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2.72 Elements of Mechanical Design  
Spring 2009

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*2.72*

*Elements of  
Mechanical Design*

*Lecture 08: Flexures*

# Schedule and reading assignment

## Quiz

- ❑ Today: Bearing layouts (mid-class)
- ❑ Thursday: Hale 6.1
- ❑ Soon: Bolted joint qualifying quiz

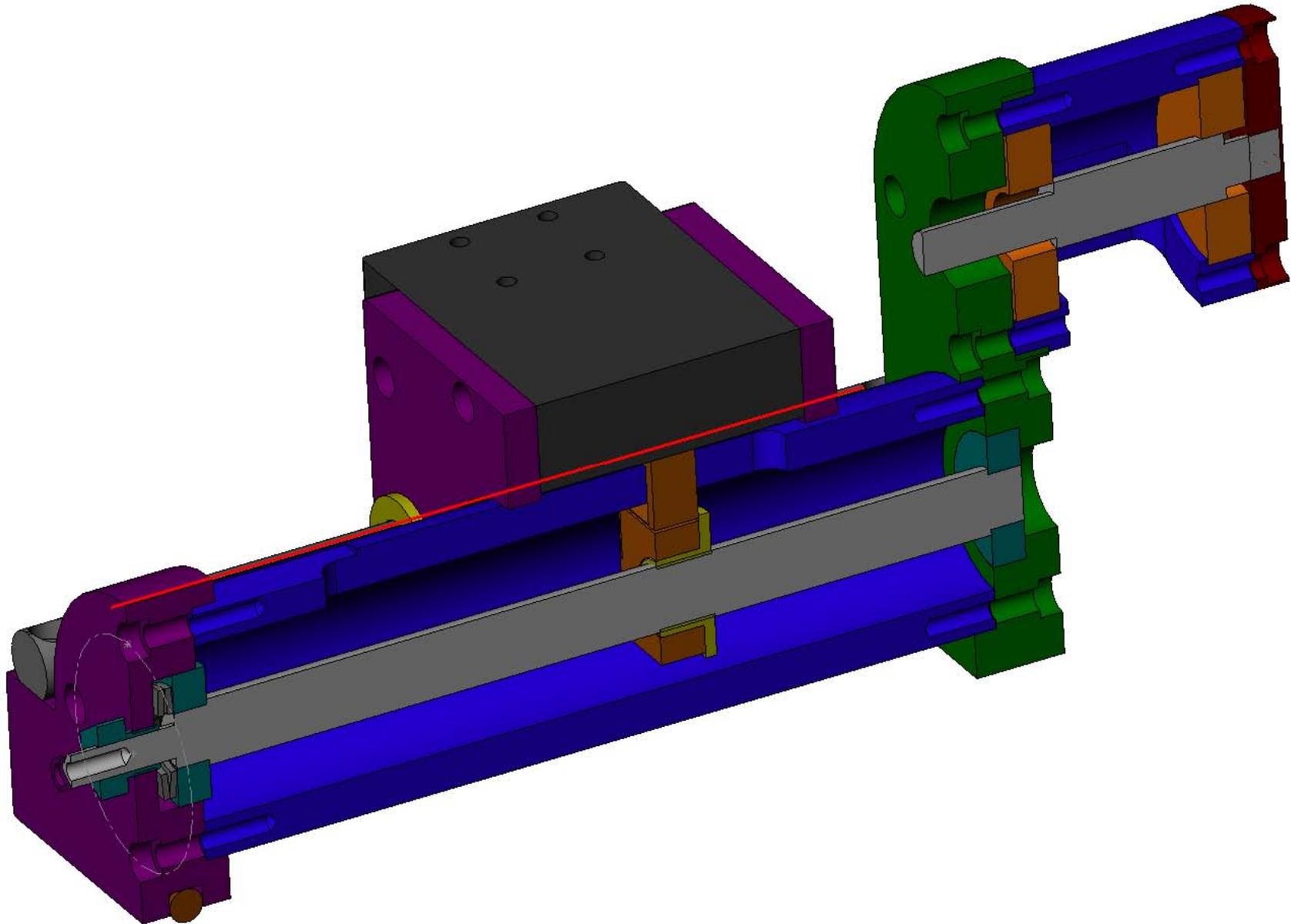
## Topics

- ❑ Flexure constraints and bearings... Degrees of Freedom

## Reading assignment

- ❑ Thursday:
  - *Layton Hale's thesis – Read 2.6, 2.7, 6.1, skim rest of Chapter 6*
  - *Chapter 7 is cool to look at*
- ❑ Tuesday:
  - *Read: 8.1, 8.3 – 8.5, 8.7, 8.9 – 8.11*
  - *Skim: 8.6, 8.8, 8.12*

# Examples drawn from your lathe



# Mechanisms: Compliant vs. rigid

## Rigid mechanisms

- ❑ Sliding joints
- ❑ 100s of nm resolution
- ❑ Large range
- ❑ kg load capacity

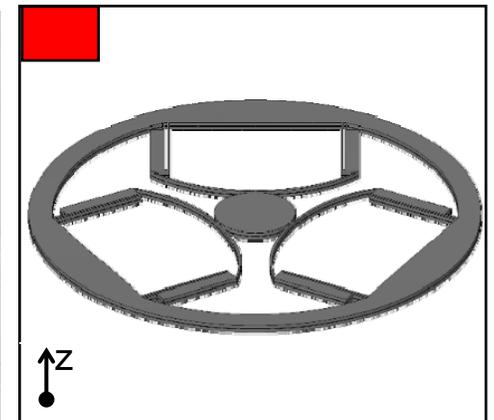
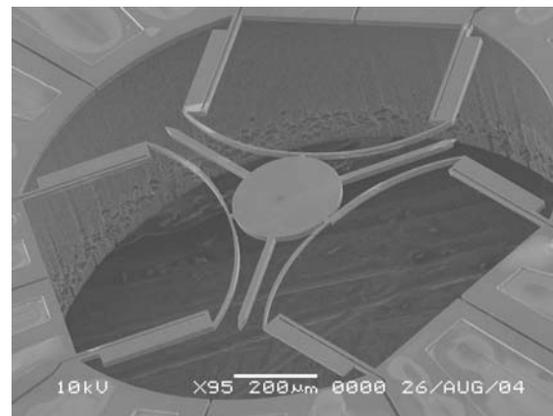
Images removed due to copyright restrictions. Please see

[http://www.physikinstrumente.com/en/primages/pi\\_m850\\_tip\\_i4c\\_o\\_eps.jpg](http://www.physikinstrumente.com/en/primages/pi_m850_tip_i4c_o_eps.jpg)

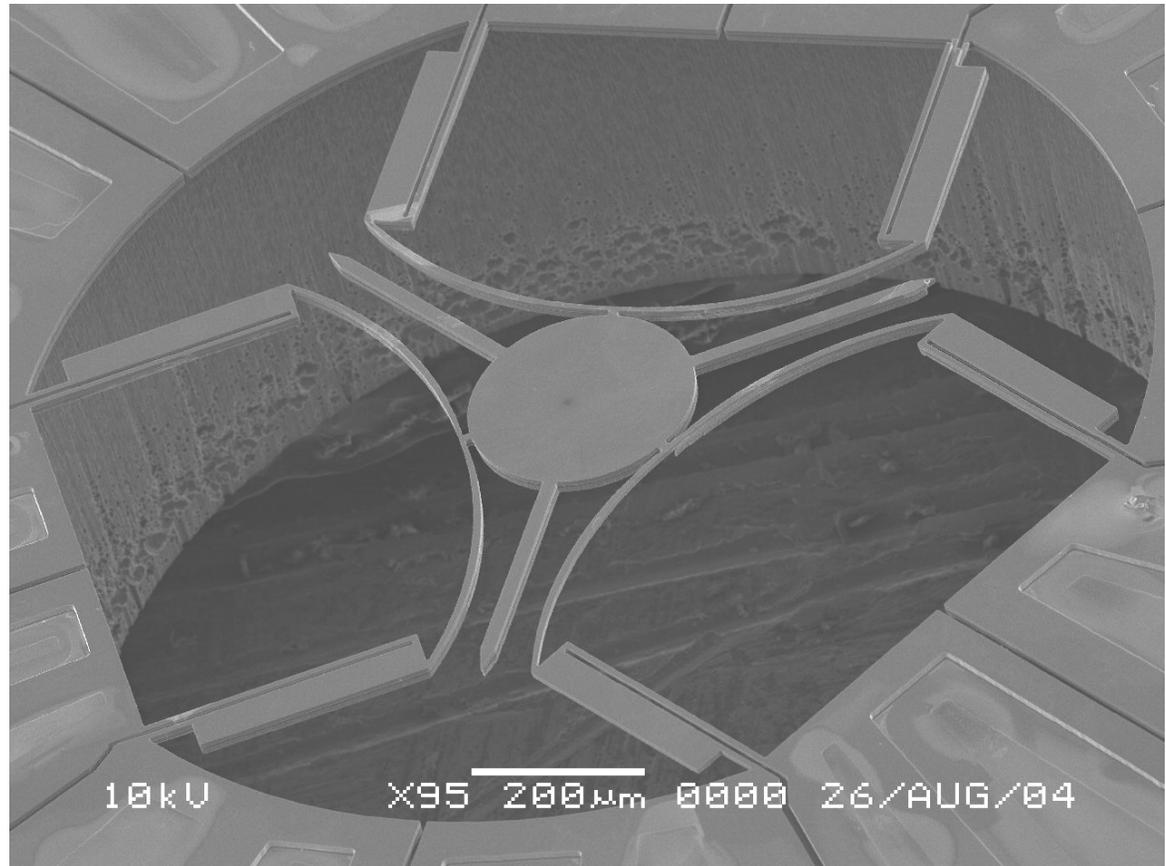
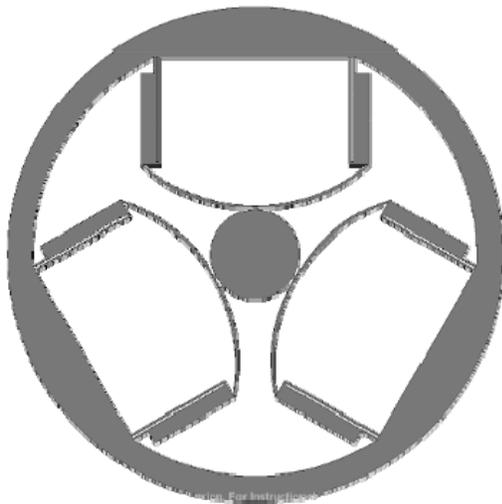
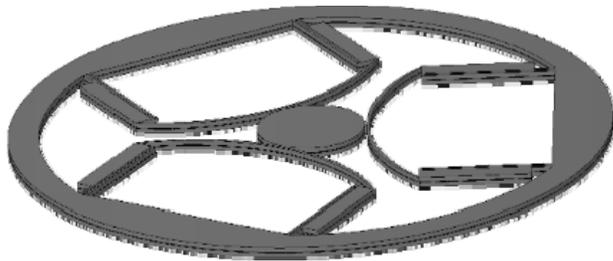
<http://www.hexapods.net/images/M850Ani160-1-slow.gif>

## Compliant mechanisms

- ❑ Motion from member compliance
- ❑ Angstrom resolution
- ❑ Limited range
- ❑ Limited load capacity

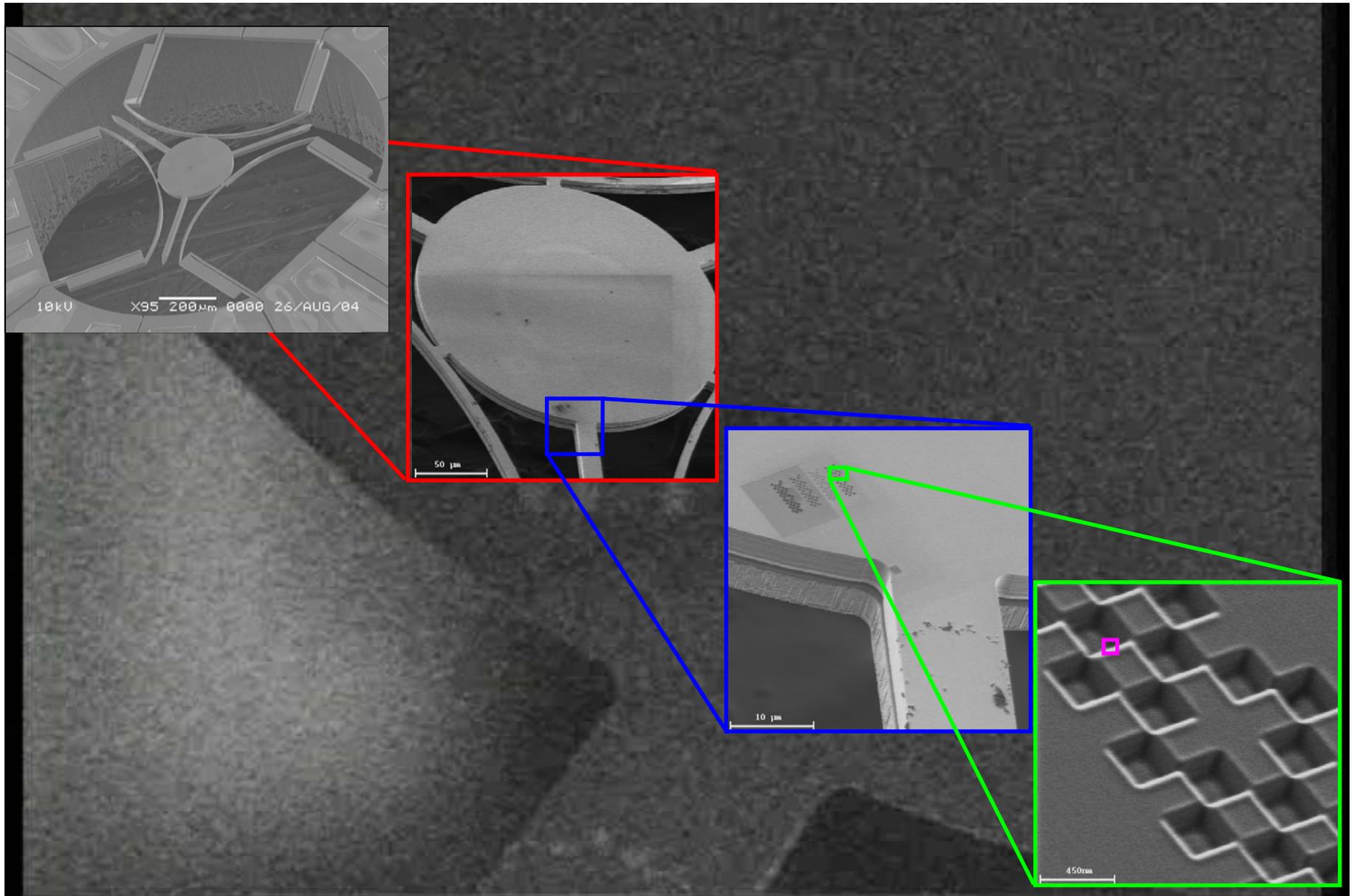


# Micro-scale precision machines



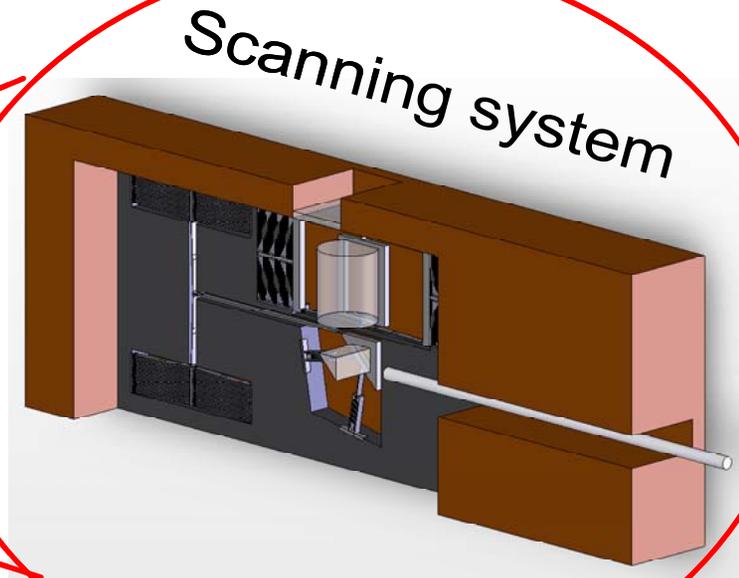
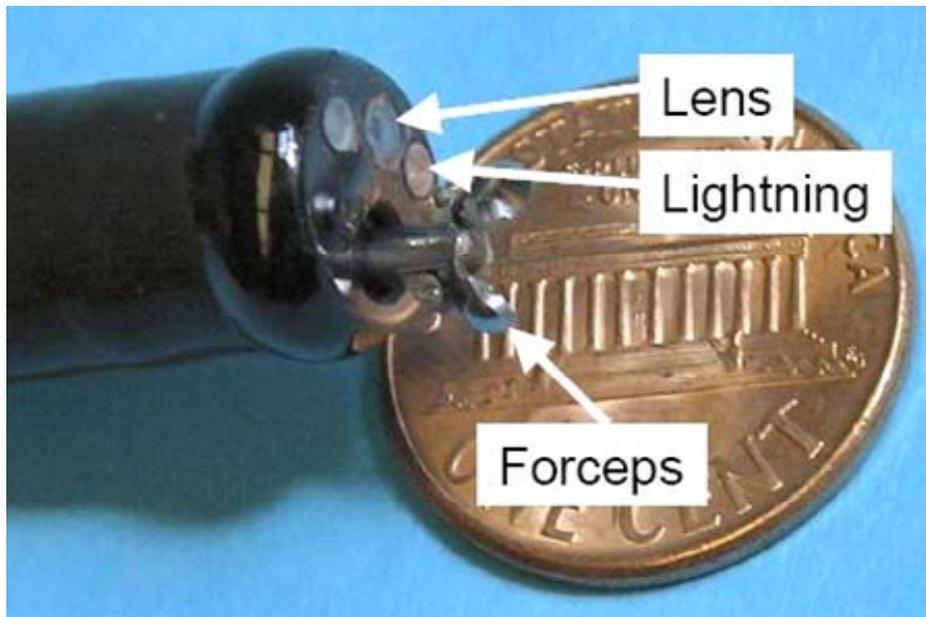
# Static

SEM: Drs. Andras Vladar & Jason Gorman (NIST)  
FIB: Dr. Konrad Jarush (Hitachi)

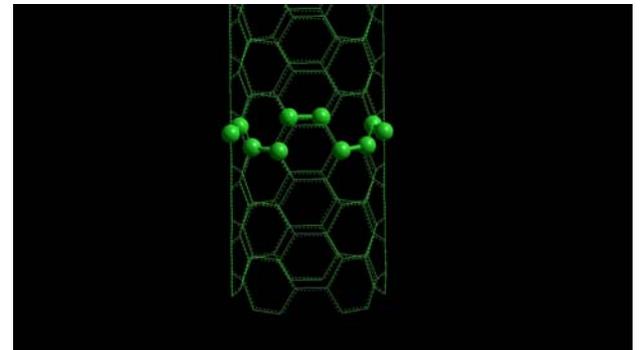
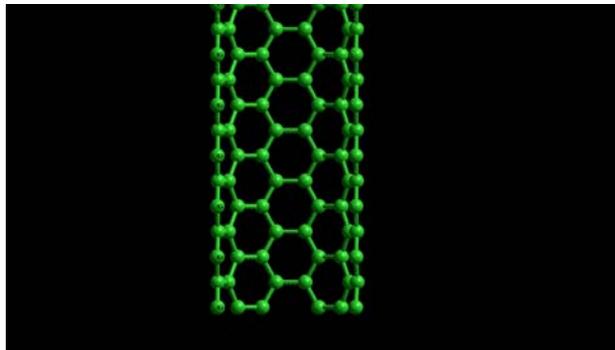
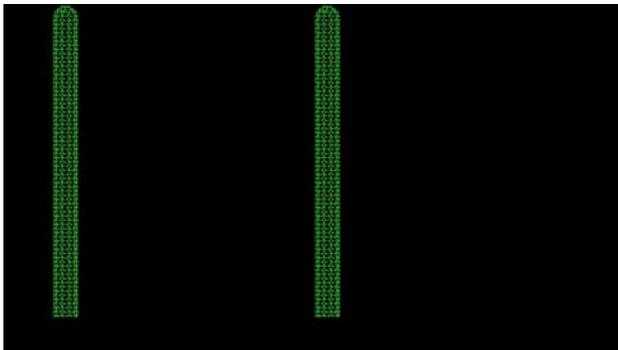
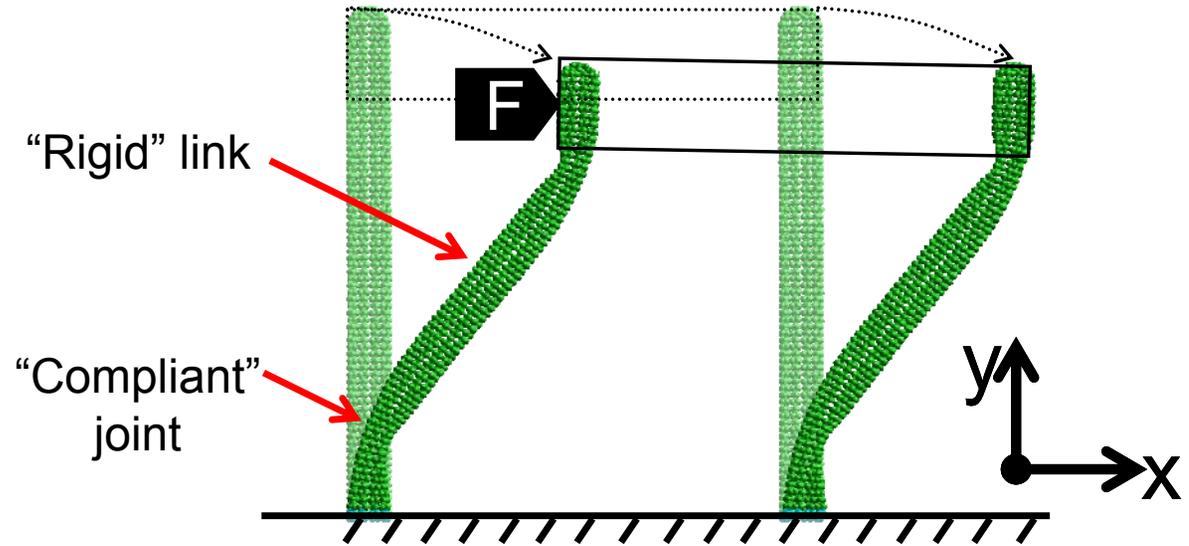
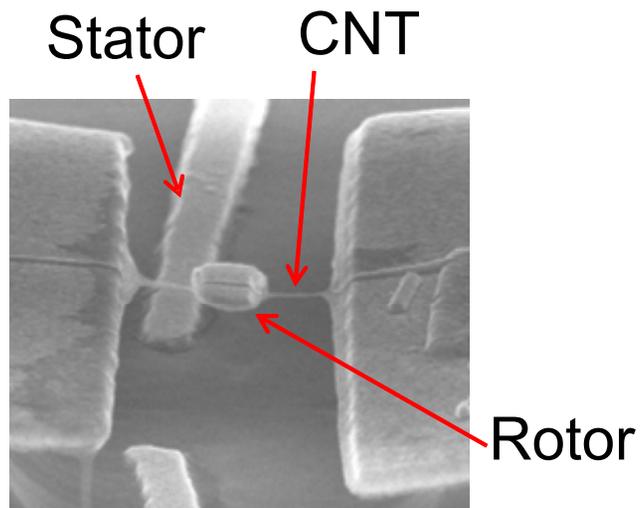


# Meso-scale devices: Biomedical

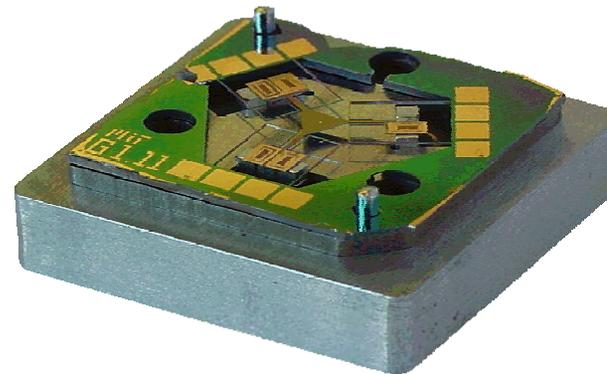
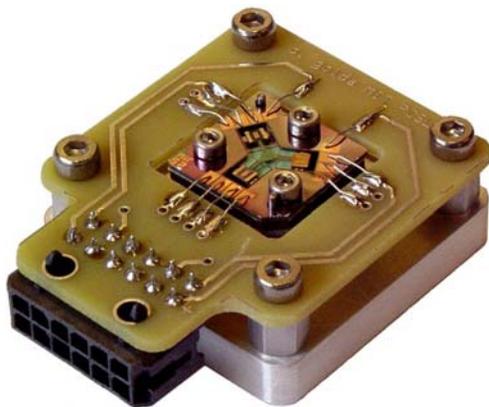
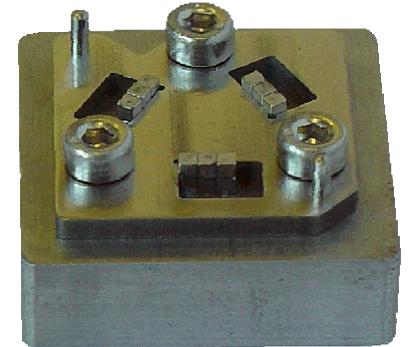
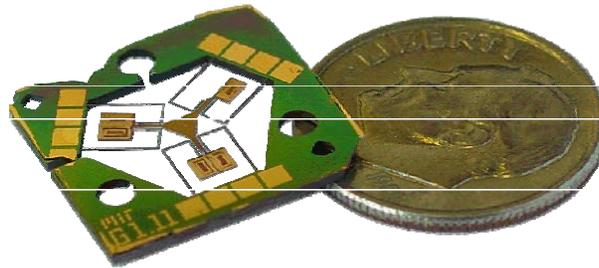
Two-photon endomicroscope



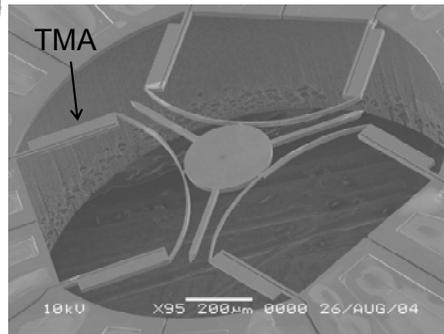
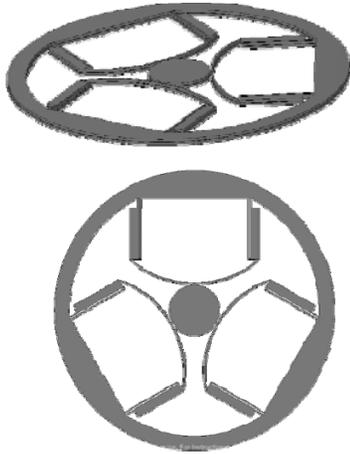
# Nano-scale devices



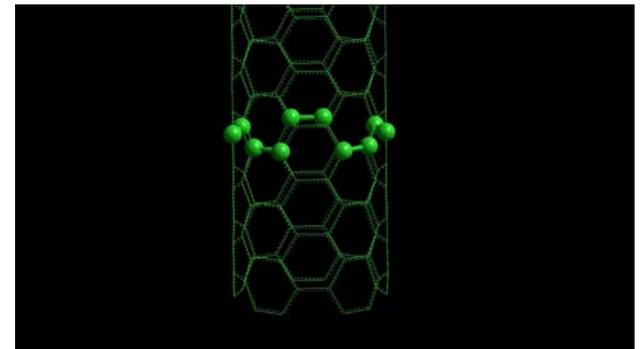
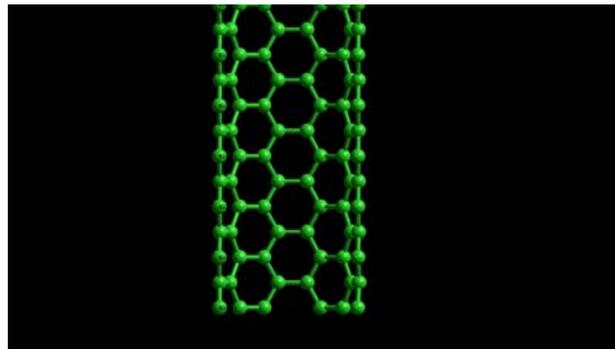
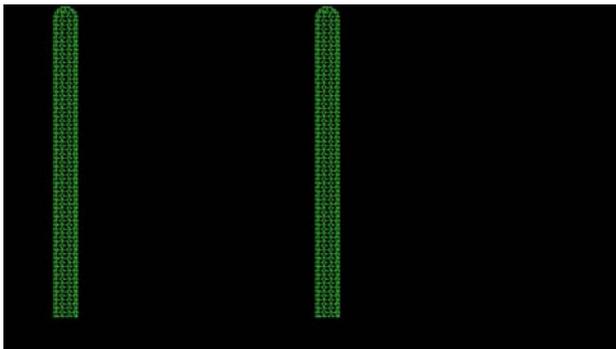
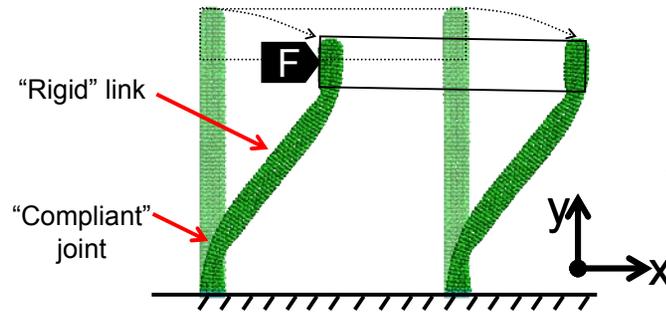
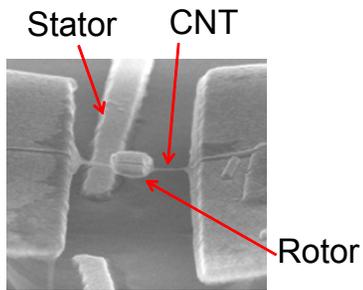
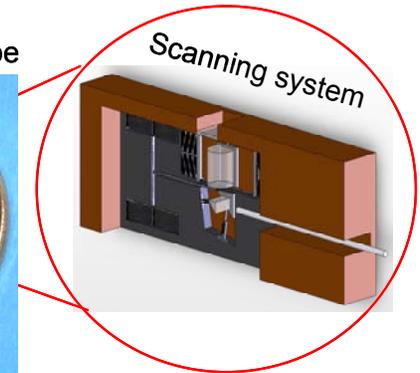
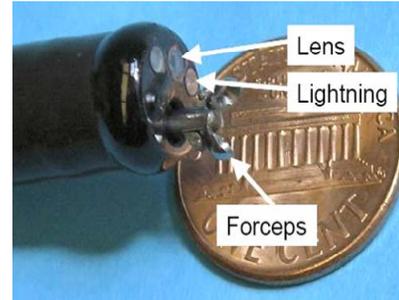
# Meso-scale precision machines



# Nano-scale devices



Two-photon endomicroscope



# Dip pen nanolithography on DNA arrays

## What is fundamentally different?

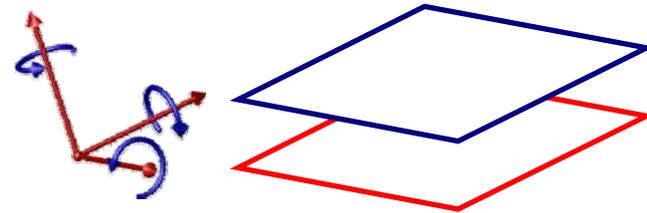
- Size → Physics → Fabrication
- Raw materials
- Surfaces vs. points or lines

Images removed due to copyright restrictions. Please see

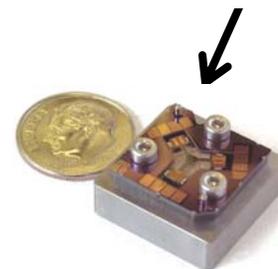
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[http://www.nanoink.net/d/Nano%20-%20Part%201\\_Sm\\_Lo-Res\\_240x180.wmv](http://www.nanoink.net/d/Nano%20-%20Part%201_Sm_Lo-Res_240x180.wmv)

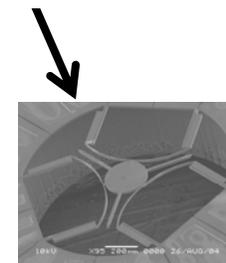
<http://images.iop.org/objects/nano/news/4/12/10/diagnal.jpg>



250 mm  
Courtesy  
PI

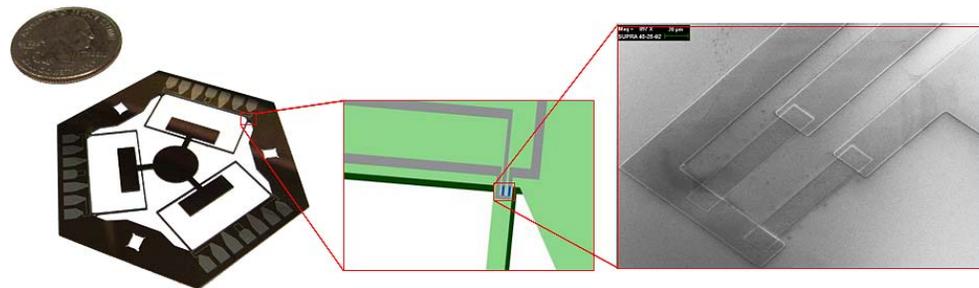
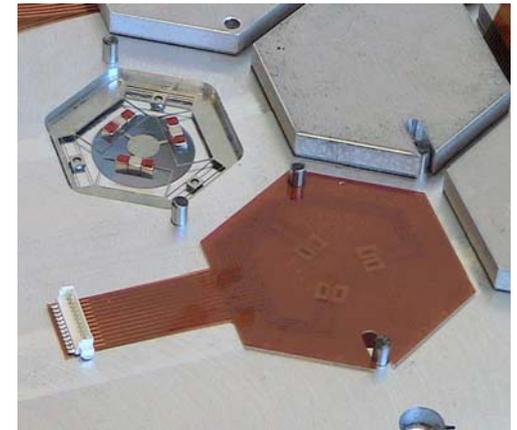
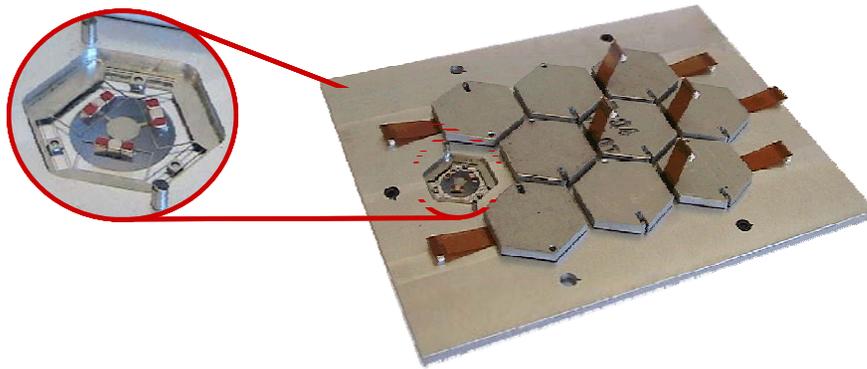
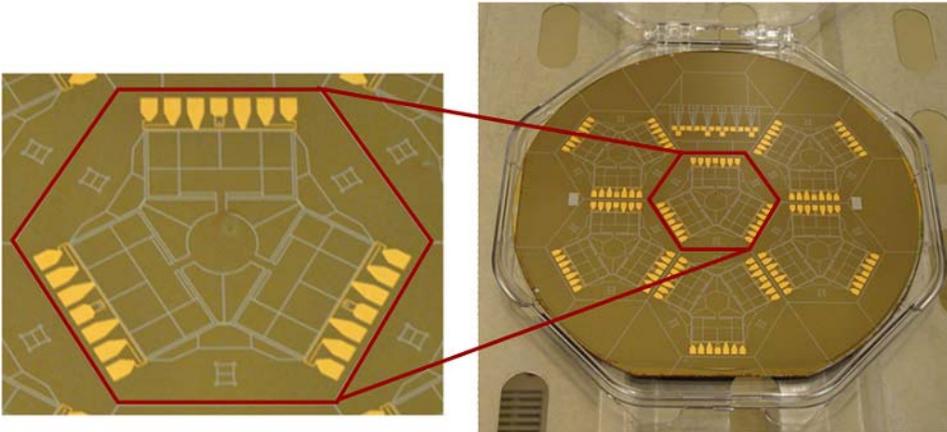
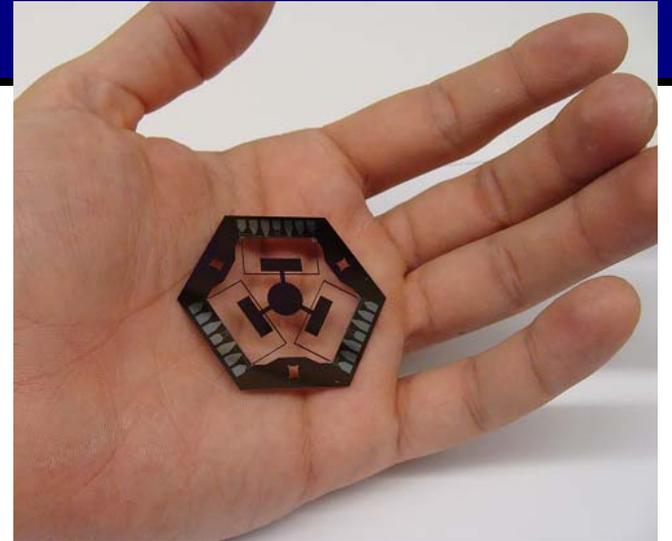


~20 mm



~1 mm

# Nanomanufacturing



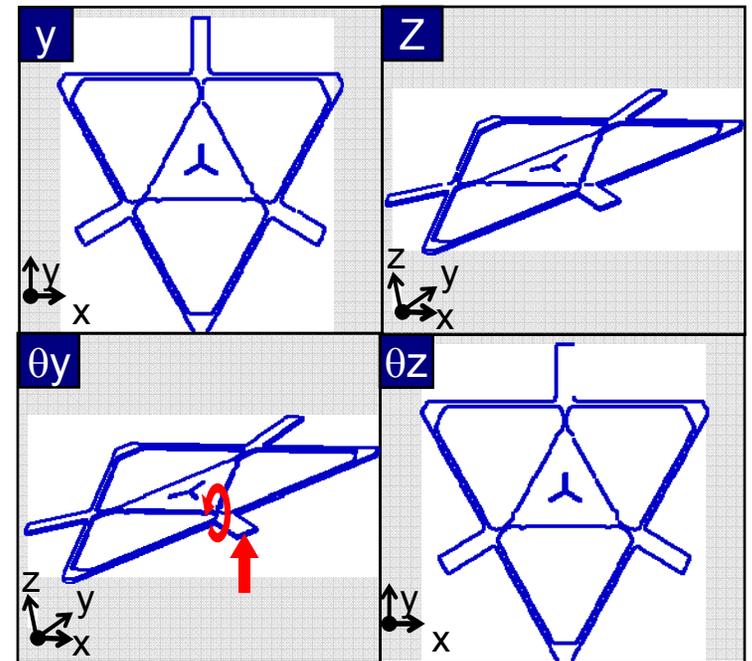
# Advantages of flexures

## Advantages

- ❑ Smooth, fine motion
- ❑ Linear/elastic operation in absence
- ❑ Failure modes are well understood
- ❑ Monolithic or assembled
- ❑ 2D nature lends to 2½D mfg.
- ❑ Miniaturization

## Disadvantages

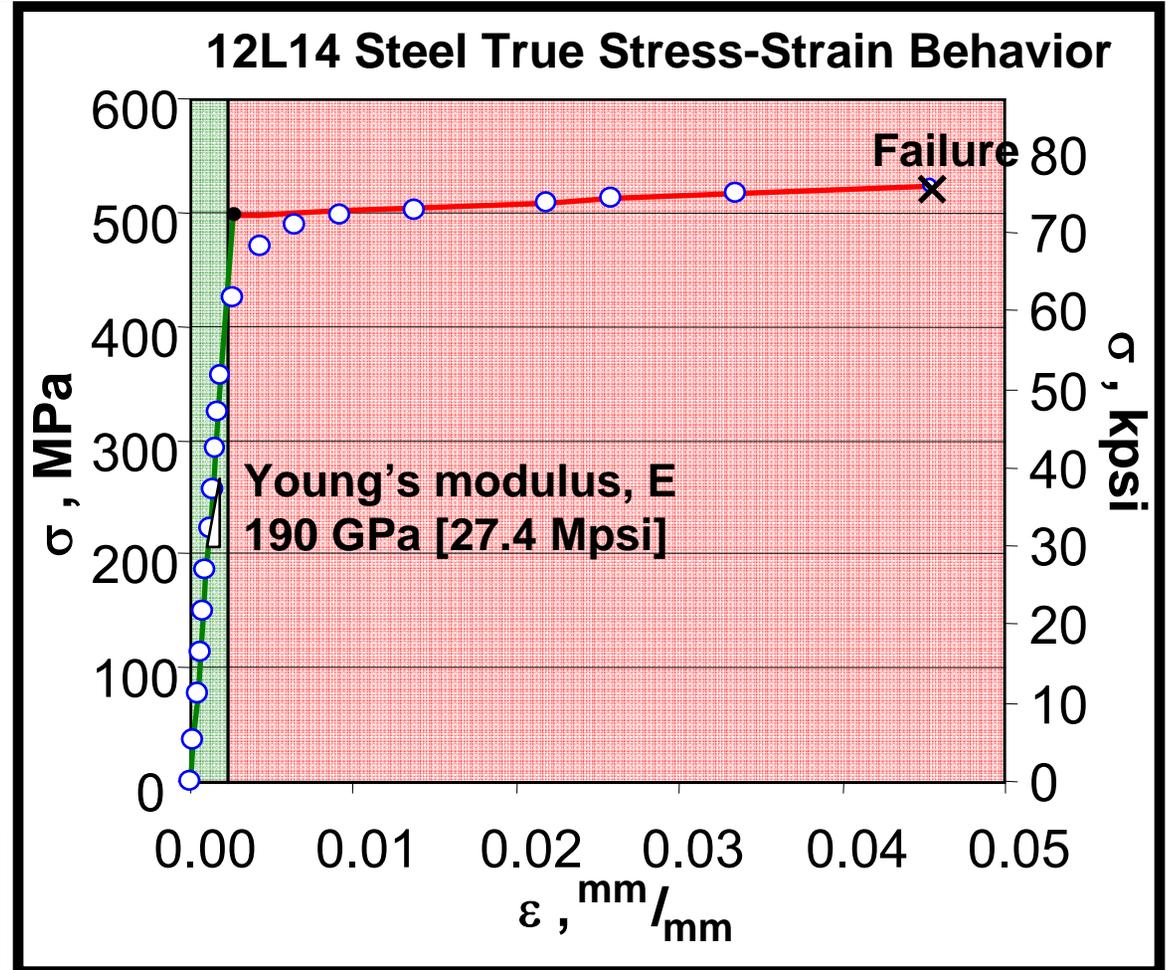
- ❑ Accuracy and repeatability sensitive to several variables
- ❑ Limited motion/stroke (usually a few to 10s % of device size)
- ❑ Instabilities such as axial or transverse buckling
- ❑ Dynamics
- ❑ Sensitivity to tolerance



# Elastomechanics ( $\sigma$ & $\varepsilon$ ) relationship

 Elastic  
 $\sigma = \varepsilon \cdot E$

 Plastic



Material	$\sigma_y/E$
Titanium V	<b>1.00</b>
Aluminum 7075	0.70
Stainless 316	0.09
Invar - Annealed	0.19

# Important material properties

## Nominal values

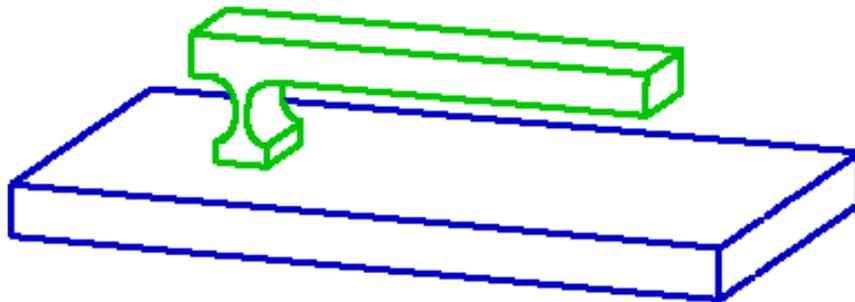
- ❑ Modulus
- ❑ Yield stress
- ❑ Coefficient of thermal expansion
- ❑ Thermal diffusivity
- ❑ Density

## Material property ratios

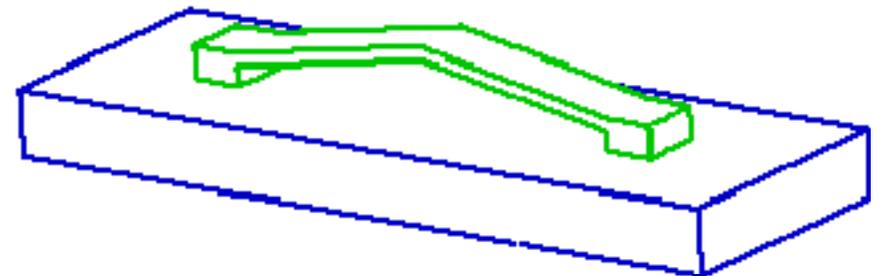
Material	Normalized Values			
	$\sigma_y/E$	$\alpha_{diff}/\alpha_{CTE}$	$E/\rho$	Cost
Titanium V	<b>1.00</b>	0.14	0.92	3.77
Aluminum 7075	0.70	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>
Stainless 316	0.09	0.13	0.94	3.50
Invar - Annealed	0.19	0.87	0.70	5.21

# Modules

## Lever

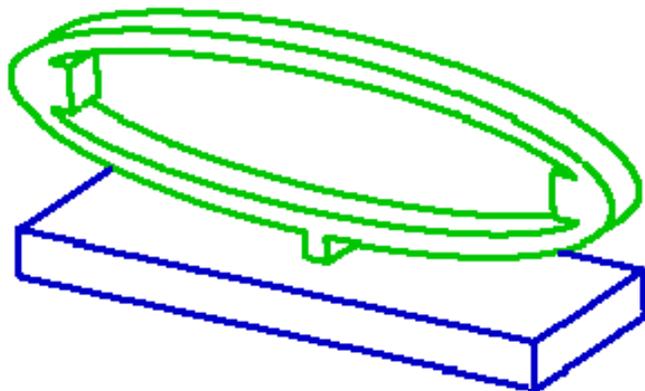
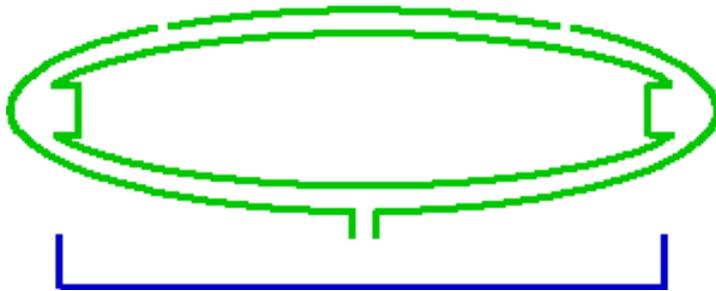


## Chevron

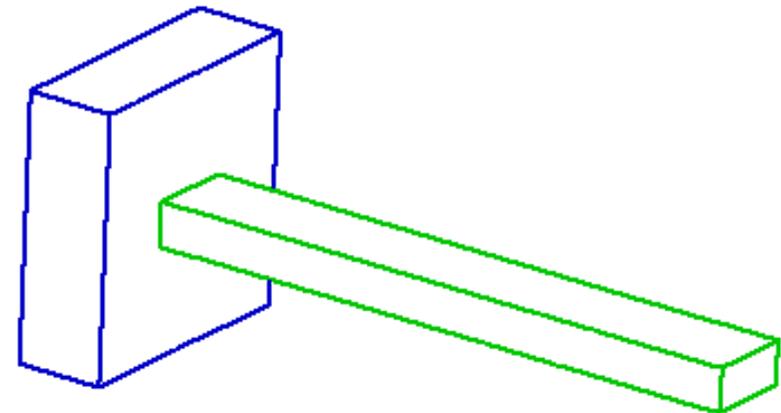
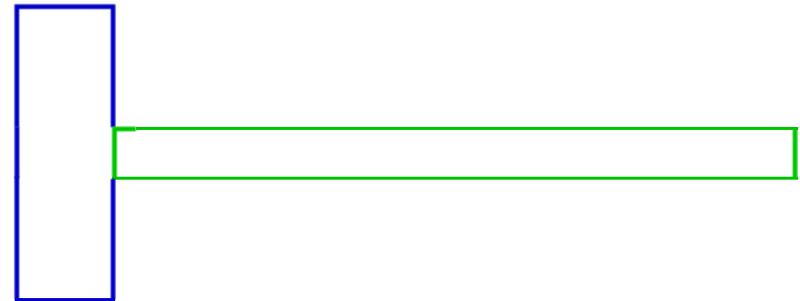


# Modules cont.

## Ellipse

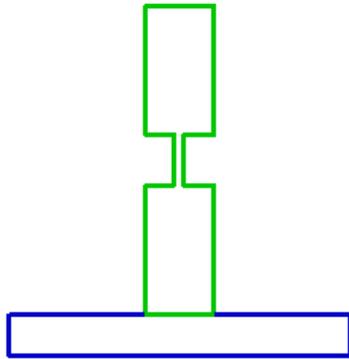


## Cantilever/flexure blade

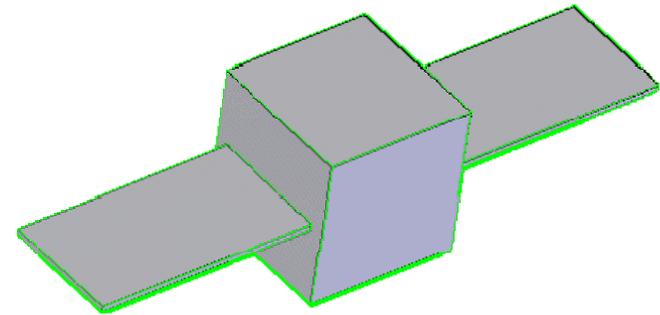


# Modules cont.

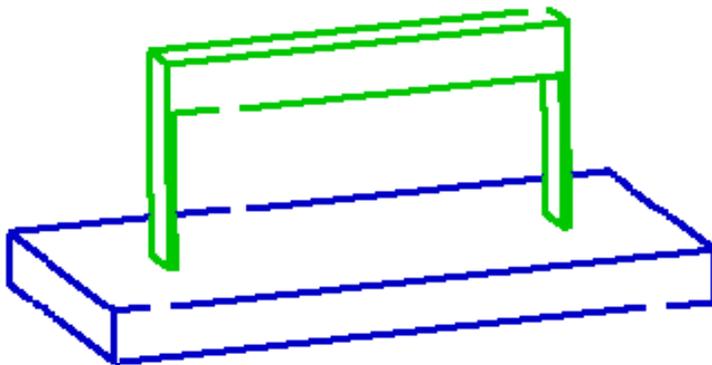
## Flexure hinge



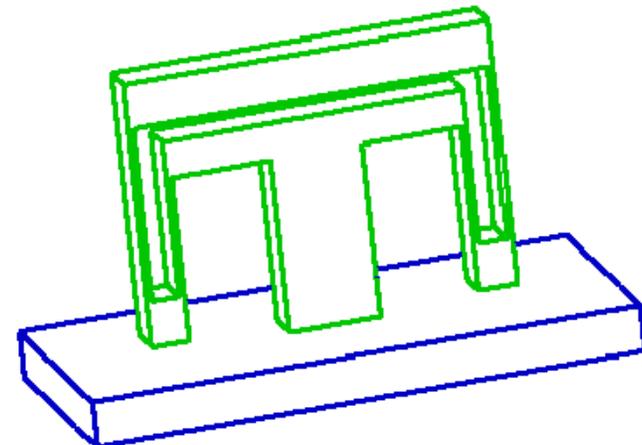
## Torsion



## Parallel four bar

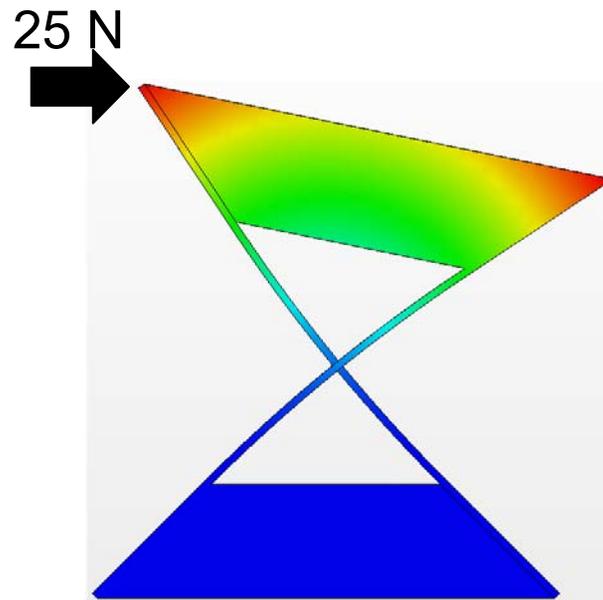
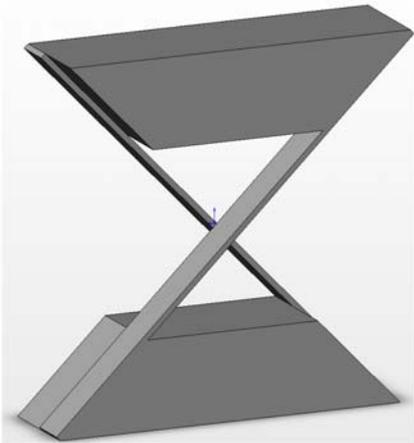


## Double parallel four bar

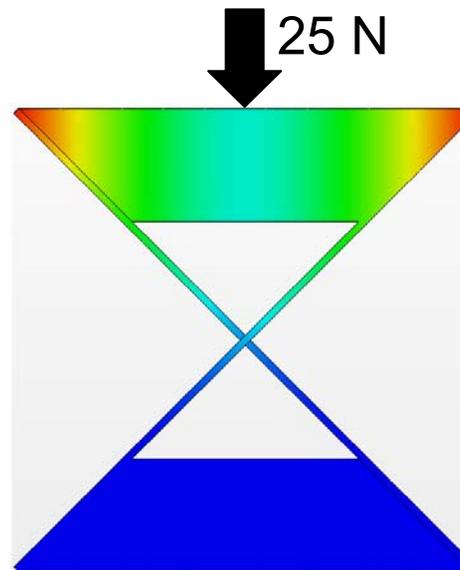
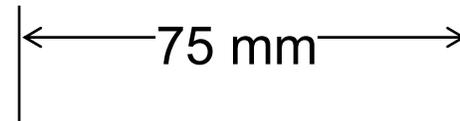


# Module cont.: Cross flexure pivot

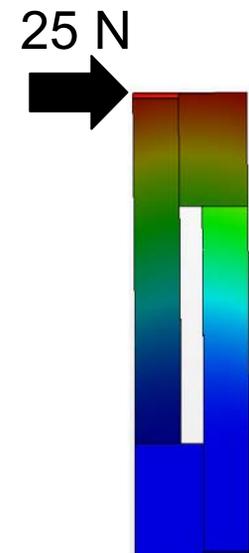
Deformation scale 1 : 1



||



⊥

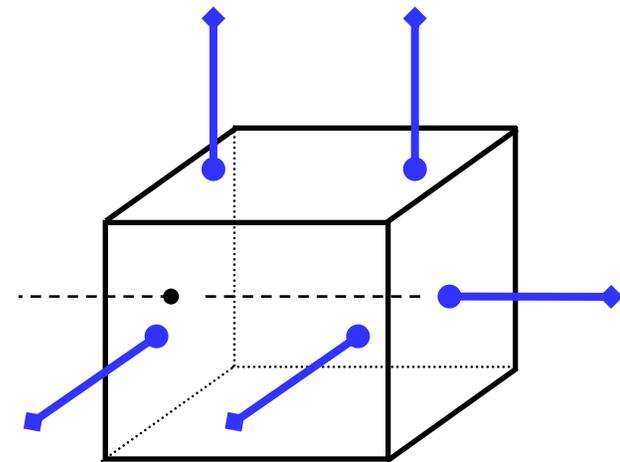
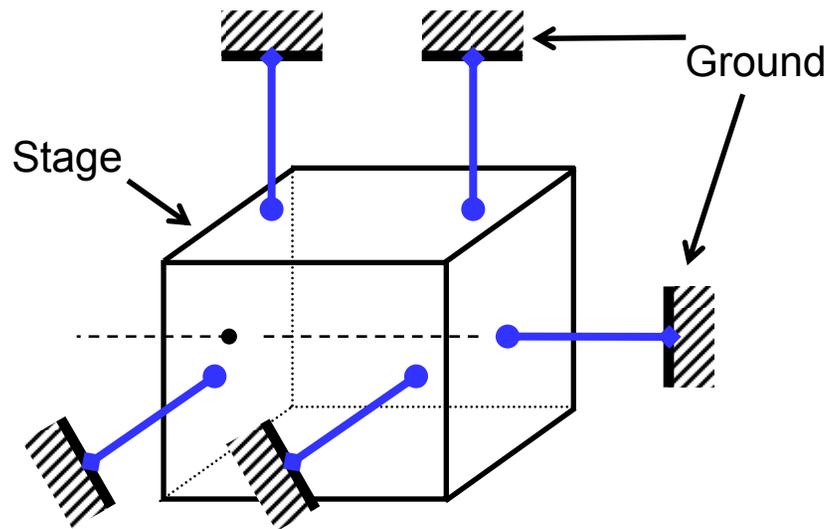
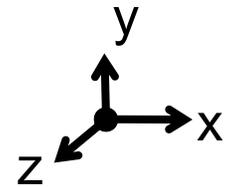


⊥op

# Review of constraint fundamentals

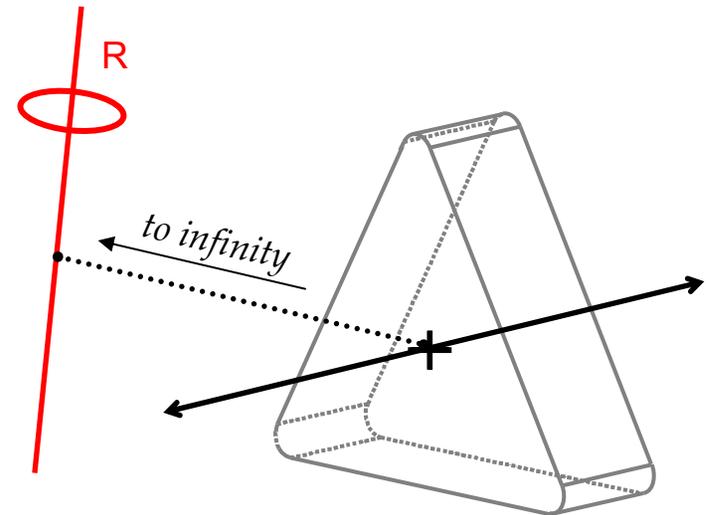
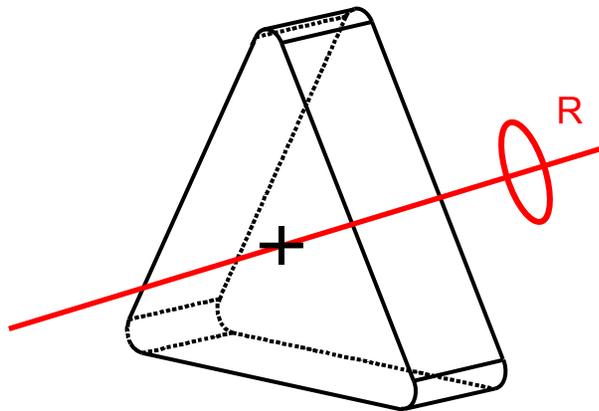
## Rigid bodies have 6 DOF

- Constraints have lines of action
- $C = \#$  of linearly independent constraints
- $\text{DOF} = 6 - C \quad \rightarrow \quad F = 6 - C$



# DOF in constraint-based design

A linear displacement may be visualized as a rotation about a point which is “far” away

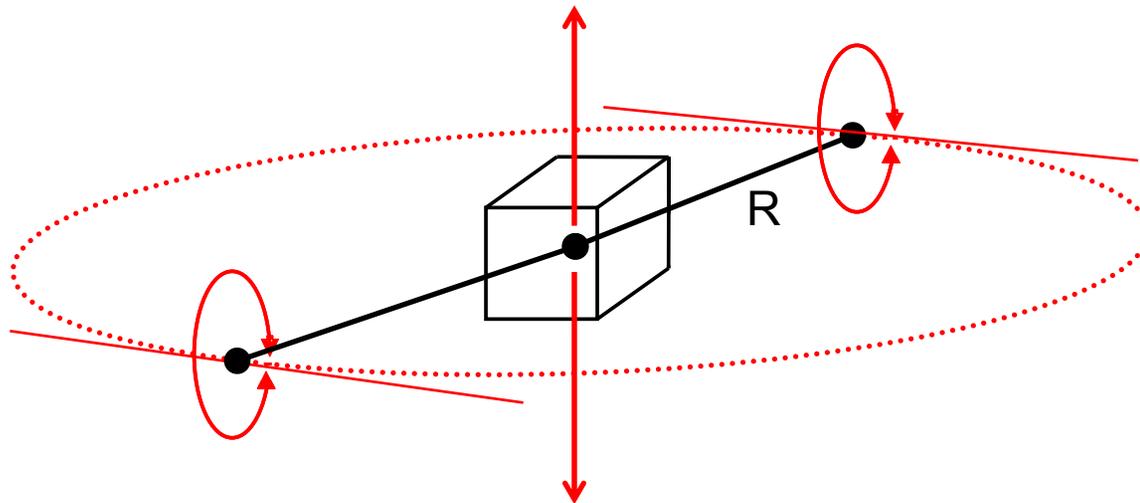


# Two principles of projective geometry

## Projective geometry comes in useful here

- Parallel lines intersect at infinity
- Translation represented by a rotation line at a hope of “infinite radius”

Image courtesy of John Hopkins  
MIT MS Thesis



# Constraint fundamentals

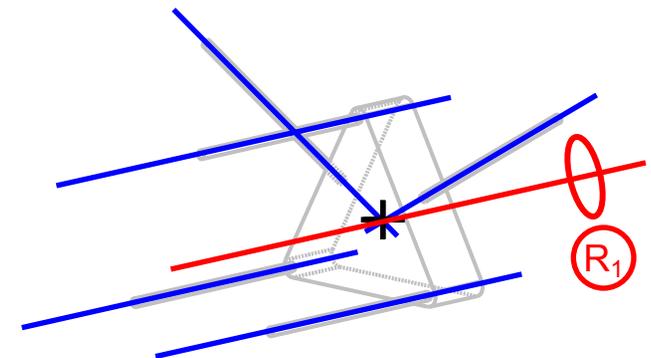
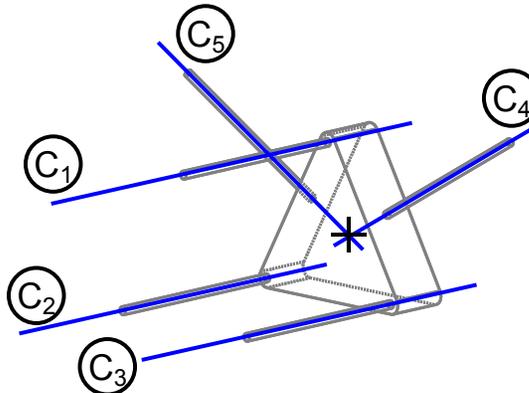
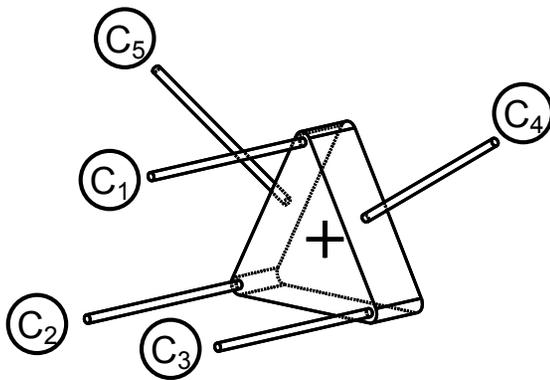
## Blanding's RULE OF COMPLIMENTARY PATTERNS

- Each permissible **Freedom (F)** is a rotation about a line and each permissible freedom rotation line must intersect each **Constraint (C)**

## Remember these principles of projective geometry

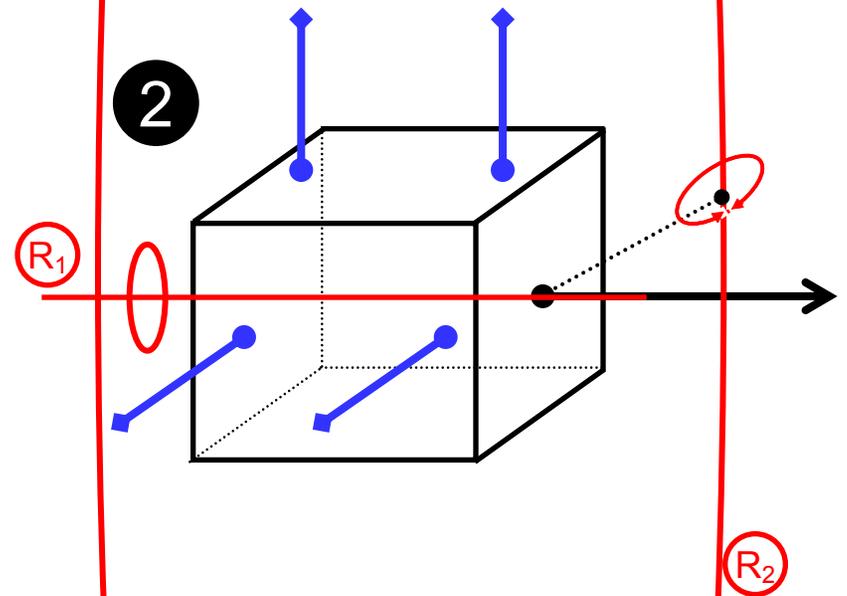
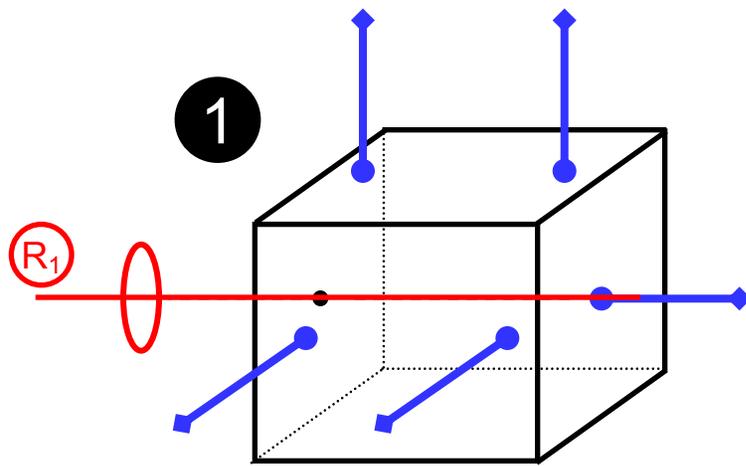
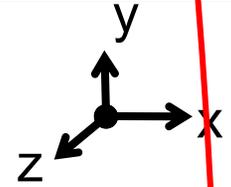
- Parallel lines intersect at infinity
- Translation represented by a rotation line at a hope of "infinite radius"

$$R = 6 - C = 6 - 5 = 1... \quad \text{so where is it?}$$



# Examples

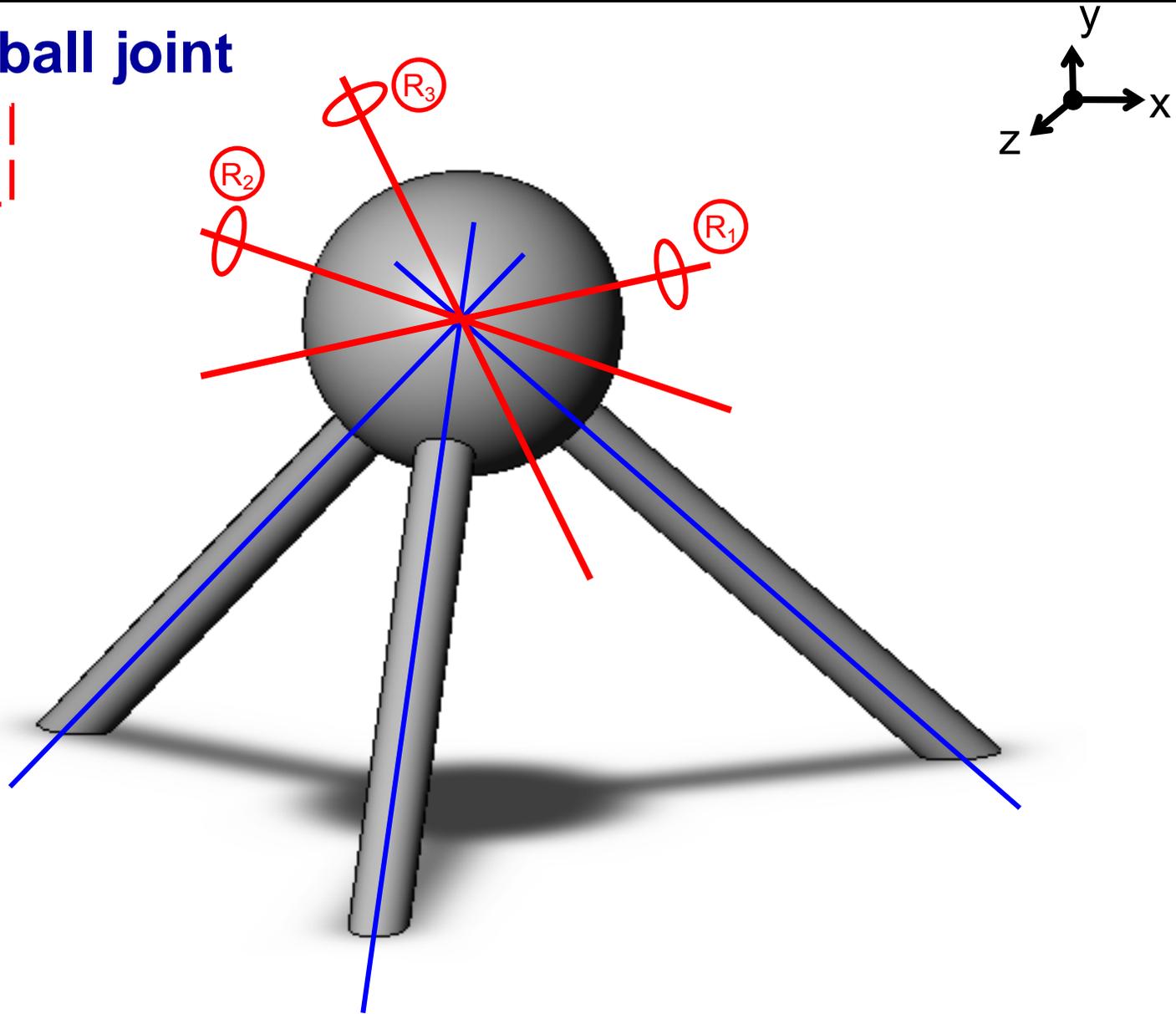
There will be a quiz on this NC



# Flexure bearing systems

## Spherical ball joint

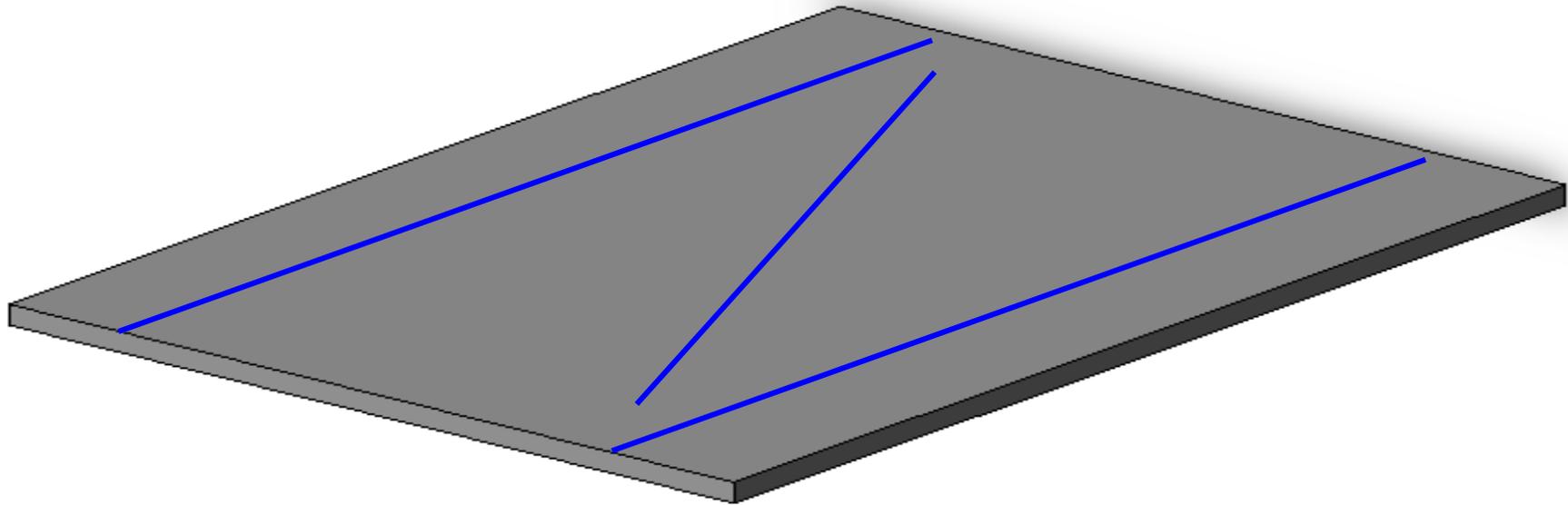
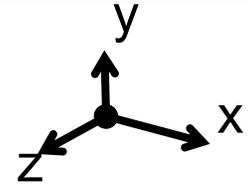
$$6 - C = F$$



# Flexure bearing systems

## Blade flexure

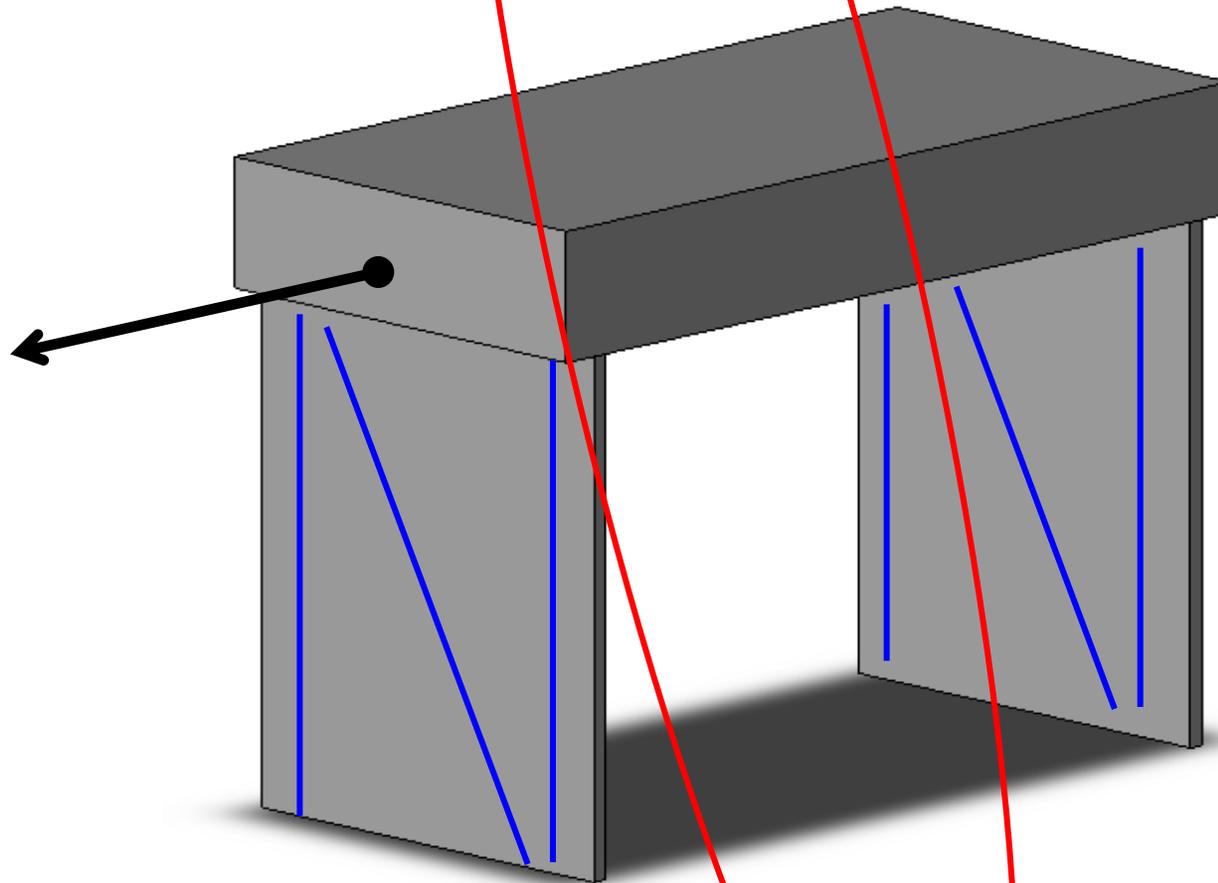
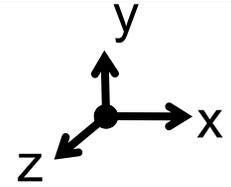
$$6 - C = F$$



# Flexure bearing systems

## Parallel guiding mechanism

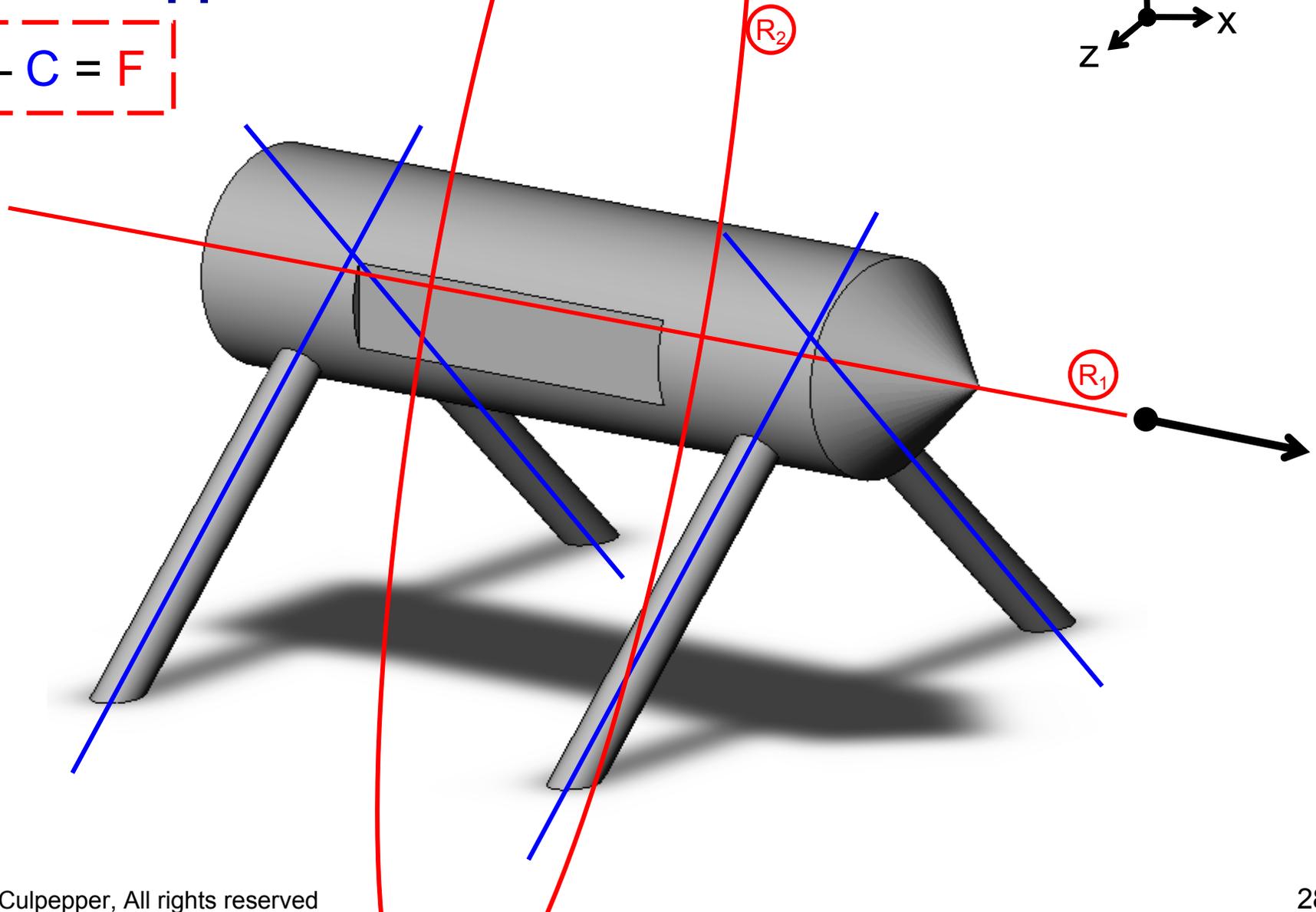
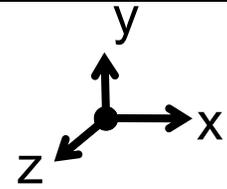
$$6 - C = F$$



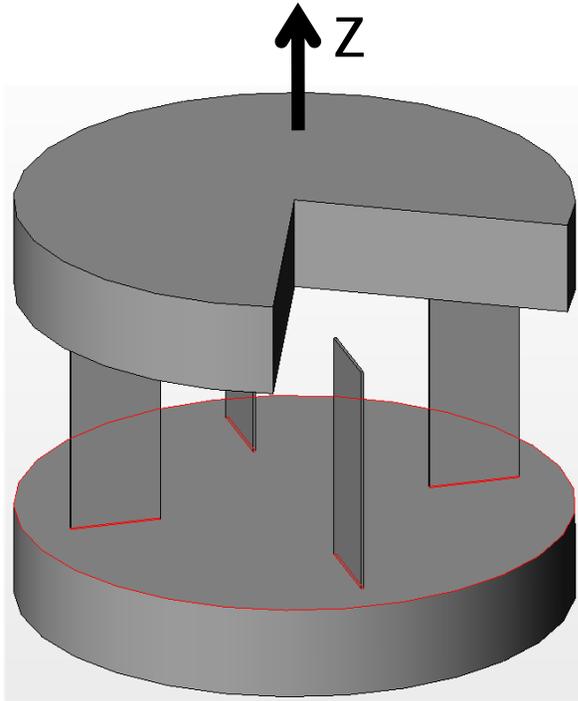
# Flexure bearing systems

Doodle hopper...

$$6 - C = F$$



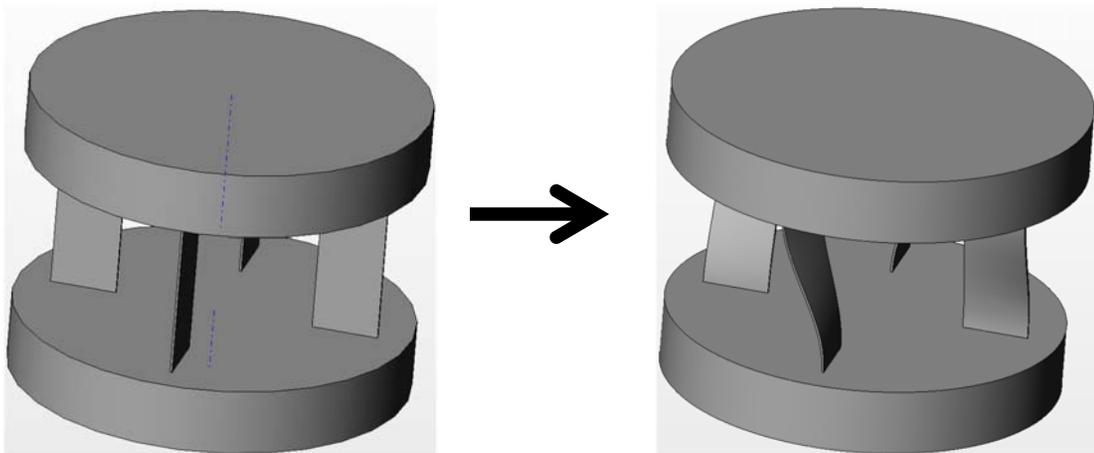
# Parallel addition rules



What is parallel ? Elements are not in the same load path. Loads are split between the elements

Add constraints so where there is a **common DOF**, then have **mechanism DOF**

Example: For instance, there are no conflicts in displacement to  $\theta_z$



Adapted from Layton Hale's Ph.D. Thesis (MIT)

# Series addition rules

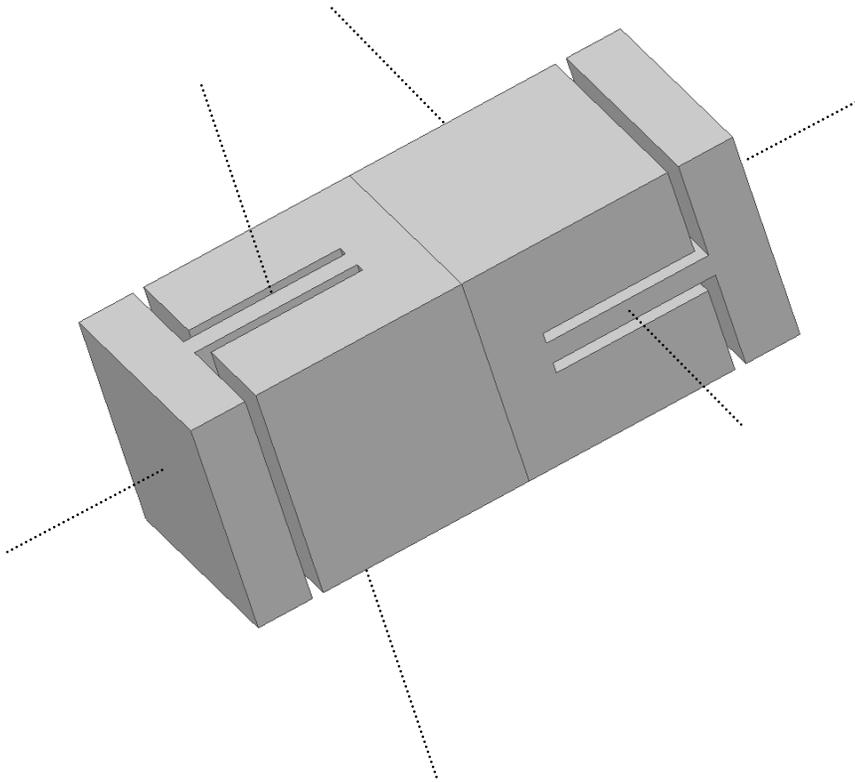
What is series?

- Differentiate series by load path
- Shared load path = series

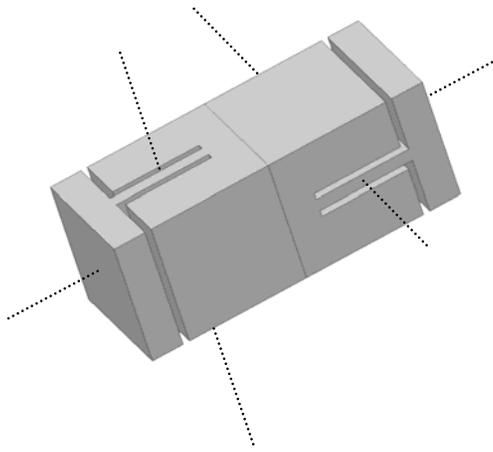
Series: Add DOF

Find common constraints

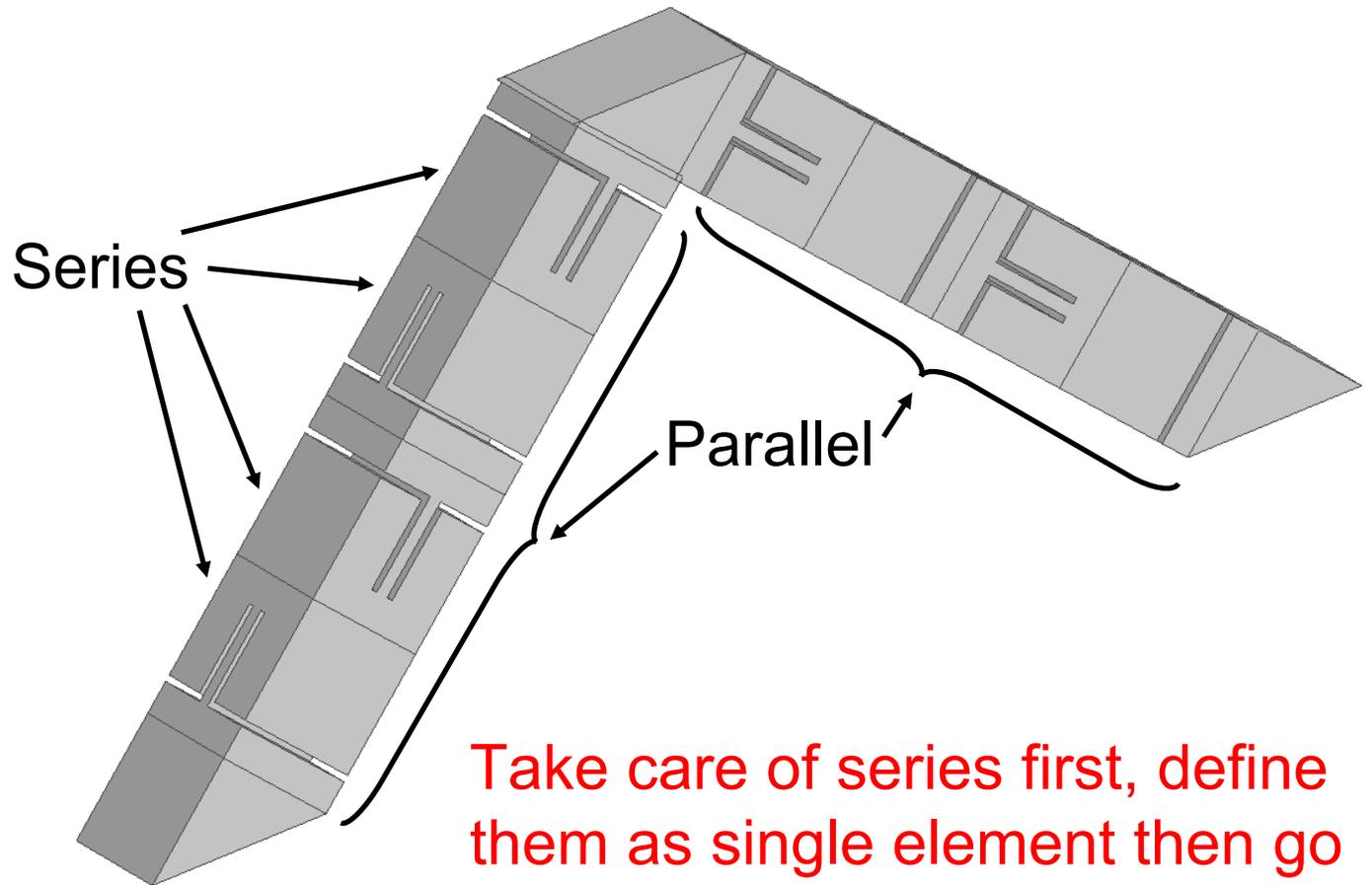
Follow the serial chain



# Parallel and series systems



Redundancy does not add  
Degrees of freedom



Take care of series first, define  
them as single element then go  
through parallel

# Accuracy

## The accuracy of most flexures is sensitive to:

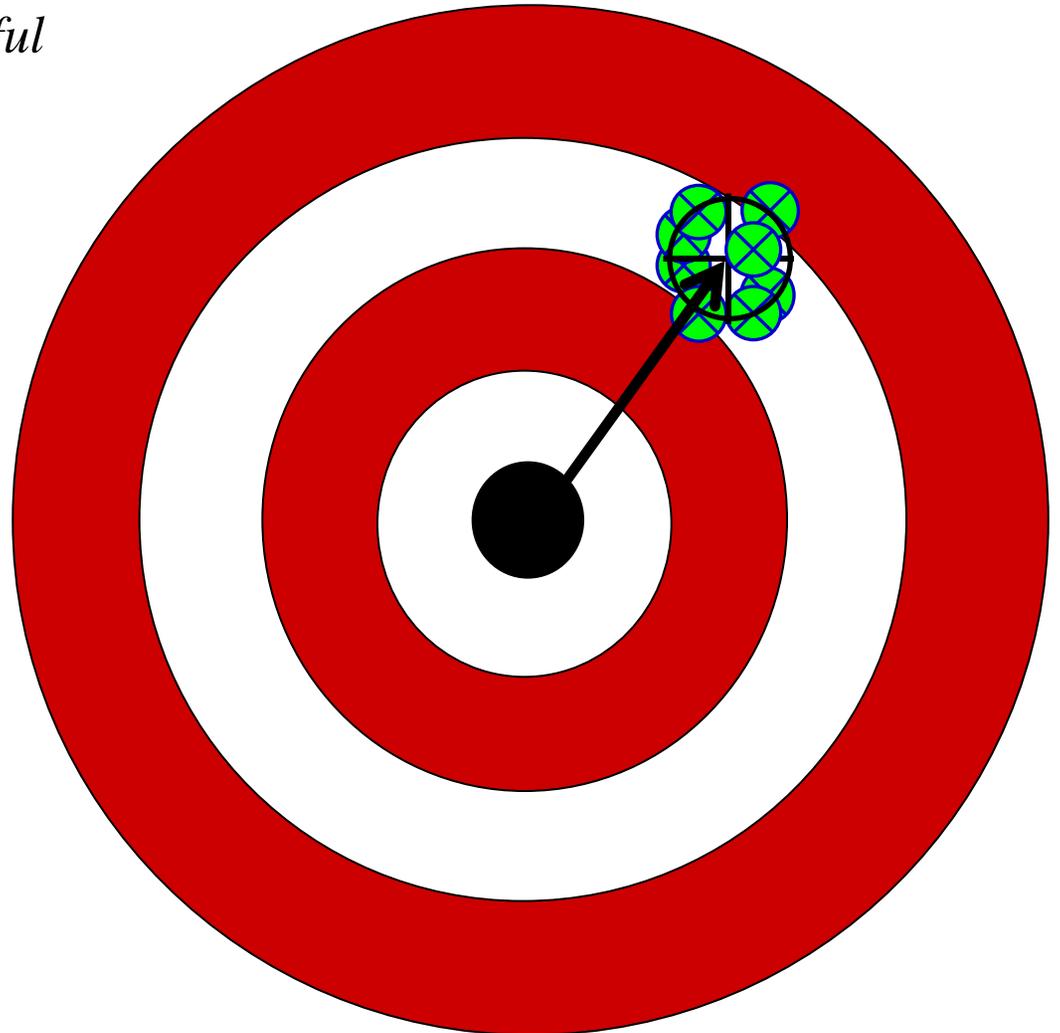
- ❑ 1. Small variations in dimensions, e.g.  $\delta_{\text{thickness}}$
  
- ❑ 2. Young's Modulus (E)
  
- ❑ 3. Time variable errors
  - *Creep*
  - *Stress relaxation*
  - *Thermal*
  - *Dynamic/vibration*



# Repeatability

## Flexures can exhibit Angstrom-level repeatability if:

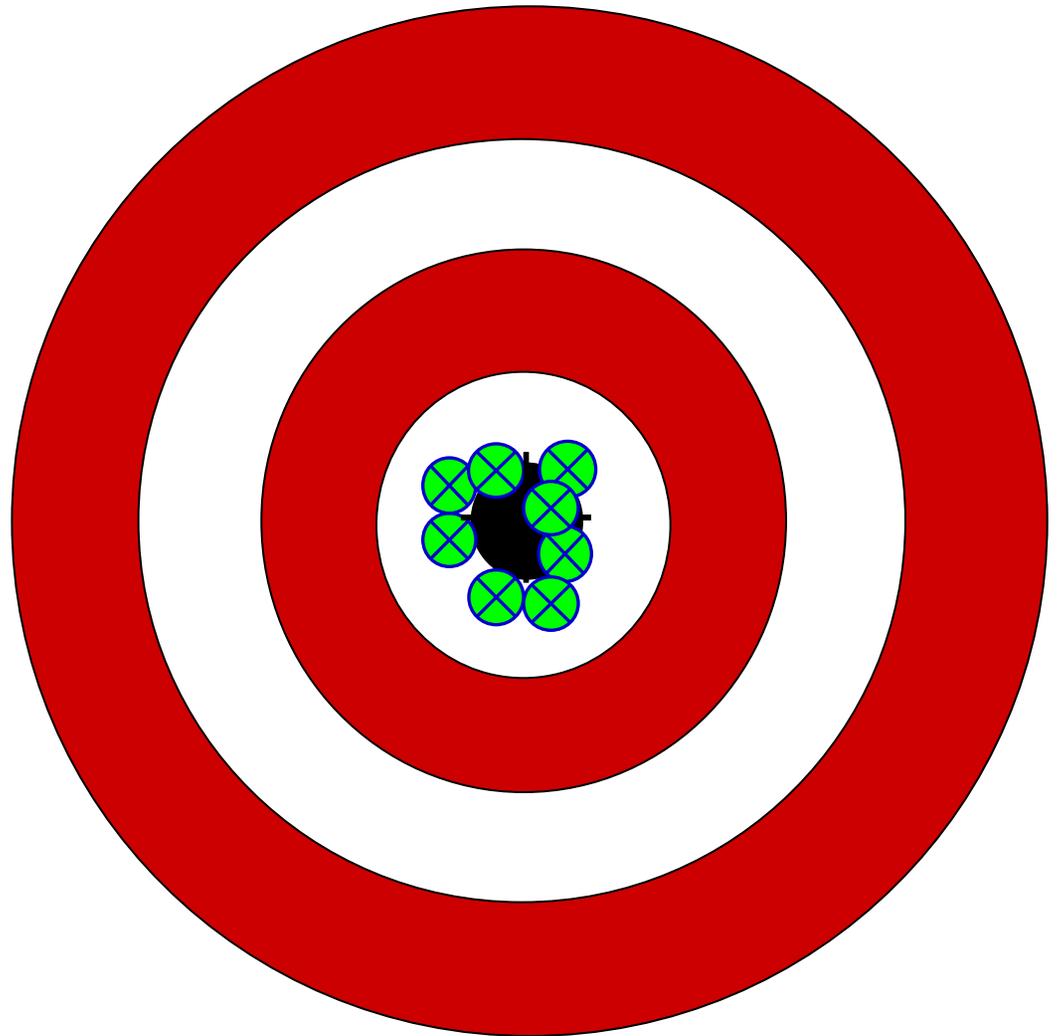
- ❑ Low material hysteresis
  - *Single crystal materials useful*
  
- ❑ No dislocation motion
  - $\sigma \ll \sigma_{yield}$
  
- ❑ Load is repeatable
  - *Magnitude*
  - *Direction*
  
- ❑ Assembly is correct
  - *No micro-slip*
  - *No friction in assembly*
  - *No yield during assembly*



# Accuracy and repeatability

**Difficult to obtain without calibration or adjustment**

- ❑ Geometry
- ❑ Materials
- ❑ Loading
- ❑ Assembly/integration
- ❑ Environmental

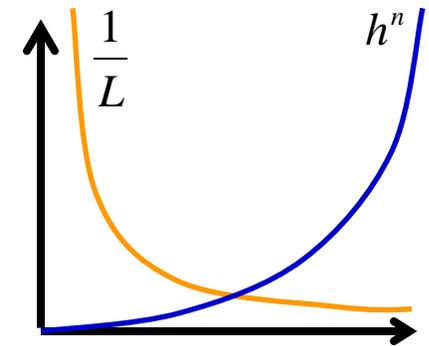
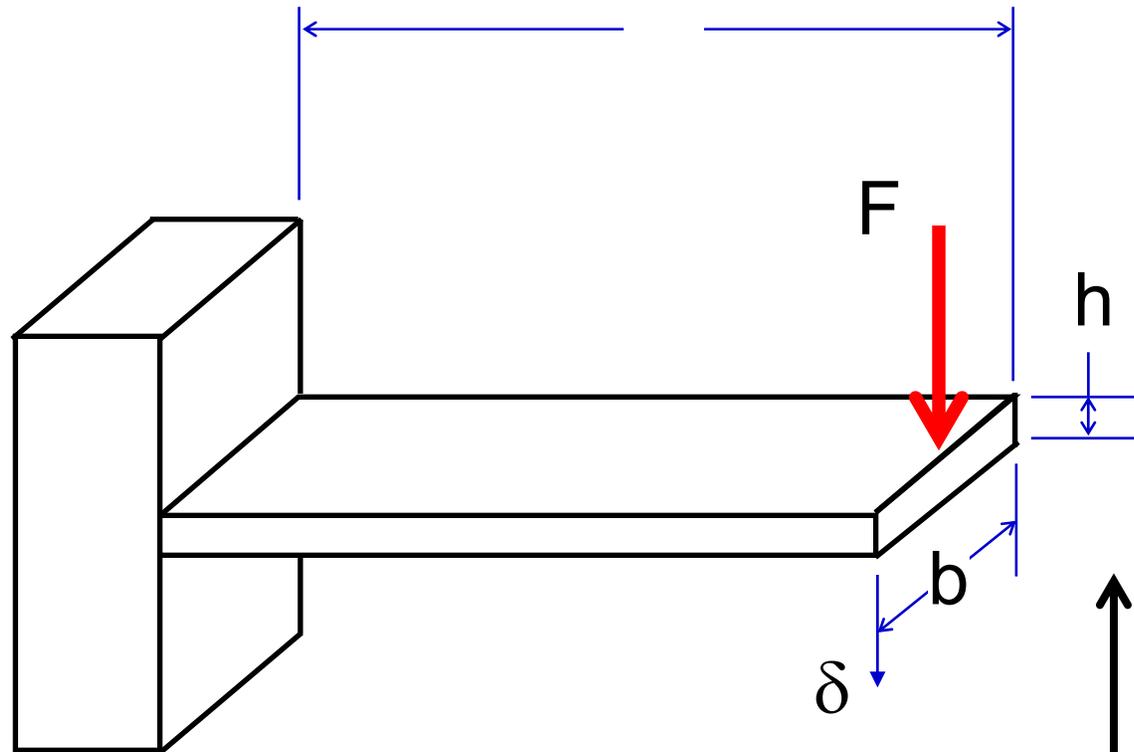


# Links between kinematics and elasticity

## Cantilever

$$\delta = \frac{F \cdot L^3}{3 \cdot E \cdot I}$$

$$I = \frac{1}{12} \cdot b \cdot h^3$$

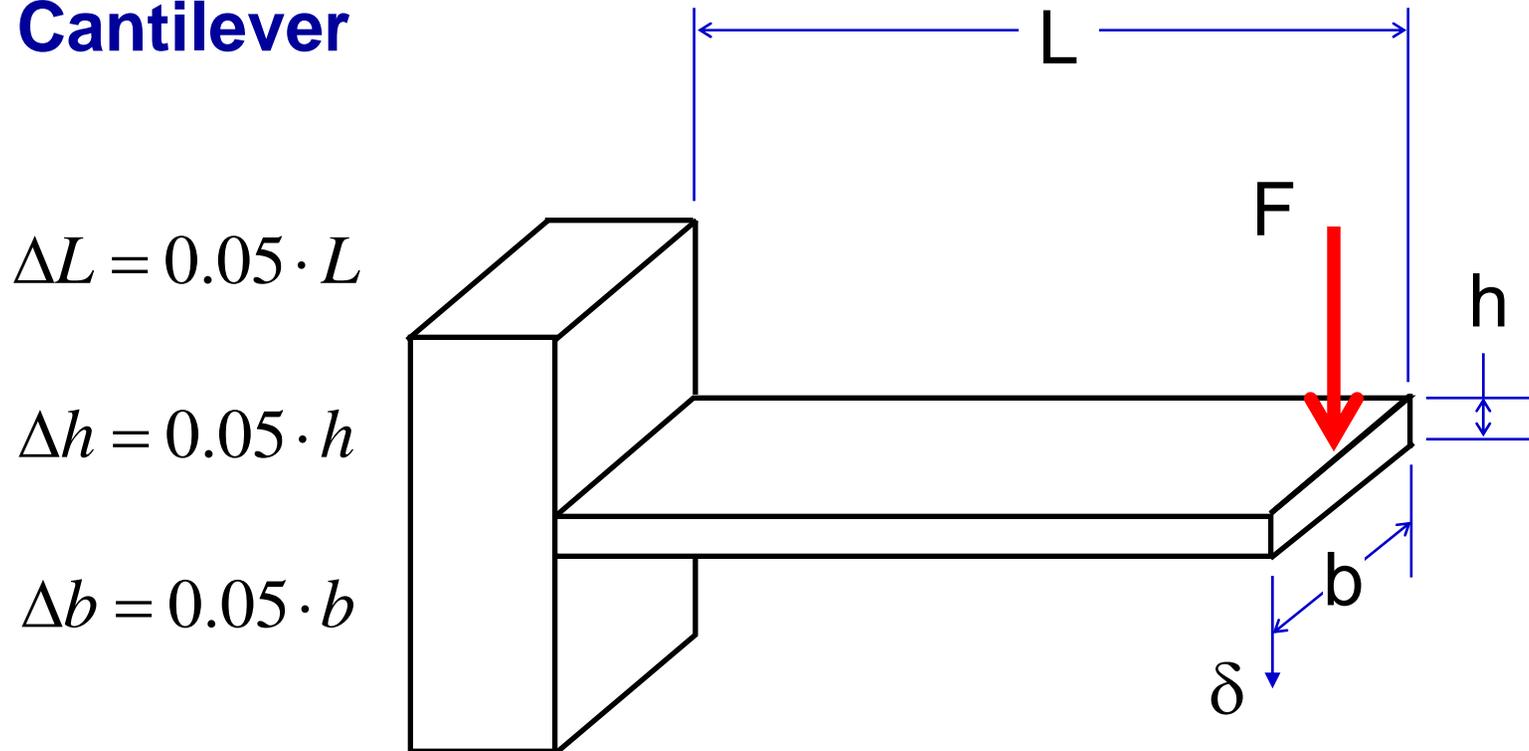


$$F = \left( \frac{E \cdot b}{4} \cdot \left[ \frac{h}{L} \right]^3 \right) \cdot \delta$$

$$k = \frac{dF}{d\delta} = \frac{d}{d\delta} \left\{ \frac{E \cdot b}{4} \cdot \left[ \frac{h}{L} \right]^3 \cdot \delta \right\} \rightarrow \frac{E \cdot b}{4} \cdot \left[ \frac{h}{L} \right]^3$$

# Links between kinematics and elasticity

## Cantilever



$$k + \Delta k = \frac{E \cdot (b + \Delta b)}{4} \cdot \left[ \frac{h + \Delta h}{L - \Delta L} \right]^3 \rightarrow \frac{E \cdot b}{4} \cdot \left[ \frac{h}{L} \right]^3 \cdot \left( 1.05 \cdot \left[ \frac{1.05}{0.95} \right]^3 - 1 \right) = k \cdot (1 + 0.42)$$

$$\Delta k = 0.42 \cdot k$$

# Fabrication processes: EDM

## EDM positives

- ❑ Accuracy (micrometers)
- ❑ 3D
- ❑ Surface finish (sub-micrometers)

Image removed due to copyright restrictions. Please see

[http://www.physikinstrumente.com/en/about/images/pi\\_WIREEDMC\\_i4c\\_K50\\_eps.jpg](http://www.physikinstrumente.com/en/about/images/pi_WIREEDMC_i4c_K50_eps.jpg)

## EDM drawbacks

- ❑ Time (mm/minute)
- ❑ Cost

# Fabrication processes: Waterjet

## Waterjet positives

- ❑ Low force
- ❑ Many materials including brittle materials and heat sensitive materials
- ❑ Rapid (inches/min)



Images courtesy of [xiaming](#) on Flickr.

## Waterjet drawbacks

- ❑ Thickness limitations
- ❑ Kerf limitations
- ❑ Draft limitations
- ❑ Accuracy ~ 125 micrometers

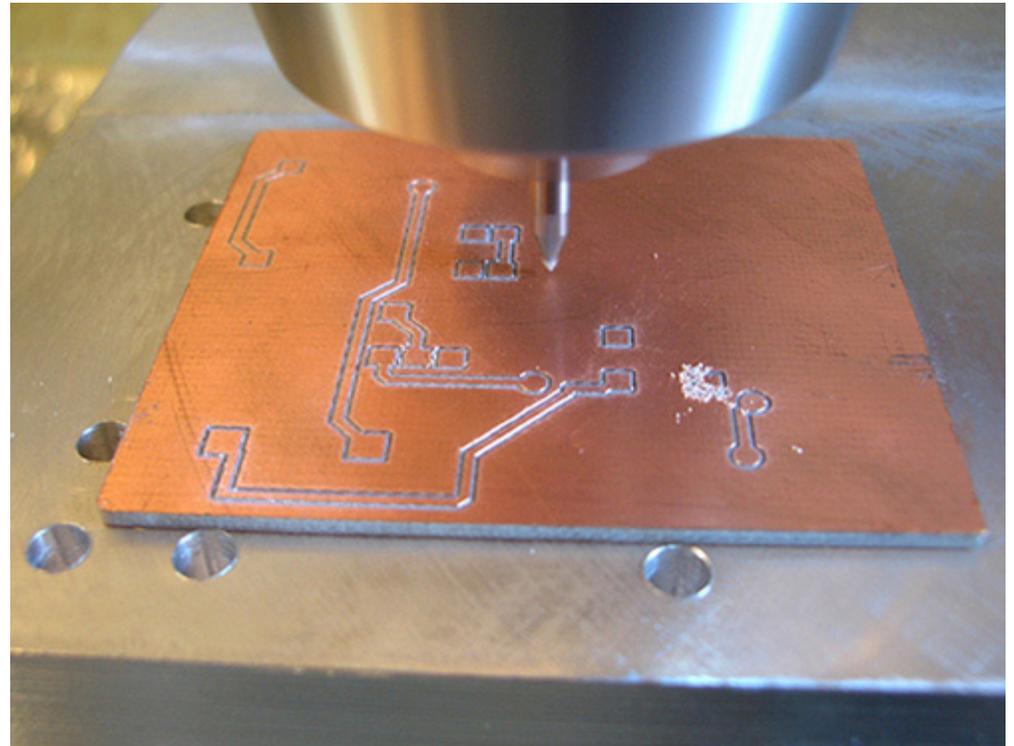


Images courtesy of [lansoper](#) on Flickr.

# Fabrication processes: Milling/cutting

## Milling/cutting positives

- ❑ Flexibility
- ❑ Any material
- ❑ Nearly any shape



## Milling/cutting drawbacks

- ❑ Fixturing
- ❑ Compliance of parts
- ❑ Work hardening
- ❑ Surface damage

Image courtesy of [jjskar](#) on Flickr.

Please see any other image of milling, such as

[http://students.washington.edu/denny/fsaecnc/wc\\_fixtplate.jpg](http://students.washington.edu/denny/fsaecnc/wc_fixtplate.jpg)

# Fabrication processes: Etching

## Etching positives

- ❑ 2½ D topologies/shapes
- ❑ Monolithic
- ❑ Micron-level features

## Etching drawbacks

- ❑ Dimensional control
- ❑ Scallops

Images removed due to copyright restrictions. Please see:

[http://www.ee.ucla.edu/~dejan/ee115c/ucla-graphics/IBM\\_metal\\_stack.jpg](http://www.ee.ucla.edu/~dejan/ee115c/ucla-graphics/IBM_metal_stack.jpg)

[http://www.stsystems.com/uploaded\\_files/1101/images/scallops.jpg](http://www.stsystems.com/uploaded_files/1101/images/scallops.jpg)

Milanovic, Veljko, et al. "Deep Reactive Ion Etching for Lateral Field Emission Devices." IEEE Electronic Device Letters 21 (June 2000): 271-273.

Milanovic, Veljko, et al. "Micromachining Technology for Lateral Field Emission Devices." IEEE Transactions on Electron Devices 48 (January 2001): 166-173.

Please see 371762. "How Microprocessor Work." February 14, 2009. YouTube. Accessed October 28, 2009.

[http://www.youtube.com/watch?v=loMz\\_I\\_Fpx4](http://www.youtube.com/watch?v=loMz_I_Fpx4)

# Assembly

## Stress and energy

- ❑ Proper thickness of clamps and clamping load distribution
- ❑ Spring washer provide force source

## Fusing

- ❑ Clamps members should “yield” before flexure
- ❑ Spring washer provide force source

## Surface conformity

- ❑ Micro-slip is a major cause of hysteresis
- ❑ Deburring and potting/bonding

## Misalignment = systematic errors

Images removed due to copyright restrictions.  
Please see Fig. 8.5 and 8.6 in Smith, Stuart.  
*Flexures: Elements of Elastic Mechanisms.*  
Amsterdam, Holland: Gordon & Breach, 2000.