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2.72 Elements of Mechanical Design  
Spring 2009

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*2.72*

*Elements of  
Mechanical Design*

*Lecture 07: Rolling  
contact bearings*

# Schedule and reading assignment

## Quiz

- ❑ Constraints

## Topics

- ❑ Bearing types & failure modes
- ❑ Experimental results to modeling
- ❑ Bearing load-life-reliability
- ❑ Start spindle exercise

## Reading assignment

- ❑ None, work on getting bearings de

## Quiz Tuesday

- ❑ Bearings (conceptual)



[www.renault4.co.uk/](http://www.renault4.co.uk/)

Courtesy of [Clementine's Renault 4 Garage](#). Used with permission.

Besides the children's toy (sorry bout that), what do you notice about the bearing that should be of import?

# Loose ends

## Thermal stability

- ❑ Function of separation distance and contact geometry - cancellation
- ❑ Will you always have bearing death if it is not thermally stable?

$$\dot{q} = \frac{\Delta T}{R_T}$$

$$R_{T-Conv} = \frac{1}{h A_{surface}}$$

$$R_{T-Cond} = \frac{t}{k A_{cross}}$$

## Mounting of races: Inside rotating vs. outside rotating

- ❑ Friction torques
- ❑ Look at the tapered roller example from lab

# Rough design steps – Inherently iterative

## Step 1: Functional requirements

- ❑ DOF
- ❑ Stiffness
- ❑ Lifetime/reliability
- ❑ Etc...

## Step 2: Bearing type/layout

## Step 3: System design & mfg issues

- |                    |                |
|--------------------|----------------|
| ❑ Housing geometry | Shaft geometry |
| ❑ Shaft deflection | Preload        |
| ❑ Thermal          | Tolerances     |

## Step 4: Assembly specifications

Iteration

*Types of rolling contact  
bearings*

# Examples of rolling element bearings

Images removed due to copyright restrictions. Please see:

[http://www.timken.com/en-us/products/bearings/productlist/ball/PublishingImages/radialball\\_150.jpg](http://www.timken.com/en-us/products/bearings/productlist/ball/PublishingImages/radialball_150.jpg)

[http://images.machinedesign.com/images/archive/news1300jpg\\_00000036367.jpg](http://images.machinedesign.com/images/archive/news1300jpg_00000036367.jpg)

<http://www.bearingsworld.com/image/pic-cp02.jpg>

<http://www.rlmc.com/image-product/23134mbw33.jpg>

# Elements: Rotary rolling contact bearing

## Inner

- Ring Race

## Outer

- Ring Race

## Diameters

- Outer Bore

## Ball/roller

## Cage/separator

## Face

## Width



*Bearing failure:  
Causes and  
failure mode*

# Cracks in bearings elements

Images removed due to copyright restrictions. Please see

[http://www.amstedrail.com/techinfo/media/94\\_1f5b.jpg](http://www.amstedrail.com/techinfo/media/94_1f5b.jpg)

[http://www.amstedrail.com/techinfo/media/94\\_1f2d.jpg](http://www.amstedrail.com/techinfo/media/94_1f2d.jpg)

# Failure mode: Spalling



Image removed due to copyright restrictions. Please see

<http://www.theautoist.com/Bearing5.JPG>

<http://www.tsb.gc.ca>

Courtesy the Transportation Safety Board of Canada.

# Causes of spalling

## Spalling

- ❑ Surface fatigue that occurs as a result of contact

## Seeds of failure

- ❑ Crack growth
- ❑ Inclusions
- ❑ Impact
- ❑ Cyclic high stress
- ❑ Degradation of the lubricant

Steel quality key to making long-lasting bearings



<http://www.tsb.gc.ca>

Courtesy the Transportation Safety Board of Canada.

## Once it starts, what happens?

- ❑ Minor spalling + correct problem may slow/stop
- ❑ Typically increases in size with continued service

Image removed due to copyright restrictions.  
Please see:

<http://www.theautoist.com/Bearing5.JPG>

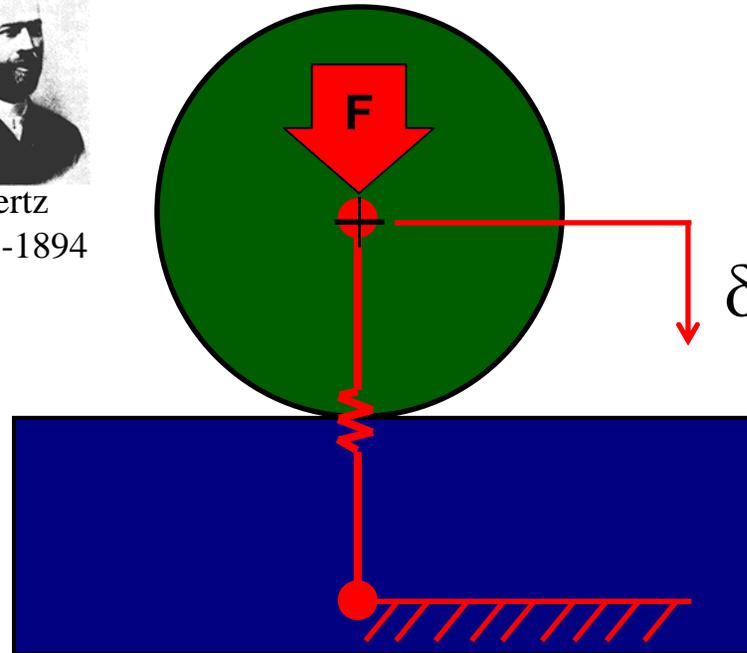
*Preload*

# Ball-flat elastomechanics: Hertz contact

## Model ball-groove contacts as six balls on flats



Hertz  
1857-1894



Hertzian contacts  
act as non-linear  
springs

Figure by MIT OpenCourseWare.

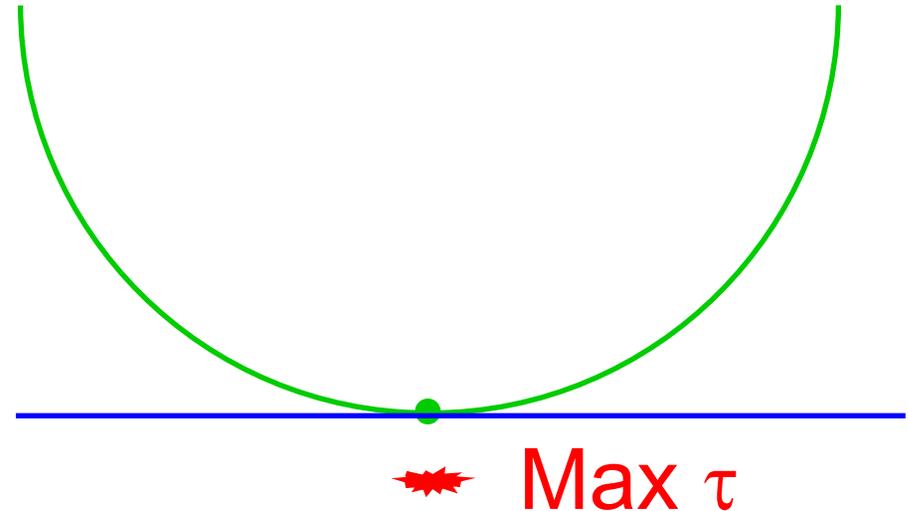
## Important relationships for ball-flat contact

$$k_n = 2 \cdot \delta^{1/2} \cdot R^{1/2} \cdot E_e$$

# Location, magnitude of max shear stress

Equivalent radius

$$R_e = \frac{1}{\frac{1}{R_{1major}} + \frac{1}{R_{1minor}} + \frac{1}{R_{2major}} + \frac{1}{R_{2minor}}}$$



Equivalent modulus

$$E_e = \frac{1}{\frac{1-\eta_1^2}{E_1} + \frac{1-\eta_2^2}{E_2}}$$

Max shear stress occurs below surface, in the member with largest R if ball and flat of same material

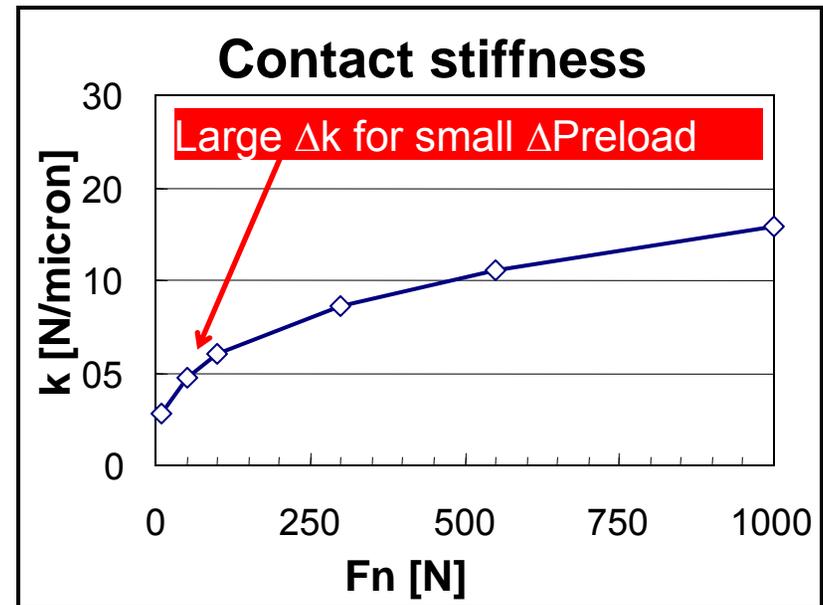
$$\tau_{\max} \left| \begin{array}{l} \text{center of contact} \\ \text{depth} = 0.48 \cdot \text{contact radius} \end{array} \right. = 0.31 \cdot \frac{1}{\pi} \cdot \left( \frac{6 \cdot F \cdot E_e^2}{R_e^2} \right)^{\frac{1}{3}}$$

# Sensitivity of contact stiffness to $\Delta F$

## Preload increases stiffness

$$k_n = K_o \cdot \left( R_e^{1/3} \cdot E_e^{2/3} \right) \cdot F_n^{1/3}$$

## A little preload goes a long way



## Classes of preload, as % of static load capacity

- ❑ Heavy                    5%
- ❑ Medium                 3%
- ❑ Light                    2%

# How do you preload a bearing?

## Direct

- ❑ Nuts pressing directly on races
- ❑ Uses compliance of contacts

## Internal

- ❑ Oversized balls
- ❑ Uses compliance of ball-race contacts

## Nuts-springs

- ❑ Spring in series with bearing
- ❑ Primarily uses compliance of spring

Many bearings come preloaded “out of the box”

Check to make sure so that you do not add preload that will act to overload



Figure by MIT OpenCourseWare.

## Think in terms of relative stiffness because...

- ❑ Sensitivity of force to the displacement...

*Bearing life and  
reliability*

# Bearing life at rated reliability

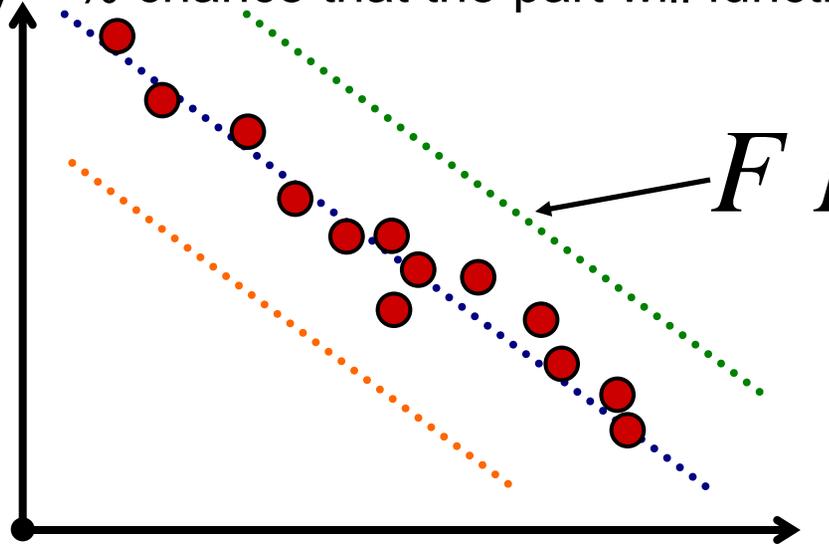
## Bearing life depends on:

- Load and revolutions

## From experimentation: For a given R, linear log behavior

- Reliability = % chance that the part will function as expected

log(F)



$$F L^{\frac{1}{a}} = \psi$$

## Where

- $a = 3$  for ball bearings
- $a = 3.333$  for roller bearings



# Regression fit to experimental data

## Manufacturers provide a match of:

- Cycle rating: Revolutions
- Load rating: Force

For example  $10^6$  or  $90^6$  revs  
Anything...

## that defines bearing failure for a given reliability

This is for common reliability

## Given these two numbers, and:

$$F L^{\frac{1}{a}} = \psi$$

Units of force →  $F$       Units of revolutions →  $L^{\frac{1}{a}}$       Constant →  $\psi$

## This may be used to extrapolate behavior at different loads and revs

Why C vs. F?

$$C_i L_i^{\frac{1}{a}} = F_{design} L_{design}^{\frac{1}{a}}$$

# Design life in terms of hours or revolutions?

We can think in terms of life as time if:

$$\text{Dynamic load rating} \quad t [hr] \quad \omega \left[ \frac{rev}{min} \right] \quad 60 \left[ \frac{min}{hr} \right] = L$$

Revs at failure

Be careful with units

$$C_{10} L_{10}^{\frac{1}{a}} = F_{design} L_{design}^{\frac{1}{a}} = F_{design} (t \omega 60)^{\frac{1}{a}}$$

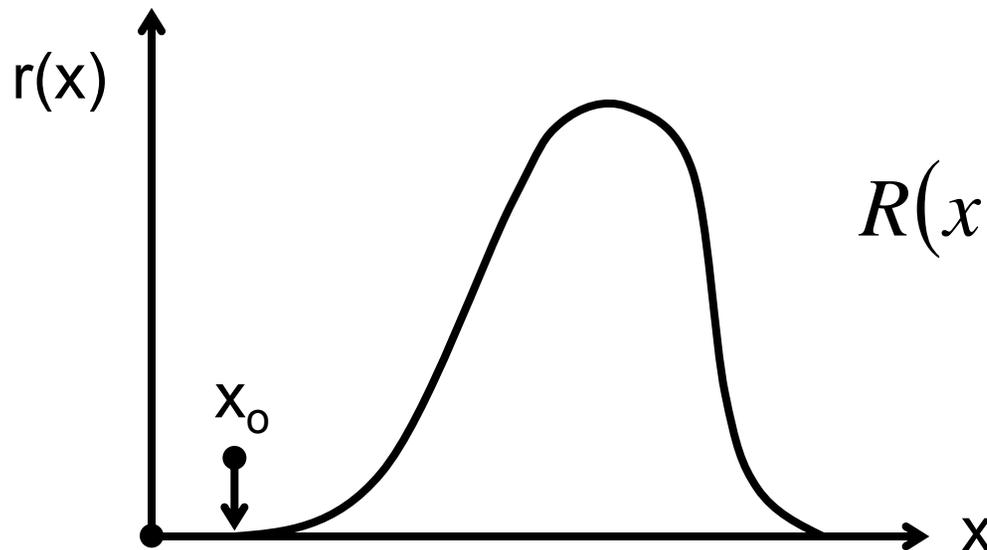
$$C_{10} (t_{rated} \omega_{rated} 60)^{\frac{1}{a}} = F_{design} (t_{design} \omega_{design} 60)^{\frac{1}{a}}$$

$$C_{10} = F_{design} \frac{(t_{design} \omega_{design} 60)^{\frac{1}{a}}}{(t_{rated} \omega_{rated} 60)^{\frac{1}{a}}}$$

# Reliability vs. life

## Reliability often well-predicted via Weibull distribution

- $x_0$  = minimum guaranteed value of  $x$
- $\theta$  = corresponds to 63.2 percentile of the variate (stochastic variable)
- $b$  = a shape parameter (controls skew, large = right)



$$R(x) = \exp \left[ - \left( \frac{x - x_0}{\theta - x_0} \right)^b \right]$$

This is for common load on bearings

- For bearings, we use this as:

$$x = \frac{L}{L_{10}}$$

Commonly used to fit experimental data;  $b$  &  $\theta$  come from fit

# Relationship between load, life and reliability

**BUT the catalogue never tells me what happens for....**

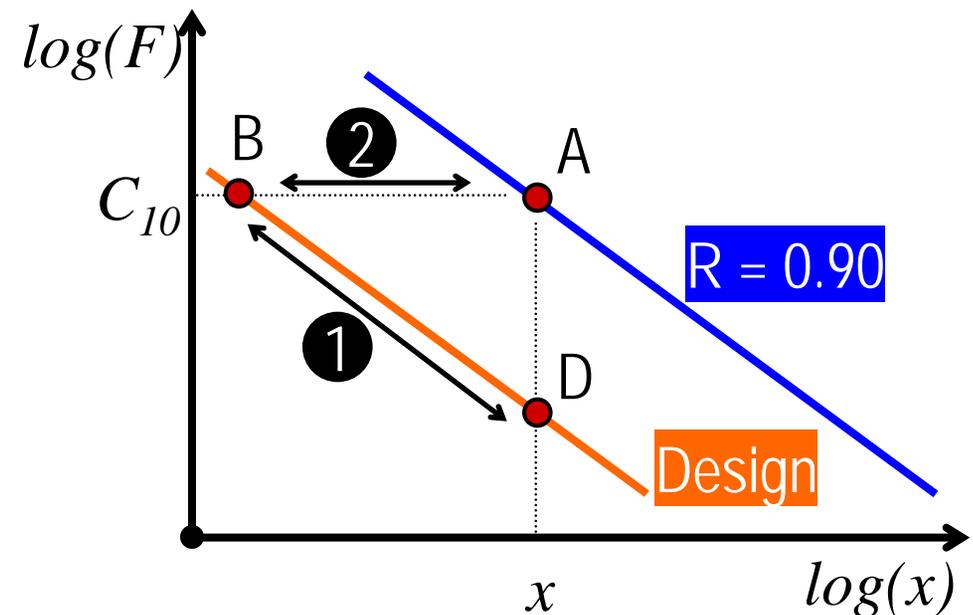
- ❑ My exact desired load
- ❑ My exact desired life
- ❑ Situations when I want a reliability that is different than  $R = 0.90$

Constant reliability :

$$\textcircled{1} \quad F_B x_B^{\frac{1}{a}} = \psi = F_D x_D^{\frac{1}{a}}$$

Constant load

$$\textcircled{2} \quad R(x) = \exp \left[ - \left( \frac{x_B - x_o}{\theta - x_o} \right)^b \right]$$

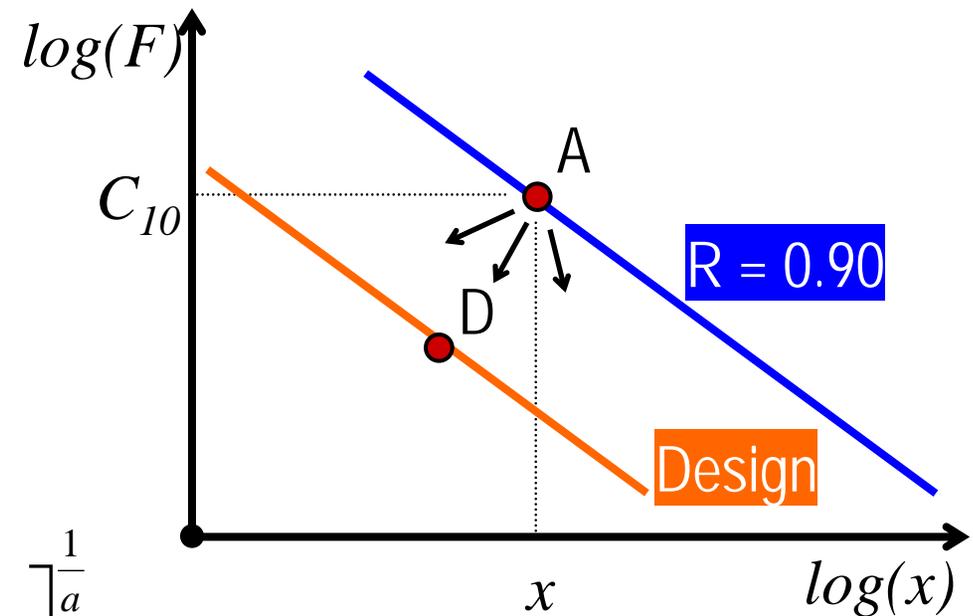


$$x = \frac{L}{L_{PF}}$$

# Relationship between load, life and reliability

Load as a function of reliability & vice versa for given:

- $C_{10}$
- $L_{10}$
- $R = 0.90$ ;  $x = 1$  when  $L = L_{10}$



$$C_{10} = F_D \left[ \frac{x_D}{x_0 + (\theta - x_0) \left( \ln \left\langle \frac{1}{R_D} \right\rangle \right)^{\frac{1}{b}}} \right]^{\frac{1}{a}}$$

$$x = \frac{L}{L_{PF}}$$

# Reliability of multi-bearing sets

## What is the reliability of:

- ❑ One bearing?
- ❑ A spindle with two bearings?
- ❑ With N bearings?

**For first order design, how should individual bearing reliability scale as a function of N?**

# How to handle combined loading

So far we have only considered radial loading...

What about combined radial,  $F_r$ , and axial loading,  $F_a$ ?

- Use an equivalent load,  $F_e$ , that does the same amount of damage.

$$F_e = X_i V F_r + Y_i F_a$$

Where  $V = 1.2$  for outer ring rotation and  $1$  for inner ring

- This has to do with the fact that outer ring fails more often

$X_i$  and  $Y_i$  are a function of the

- Axial load,  $F_a$
- Static load rating,  $C_o$



# Other issues, Shigley/Mischke covers well

## Life recommendations (hrs)

❑ Aircraft engines	0 500	–	2 000
❑ 24-hour critical service	100 000	–	200 000

## Application factors

❑ General commercial	1.1 – 1.3
❑ Moderate impact	1.5 – 3.0

# Group exercise

## Work on your spindle housing-shaft-bearing design

- ❑ Constraint layout
- ❑ Loads
- ❑ Preload
- ❑ Thermal stability
- ❑ Cost