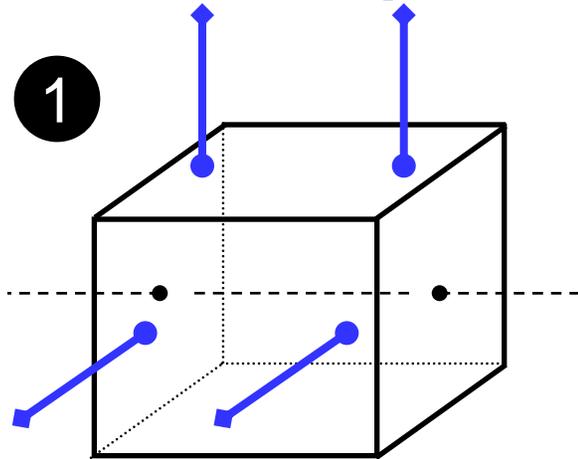
**Exact constraint design helps to prevent fights, therefore all your parts live in harmony.**

Figure by MIT OpenCourseWare.

## Penalties for over constraint

# Constraint examples

More examples:



*Exact constraint  
practice*

# Mechanical constraints: Some bearing types

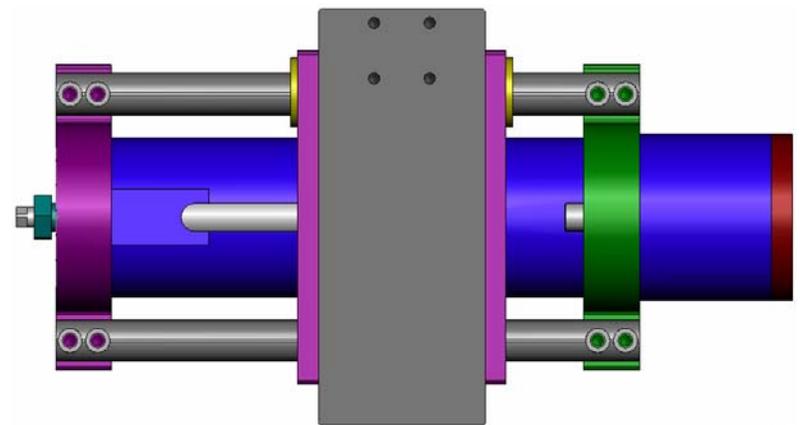
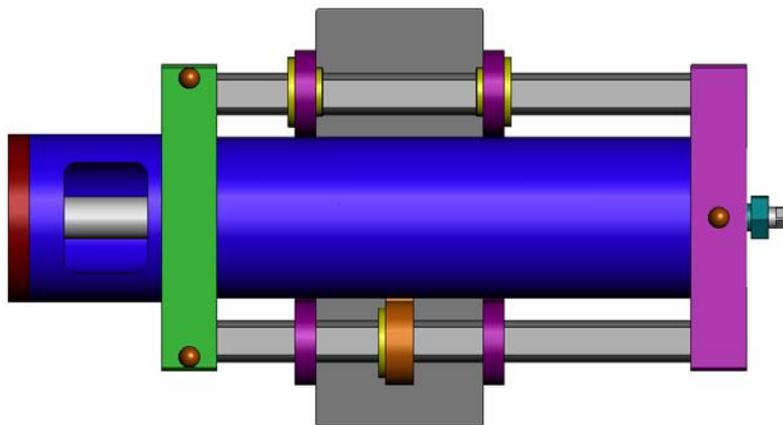
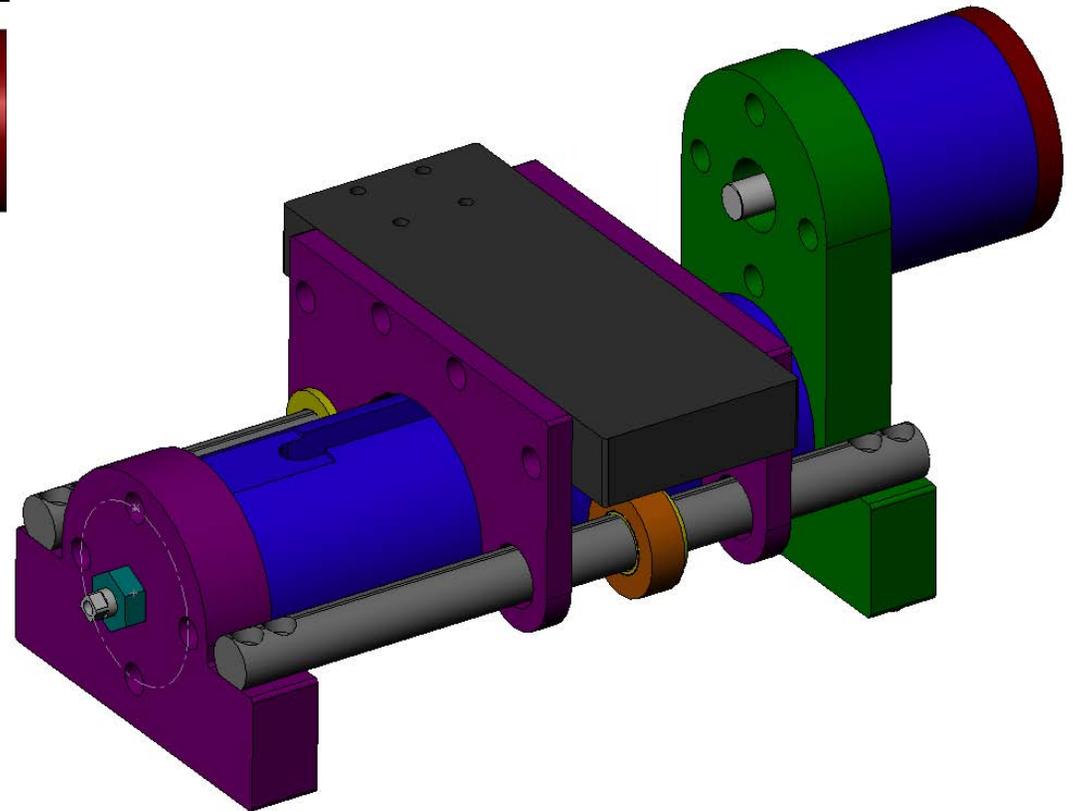
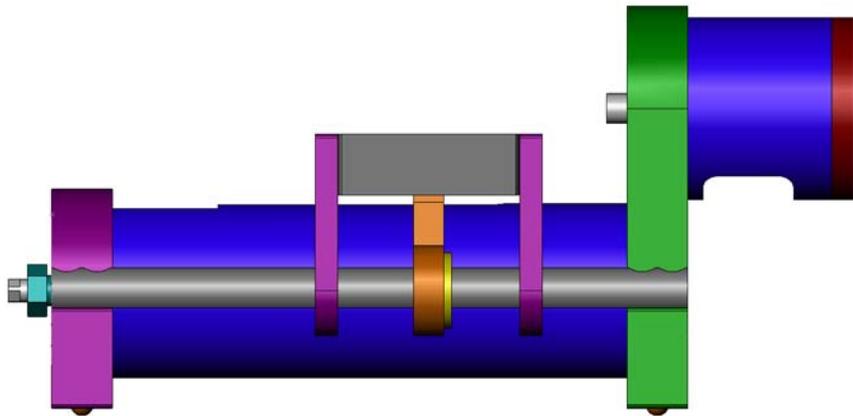
## Sliding

- ❑ Bushings
- ❑ etc...

## Radial rolling

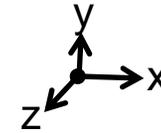
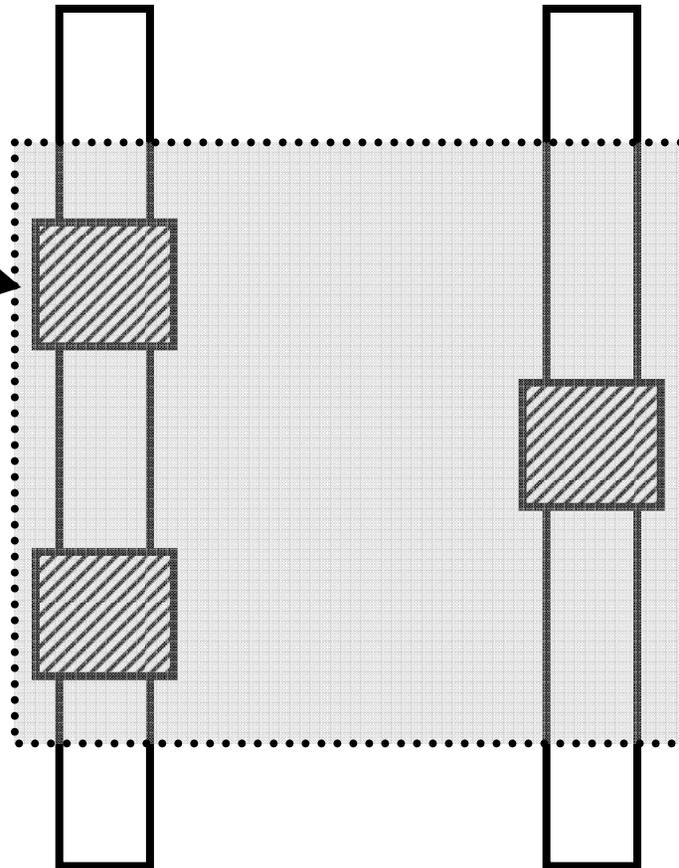
- ❑ Radial
- ❑ Shallow groove
- ❑ Deep groove (Conrad)
- ❑ Angular contact
- ❑ Tapered

# Examples drawn from your lathe

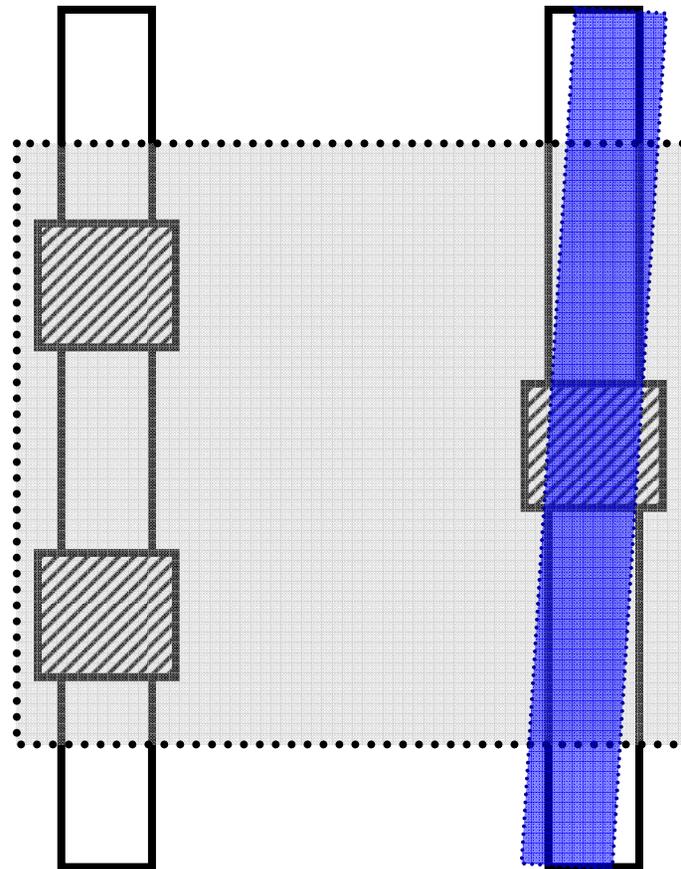


# Example: Carriage constraint

Assume these are  
bushings as in  
your lathe

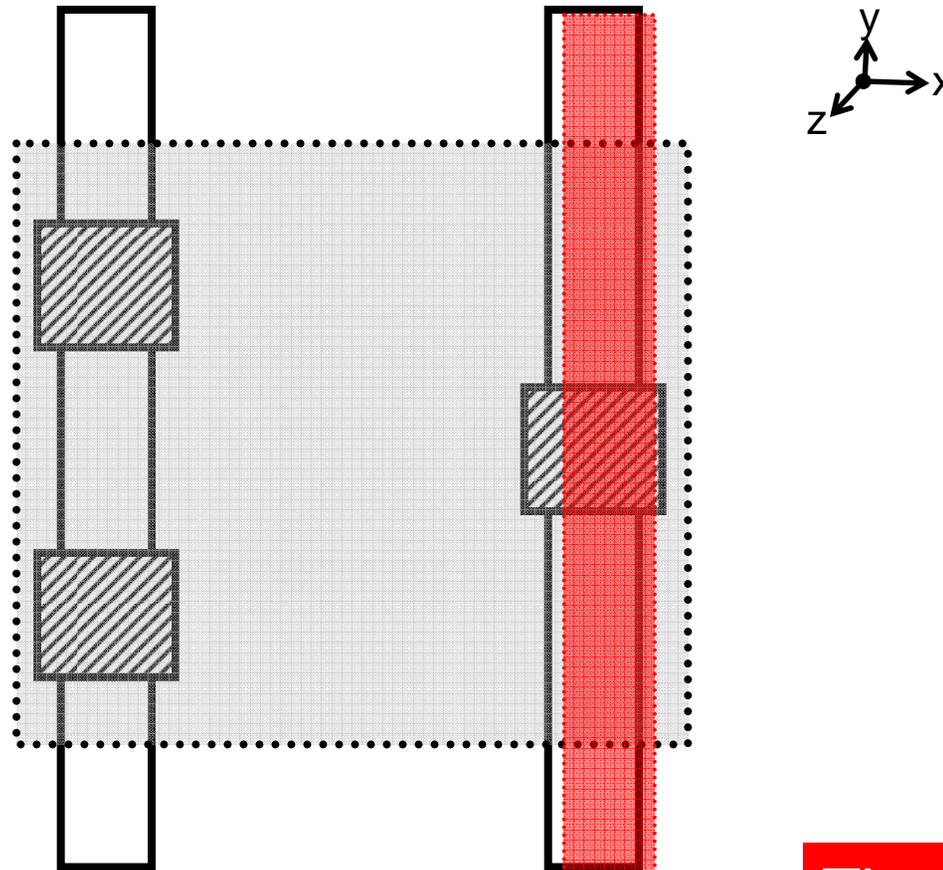


# Example cont.



Misalignment  
& assembly  
errors

# Example cont.

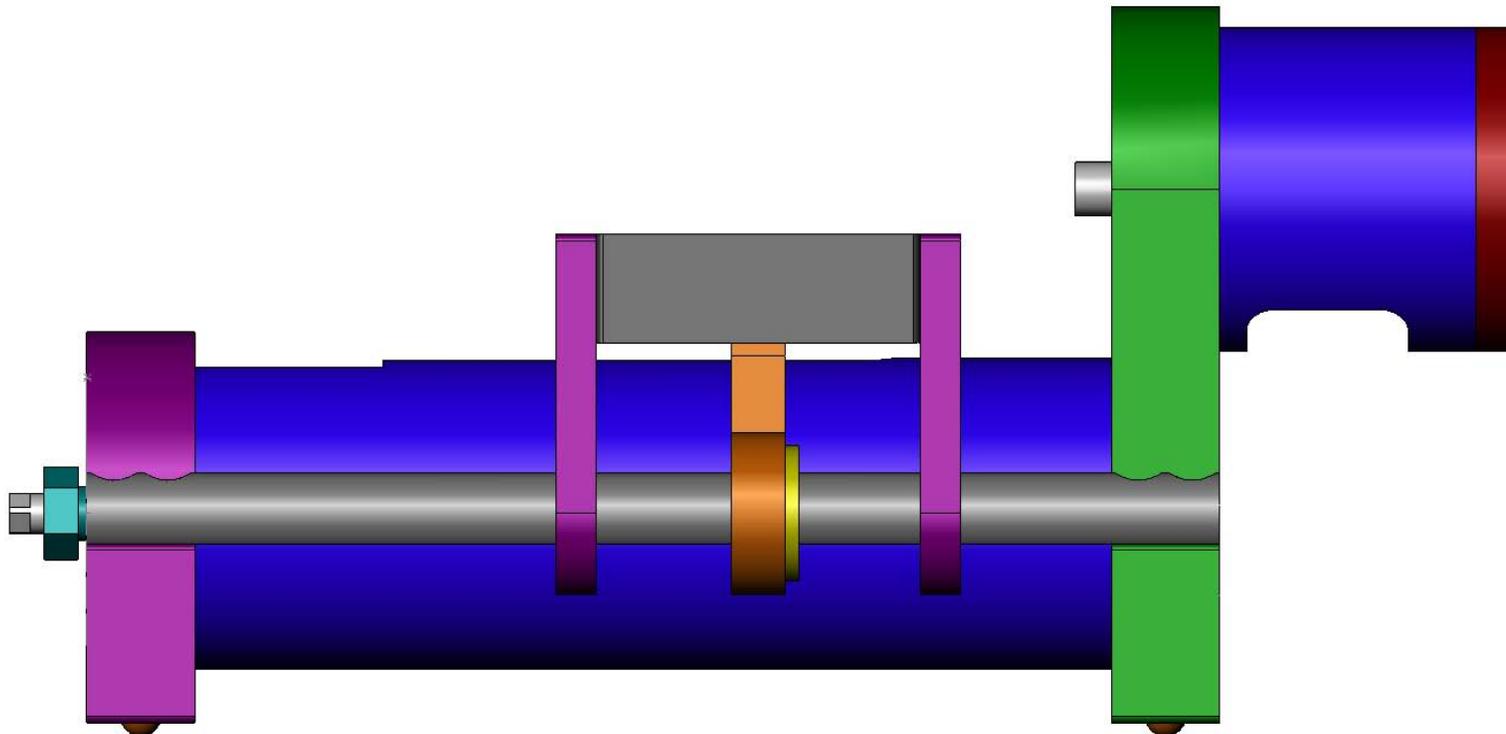


Thermally  
Induced  
growth

# Practical embodiment



Flexure that is stiff in y and z, yet compliant in x



# **Group exercise – Carriage constraints**

**Identify the motions that you desire for the carriage and the minimum # of constraints that are needed to yield only these motions.**

**Identify the constraints from each bearing set and determine how they act in concert to yield the desired motions.**

*Constraint layouts  
and  
thermal stability*

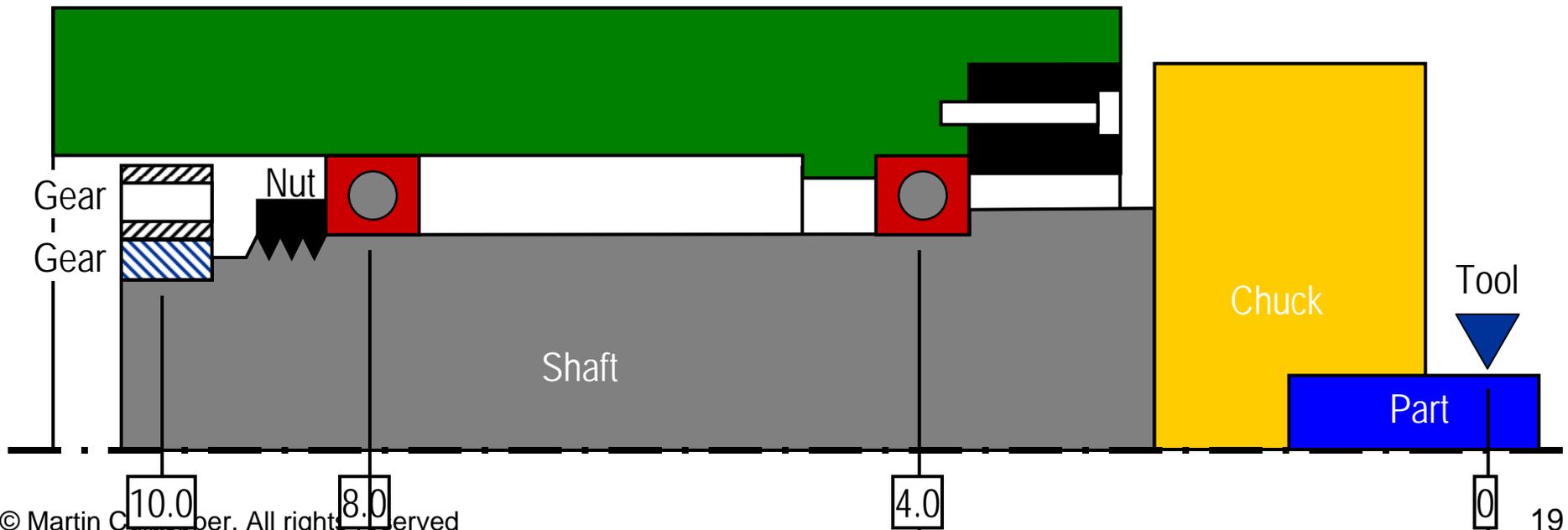
# Avoiding over constraint

## How to deal with the thermal growth issue

- ❑ The shaft typically gets hotter than the housing because the housing has better ability to carry heat away
- ❑ Whether the outer or inner race are fixed, matters...

## Constraining front and rear bearings

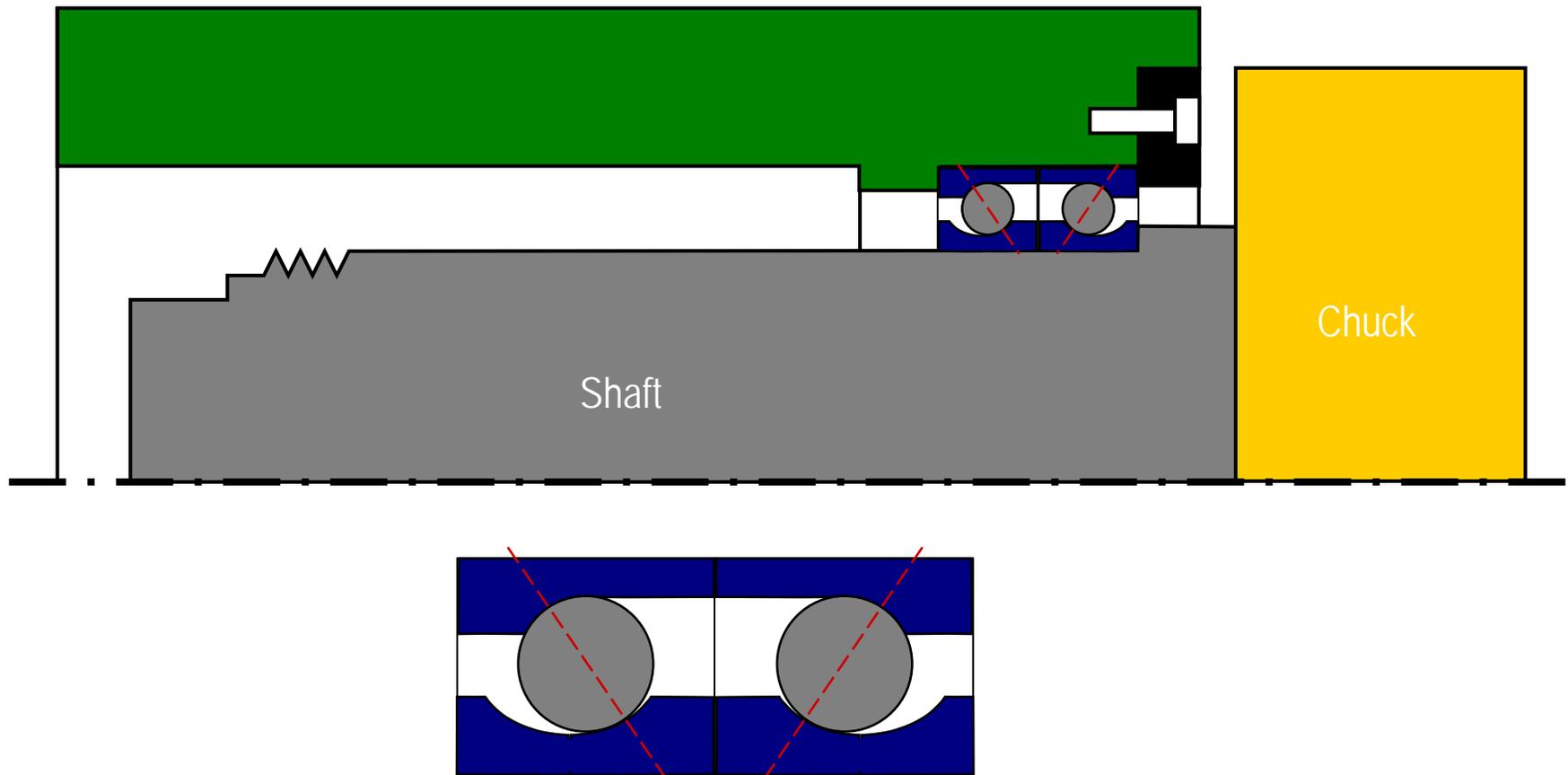
- ❑ One bearing set should be axially and radially restrained
- ❑ The other bearing set should ONLY have radial restraint



# Examples: Good or bad

Outer race fixed **axially**, if shaft heats is this bad?

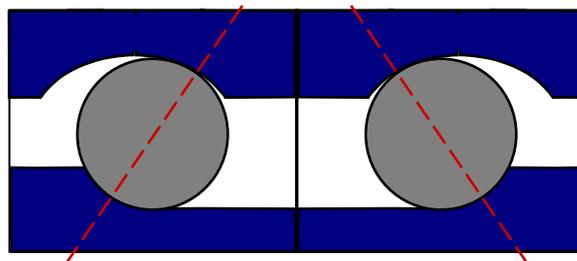
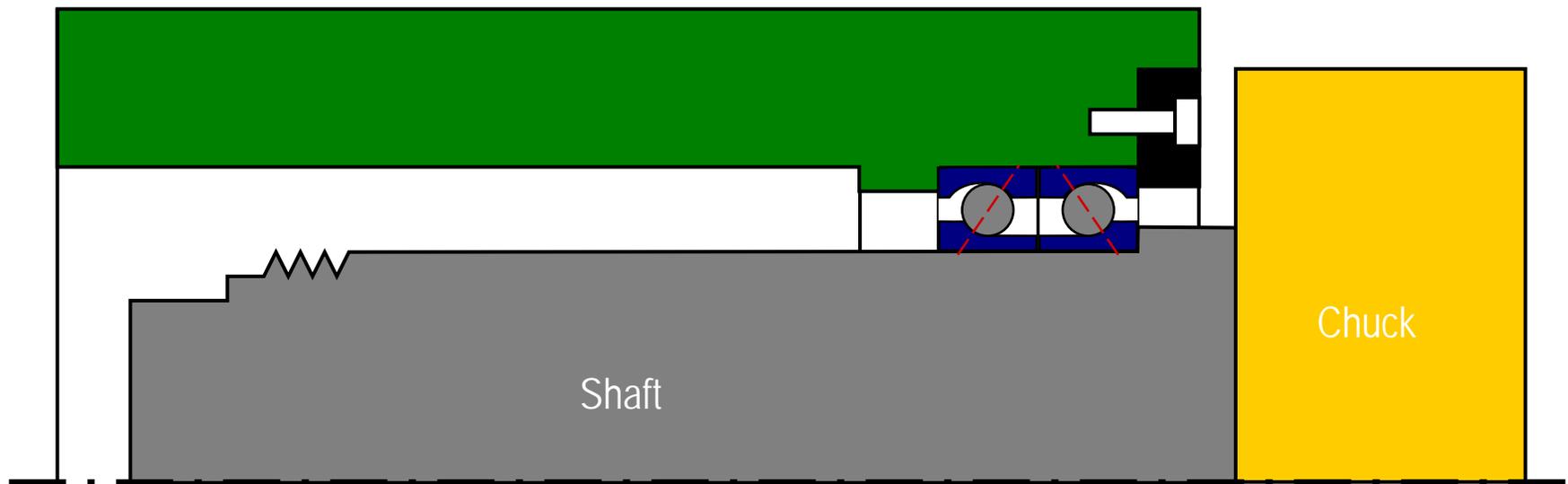
- Think about what happens to the preload...



# Examples: Good or bad

Outer race fixed **axially**, if shaft heats is this bad?

- Think about what happens to the preload...



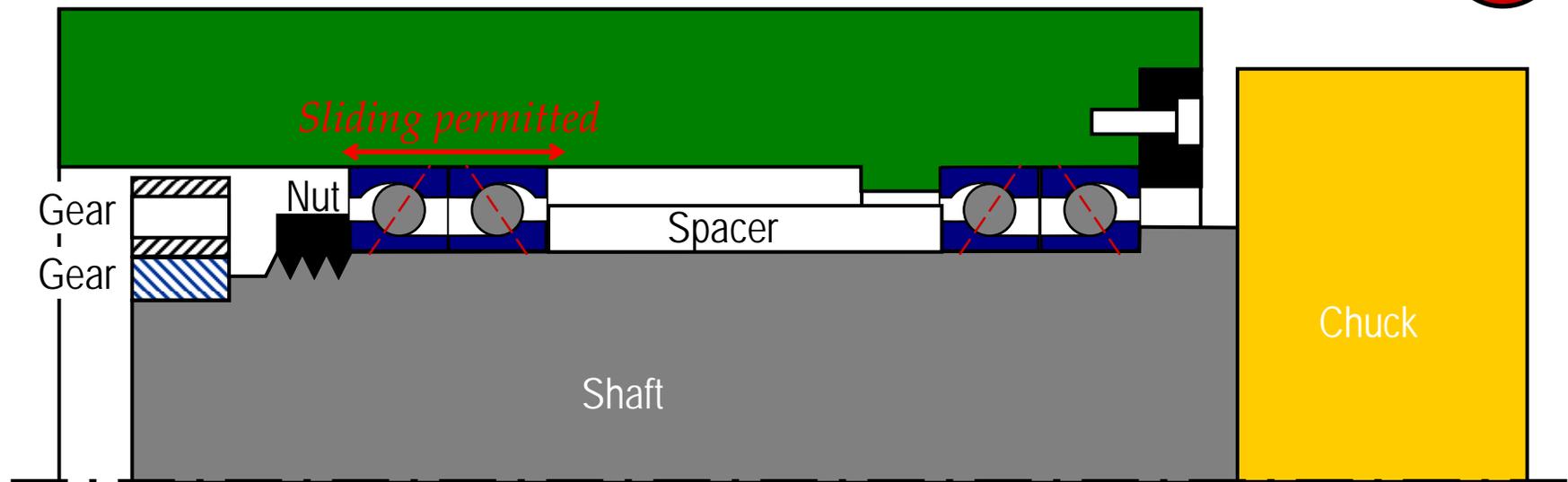
This is the back-to-back config.

It should be used when the outer race is not rotating

# Examples: Good or bad

Assume the outer race does not rotate

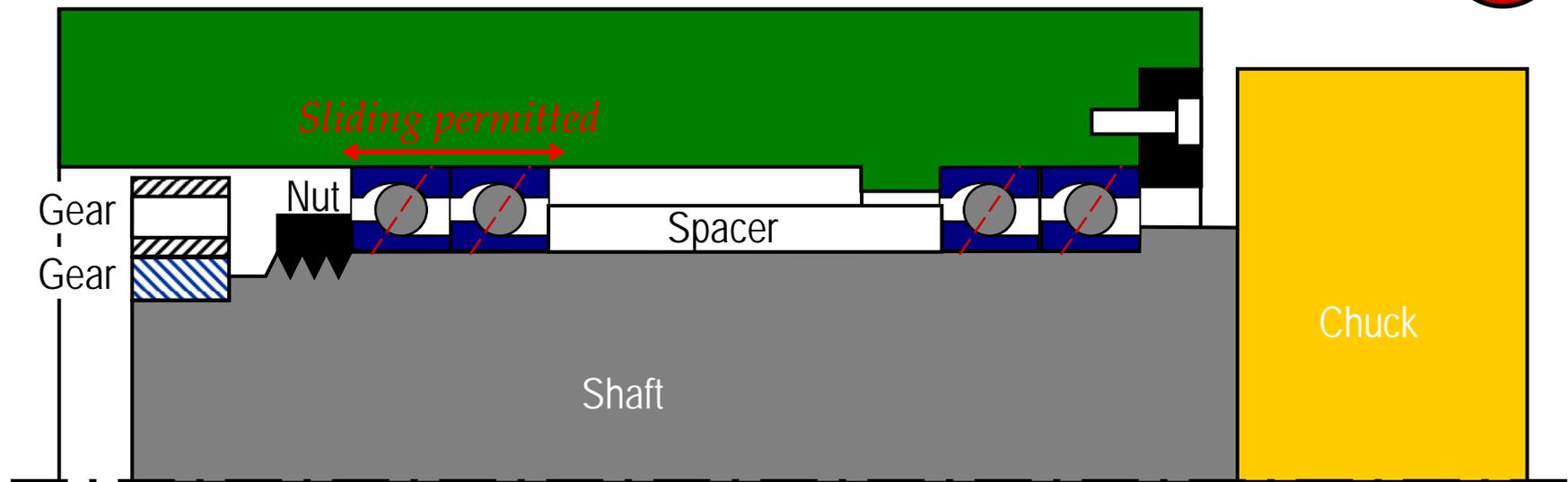
1



# Examples: Good or bad

Assume the outer race does not rotate

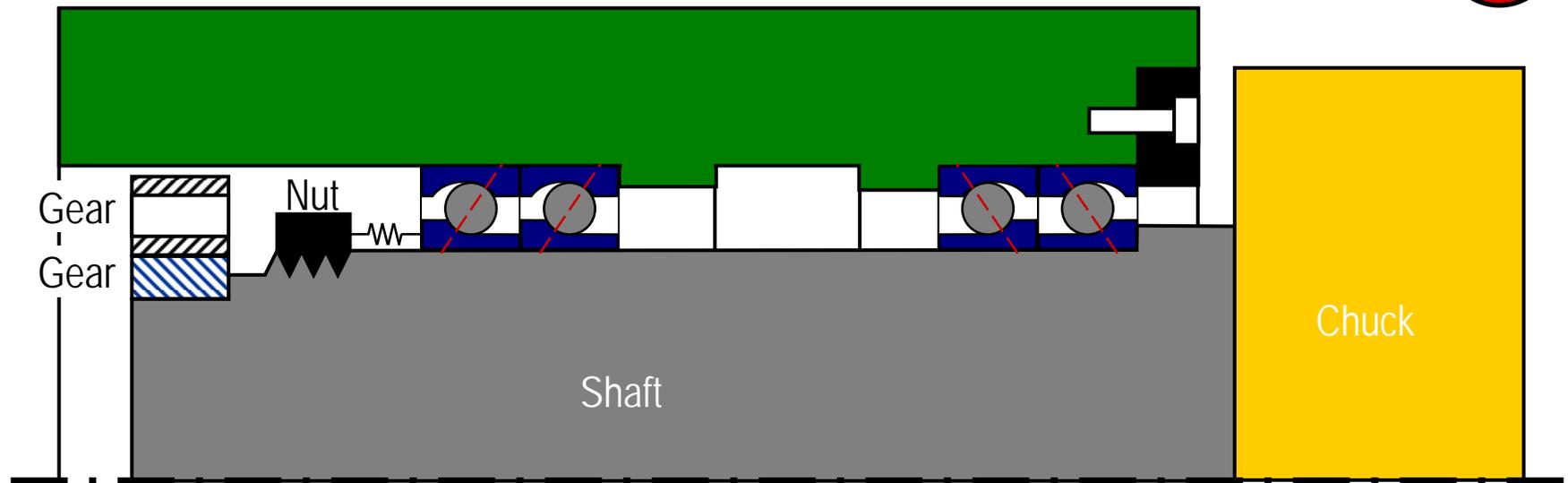
2



# Examples: Good or bad

Assume the outer race does not rotate

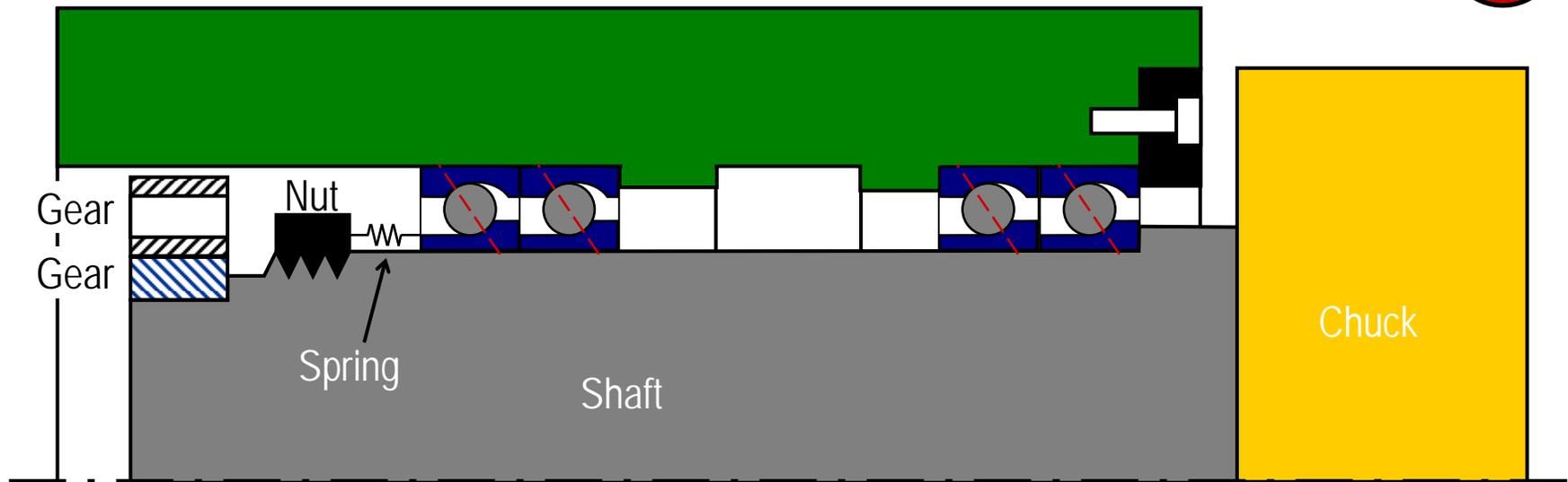
3



# Examples: Good or bad

Assume the outer race does not rotate

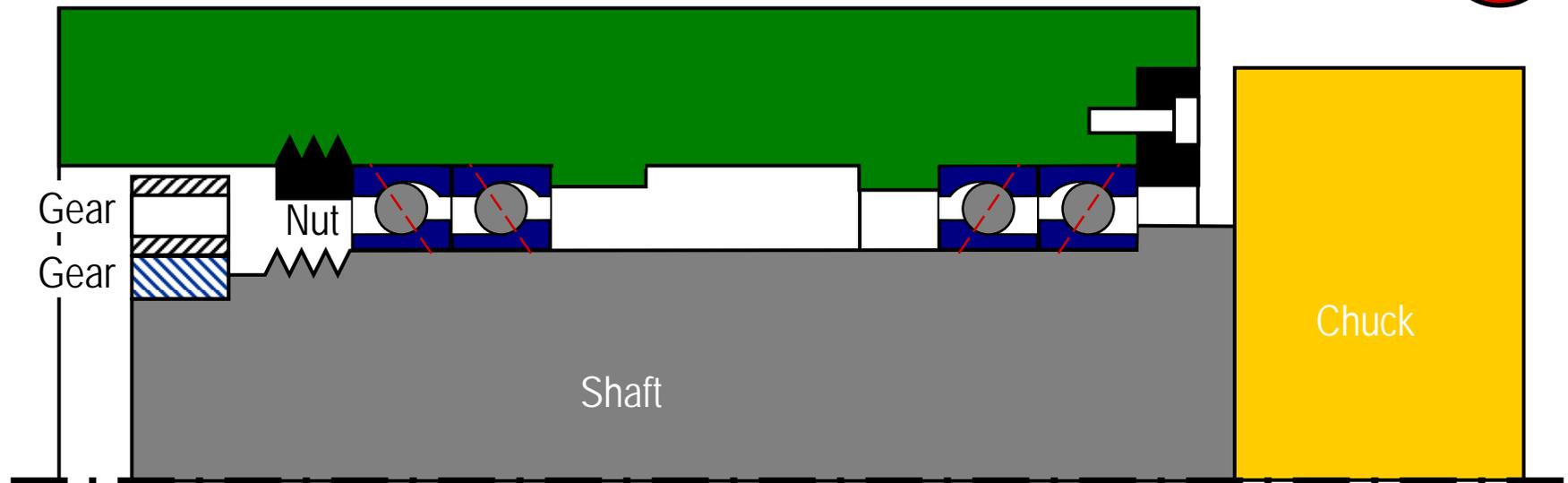
4



# Examples: Good or bad

Assume the outer race does not rotate

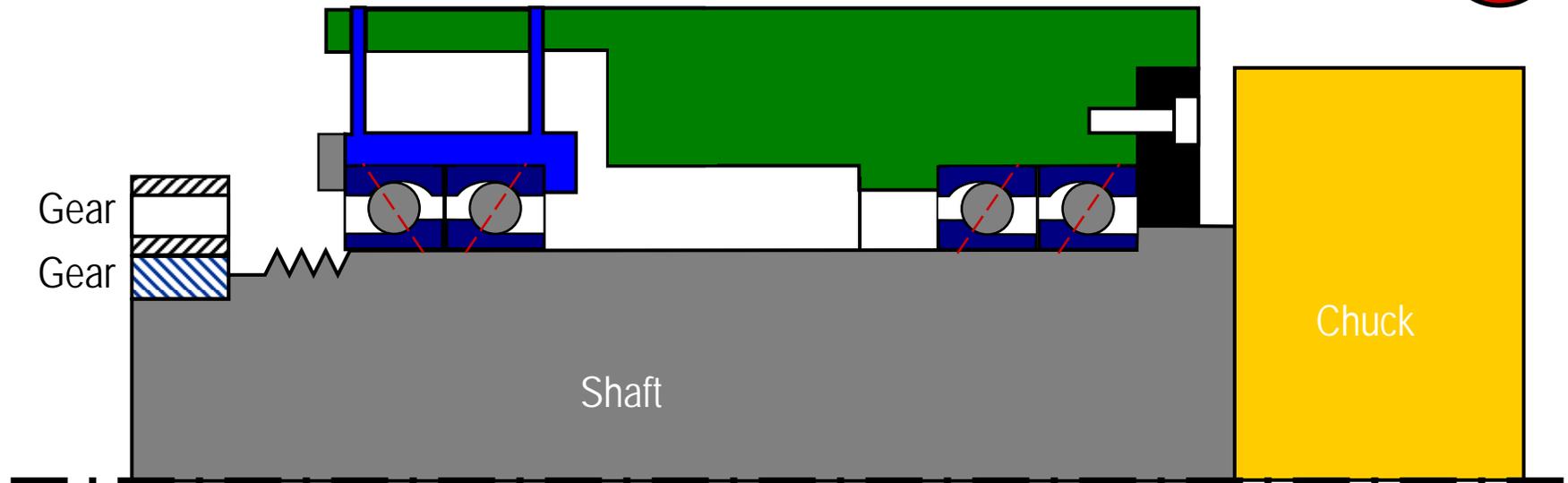
5



# Examples: Good or bad

Assume the outer race does not rotate

6



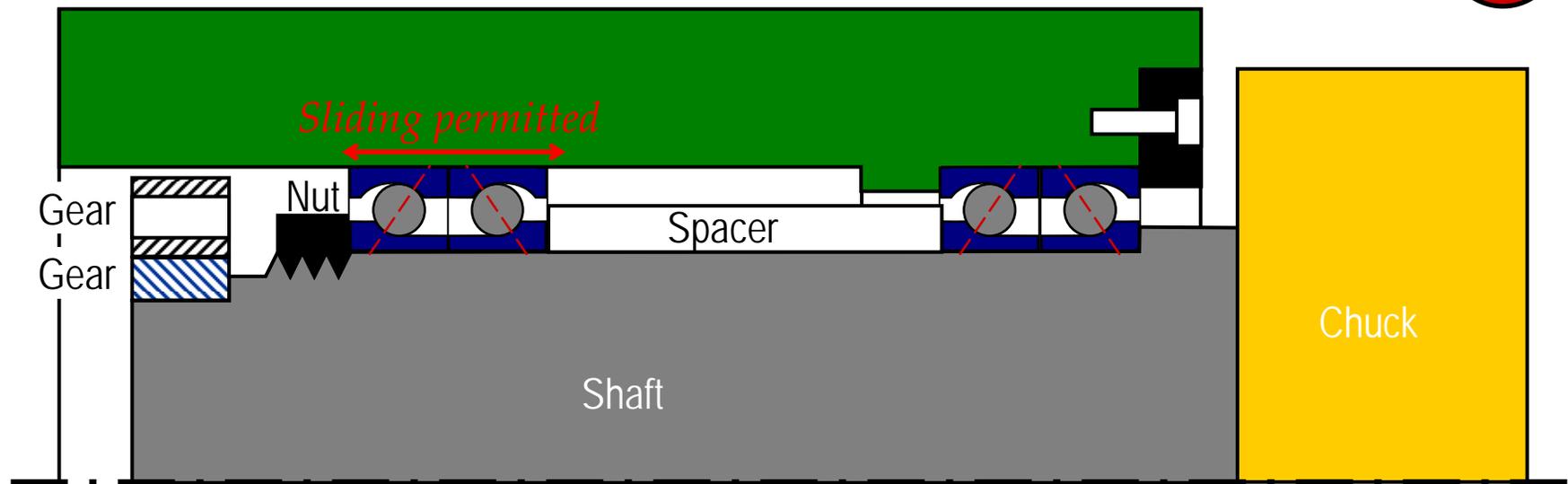
# Examples: Good or bad

## Good

- ❑ Front set constrained radially and axially
- ❑ Rear set constrained only radially

Assume the outer race does not rotate

1



**BUT, sliding means some gap must exist and therefore one must precision fabricate if a small gap is desired**

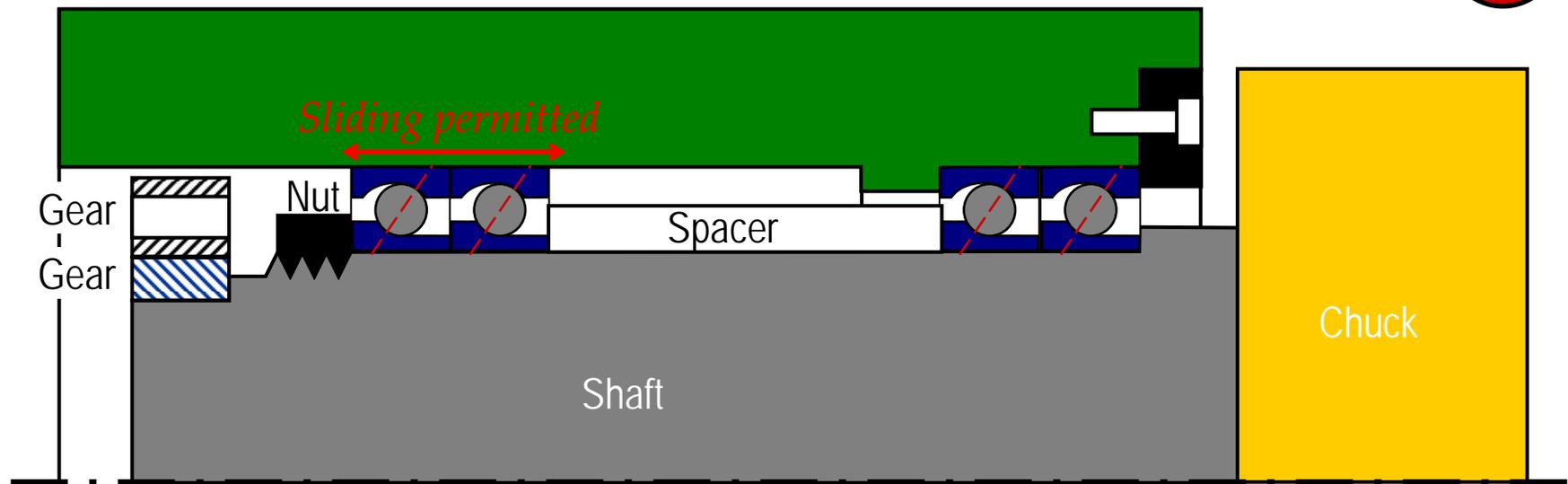
# Examples: Good or bad

## Bad

- ❑ Front set constrained only radially
- ❑ Rear set constrained only radially

Assume the outer race does not rotate

2



**This design will not work if axial loads are to be applied along both directions**

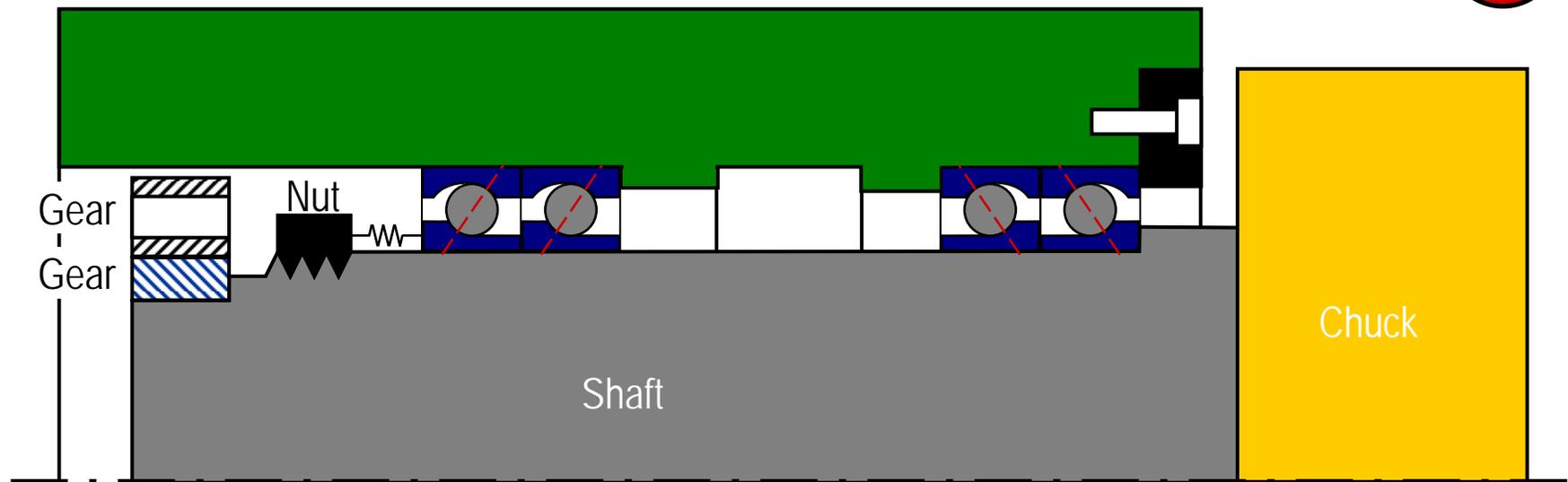
# Examples: Good or bad

## Good

- ❑ Front set constrained radially and ½ axially
- ❑ Rear set constrained radially and ½ axially

Assume the outer race does not rotate

3



**BUT, adding a spring increases part count/cost**

**Axial stiffness values (left vs. right) will be different**

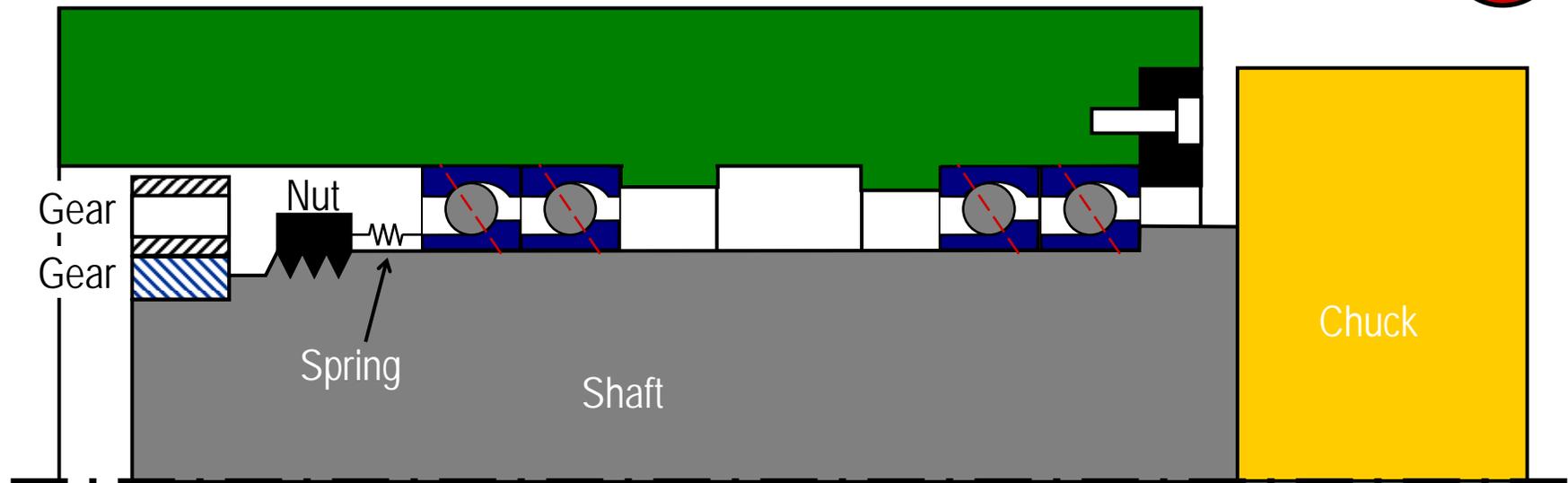
# Examples: Good or bad

## BAD

Assume the outer race does not rotate

- ❑ Front set is constrained only ½ axially
- ❑ Rear set is constrained only ½ axially

4



**This design will not work if axial loads are to be applied along both directions**

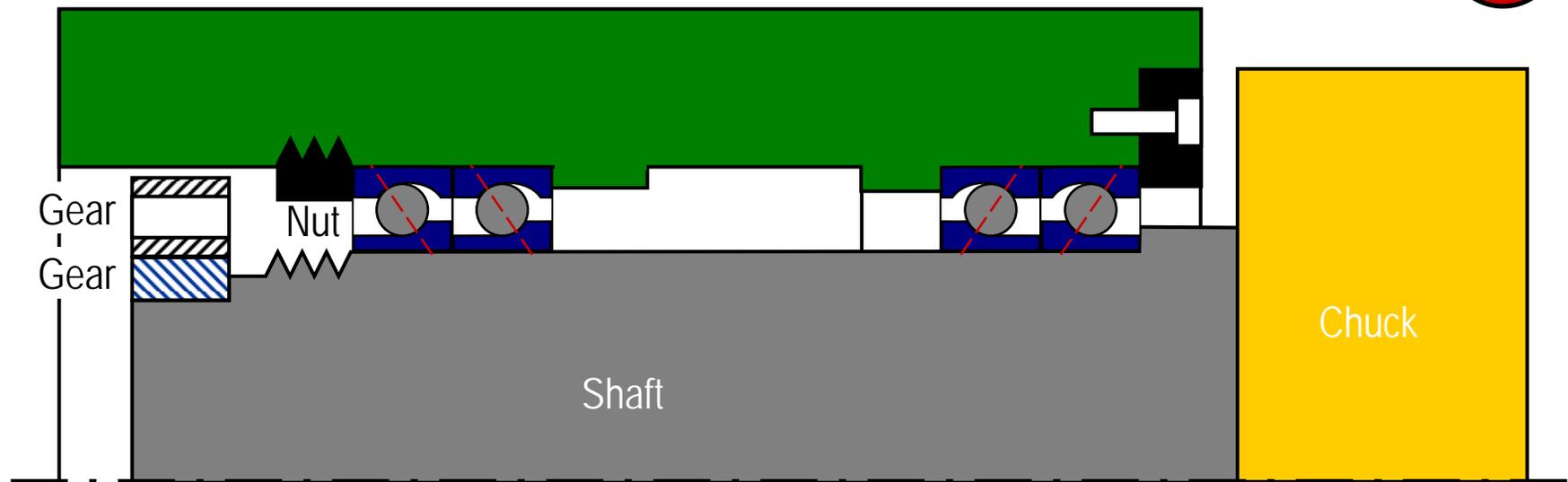
# Examples: Good or bad

## BAD

- ❑ Front set is constrained ½ axially and radially
- ❑ Rear set is constrained ½ axially and radially

Assume the outer race does not rotate

5



**At high speeds/loads, thermal growth may kill the bearing sets**

- ❑ Like a double face-to-face...

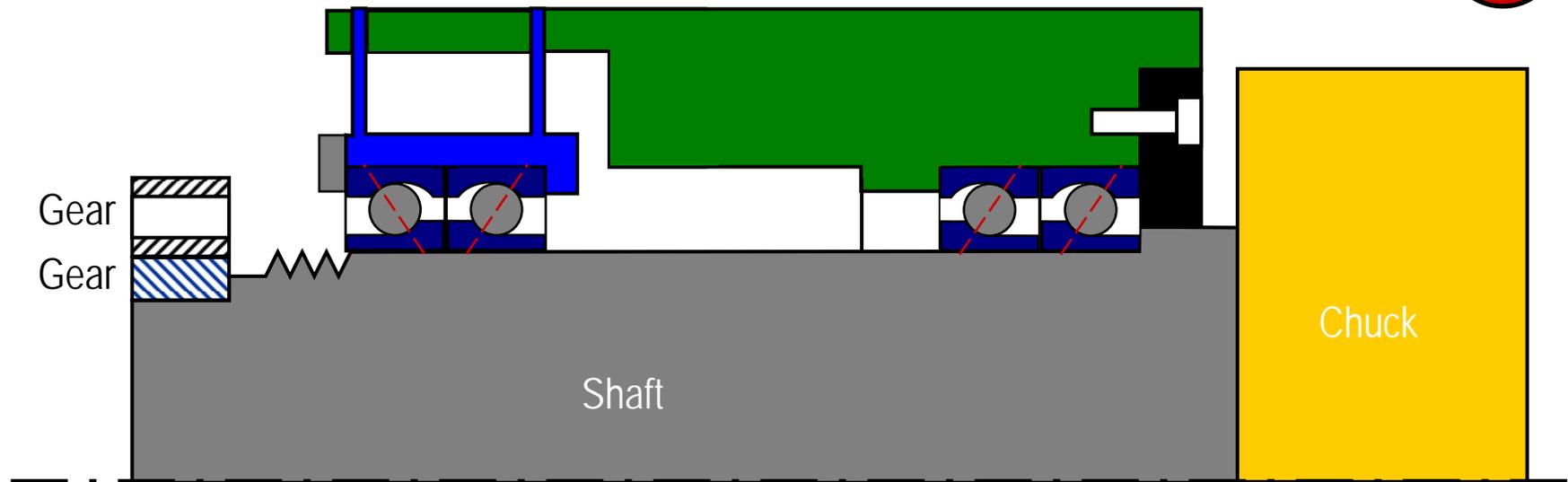
# Examples: Good or bad

## Bad

Assume the outer race does not rotate

- ❑ Front set constrained radially and ½ axially
- ❑ Rear set constrained radially (if flexure is of proper stiffness)

6



**Left bearing set is not in the back-to-back configuration**

**Shaft can pop out...**

*Group exercise*

# **Group exercise – Spindle constraints**

**The spindles you have seen use tapered roller bearings.**

**First, sketch a layout from one of the previous lathes and diagnose its layout**

**Second, generate and sketch a different way to constrain the spindle shaft.**