2.72 Elements of Mechanical Design
Spring 2009

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Elements of Mechanical Design

Lecture 09: Alignment

Schedule and reading assignment

Quiz

- Thursday: Hale 6.1
- Soon: Bolted joint qualifying quiz

Topics

- Lab notebooks
- Alignment methods
- Kinematic coupling grade bump = ½ grade for use/design

Reading assignment

- *Read: 8.2*
- *Examples: All in 8.2*

Lab notebooks

Technical quality/quantity

- Appropriate equations, codes
- Units
- Important results highlighted/boxed/noted/explained

Graphical quality/quantity

- Appropriate sketches/pictures
- Pasted CAD/etc…

Archival quality

- Can this be copied?
- Understood by others?

Best practices

- Dating and number of pages
- Permanent pen
- No blank spaces (X out)

Ideal alignment interface

Repeatable

Accuracy

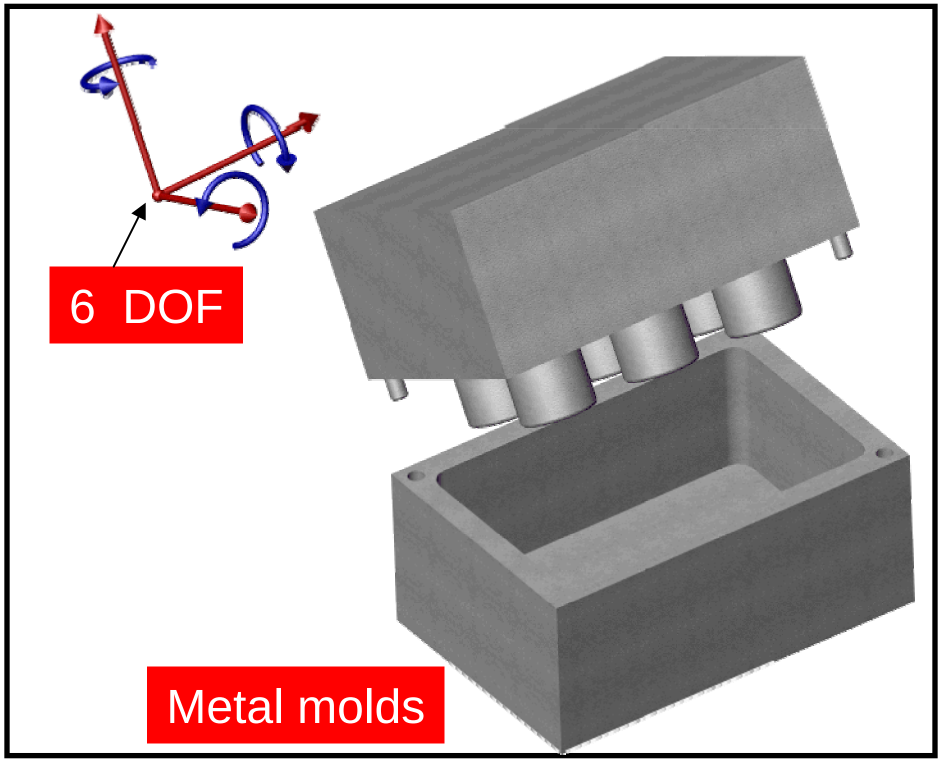
Stiffness (sensitive?)

Load capacity

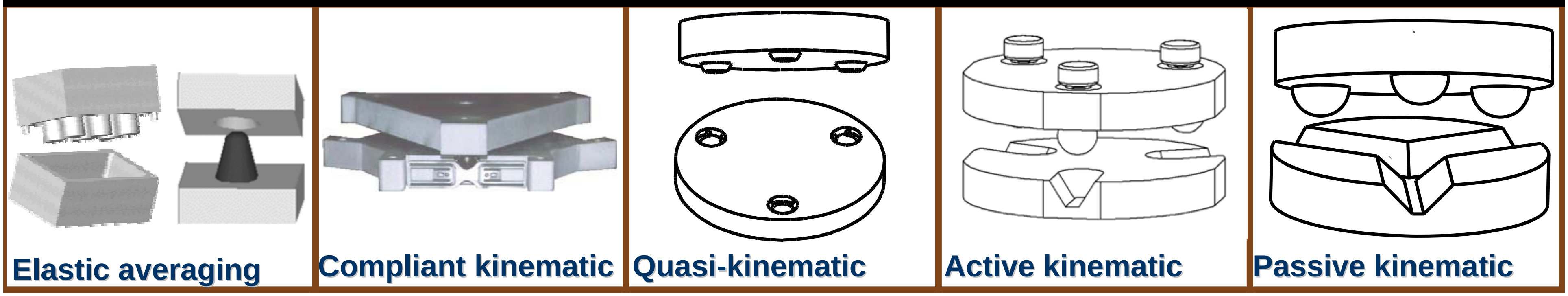
“Perfect” constraint

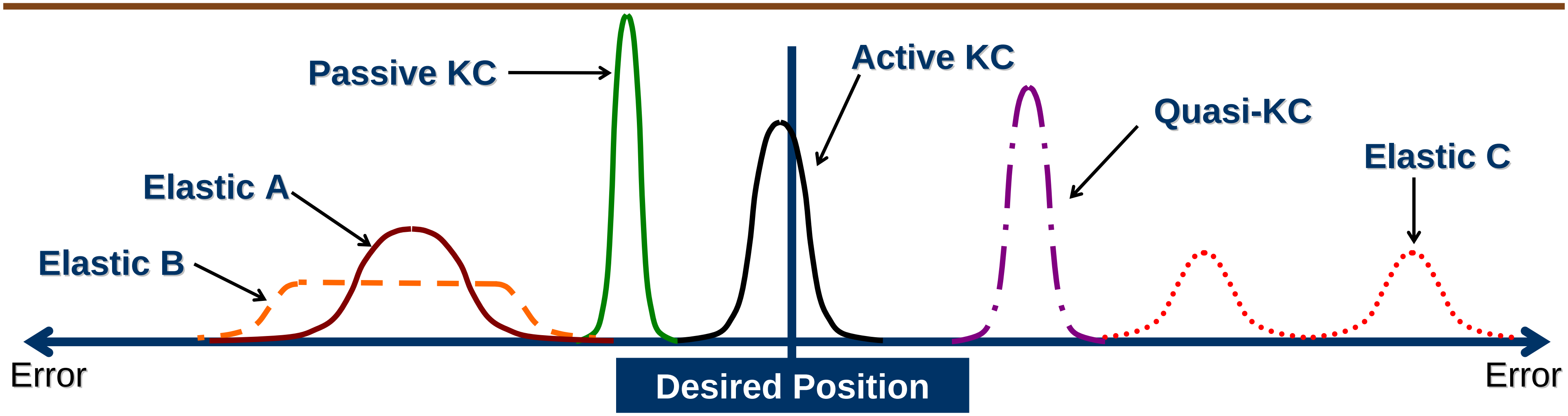
Lowest energy state

High natural frequency

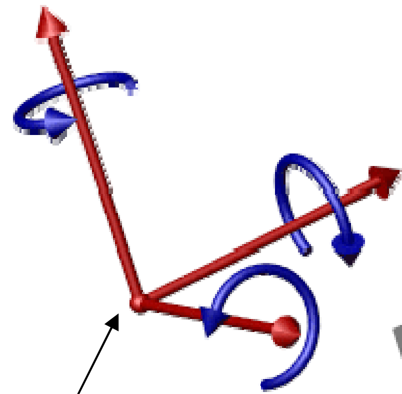


Common alignment methods

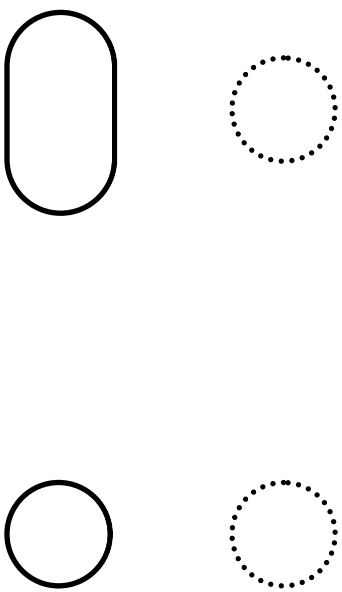


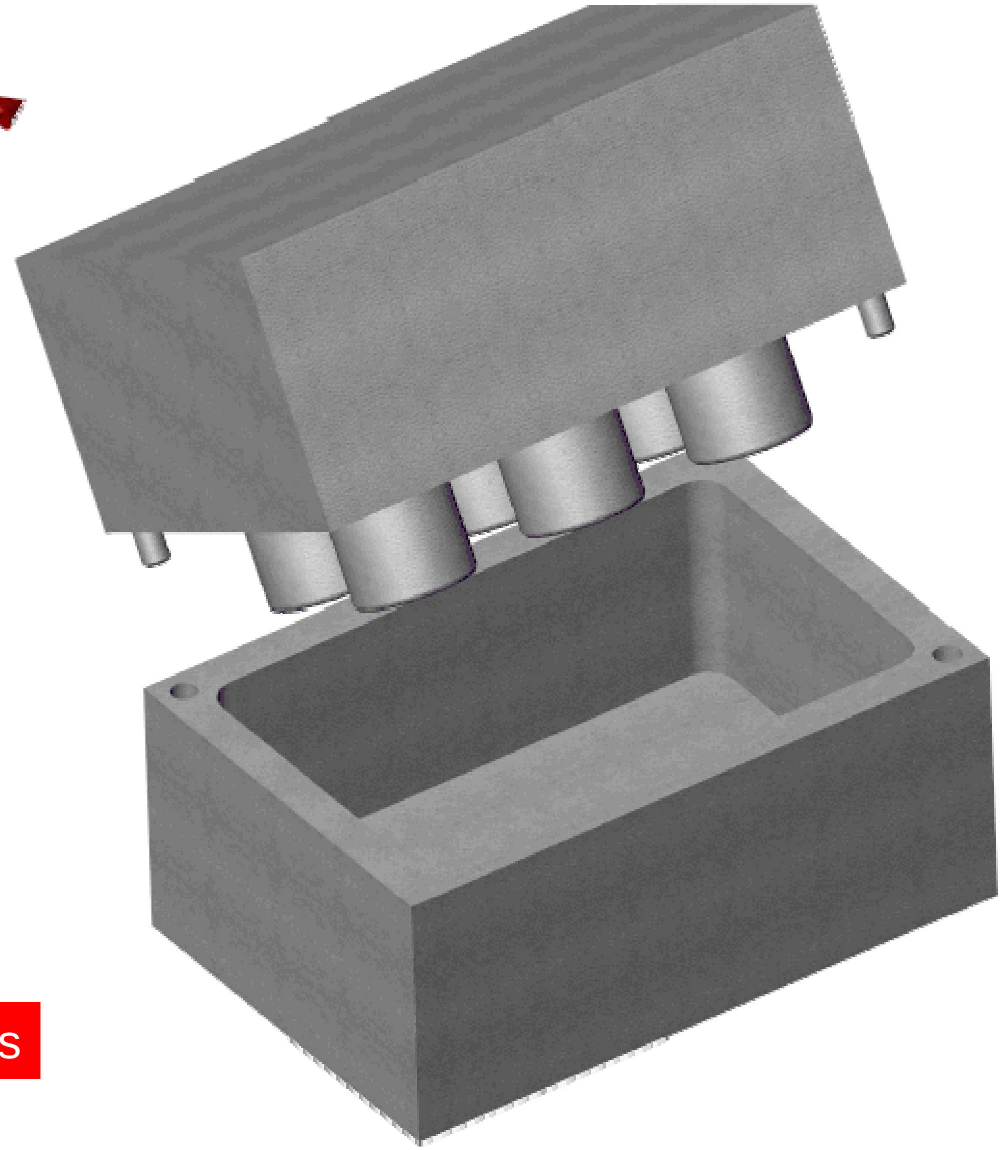


Pin-hole



6 DOF





Metal molds

3 – 2 – 1 Alignment schemes

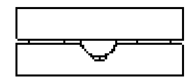
Exact constraint couplings

Exact constraint (EC):

- Constraints = DOF to be constrained
- Deterministic saves $
- Balls (inexpensive) & grooves (more difficult to make)

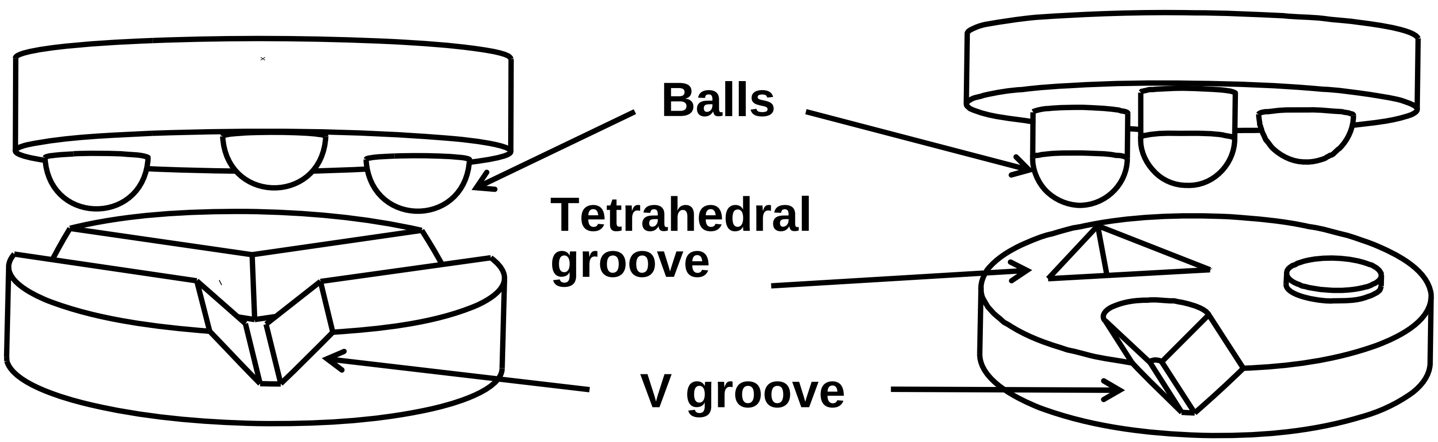
There are many types of EC couplings, our time limits us to a semi-focused study on kinematic couplings

In KC design the issues are:



- KNOW what is happening in the system (coupling)
- MANAGE forces, deflections, stresses and friction

Maxwell



Kelvin

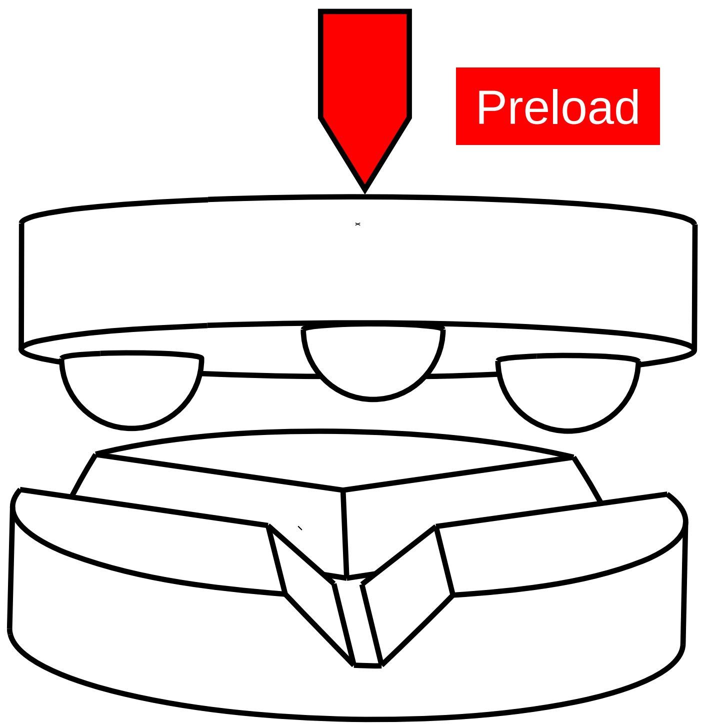
Passive kinematic couplings

Fabricate and forget

¼ micrometer with best practices, 10s of nm recently

What is important?

- Contact forces
- Contact stress
- Stiffness vs. geometry
- Stiffness vs. preload
- Friction & settling
- Thermal loading
- Preload repeatability



Preload (nesting load) is the force applied to keep the coupling components engaged and prevent tipping

Ball motions: Displacements

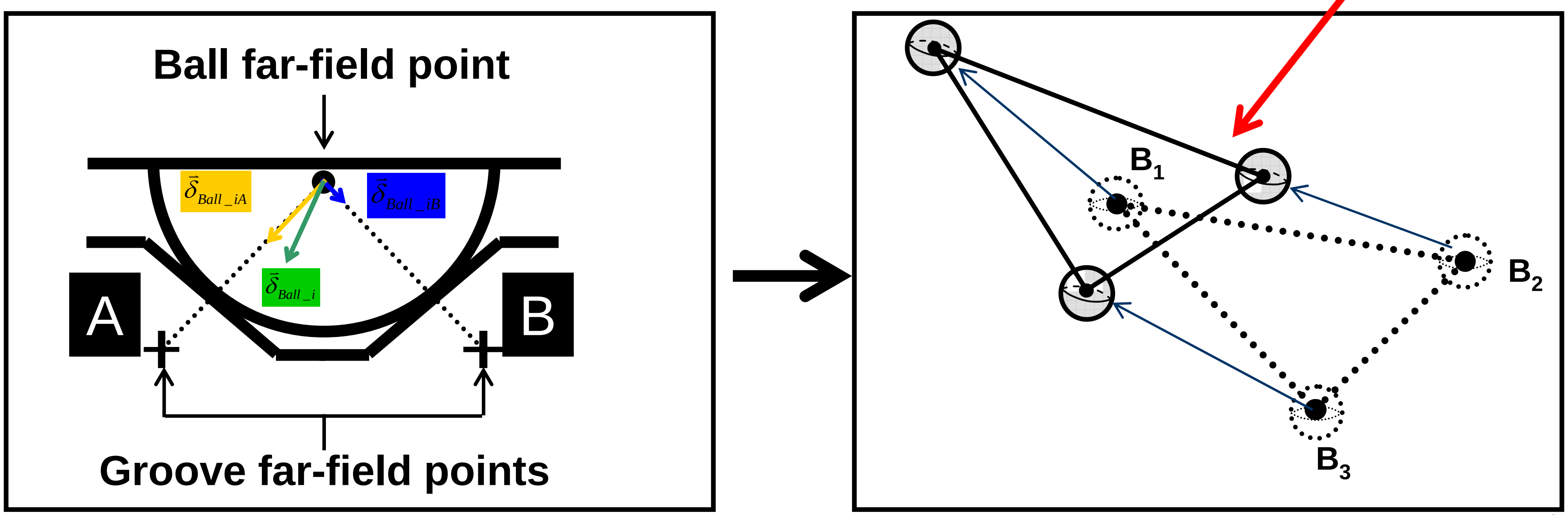
$$\Sigma\vec{\delta}_{n\_Ball\_iA} = -\left.\left(\frac{9}{16}\cdot\frac{\left|F_n{}^2\right|}{R_e\cdot E_e{}^2}\right)^{1/3}\right|_{Ball\_iA}\cdot\hat{n}_{Ball\_iA}$$

$$\Sigma\vec{\delta}_{n\_Ball\_iB} = -\left.\left(\frac{9}{16}\cdot\frac{\left|F_n{}^2\right|}{R_e\cdot E_e{}^2}\right)^{1/3}\right|_{Ball\_iB}\cdot\hat{n}_{Ball\_iB}$$

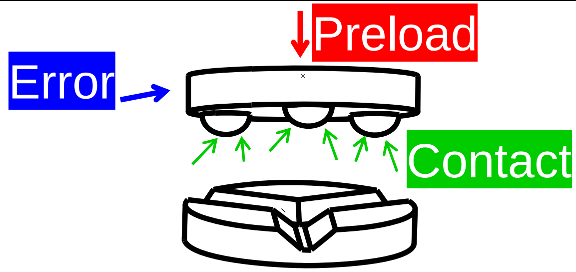
Hertz
1857-1894

This assumes that the ball-ball stiffness is > ~10x ball-groove stiffness

$$\Sigma\vec{\delta}_{Ball\_i} = \vec{\delta}_{Ball\_iA} + \vec{\delta}_{Ball\_iB}$$



Load balance: Force and moment



Force balance (3 equations)

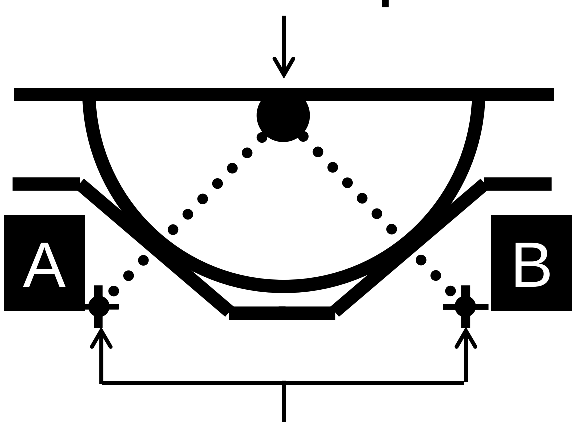
$$\Sigma\vec{F}_{relative} = 0 = \left(\vec{F}_{preload} + \vec{F}_{Error}\right) + \left(\vec{F}_{Ball\_1} + \vec{F}_{Ball\_2} + \vec{F}_{Ball\_3} + \vec{F}_{Ball\_4} + \vec{F}_{Ball\_5} + \vec{F}_{Ball\_6}\right)$$

Moment balance (3 equations)

$$\vec{M}_{relative} = \Sigma^{6}_{i=1}\left(\vec{M}_{Ball\_i}\right) + \left(\vec{M}_{preload}\right) + \left(\vec{M}_{error}\right) = \left(\vec{r}_{preload} \times \vec{F}_{preload} + \vec{r}_{error} \times \vec{F}_{Error}\right) + \Sigma\vec{r}_{Ball\_i} \times \vec{F}_{Ball\_i}$$

Goal:
1. Solve 6 equations for contact forces
2. Solve normal displacements
3. Solve relative displacements/rotations

Given geometry, materials, preload force, error force, solve for local distance of approach



Modeling round interfaces

Equivalent radius

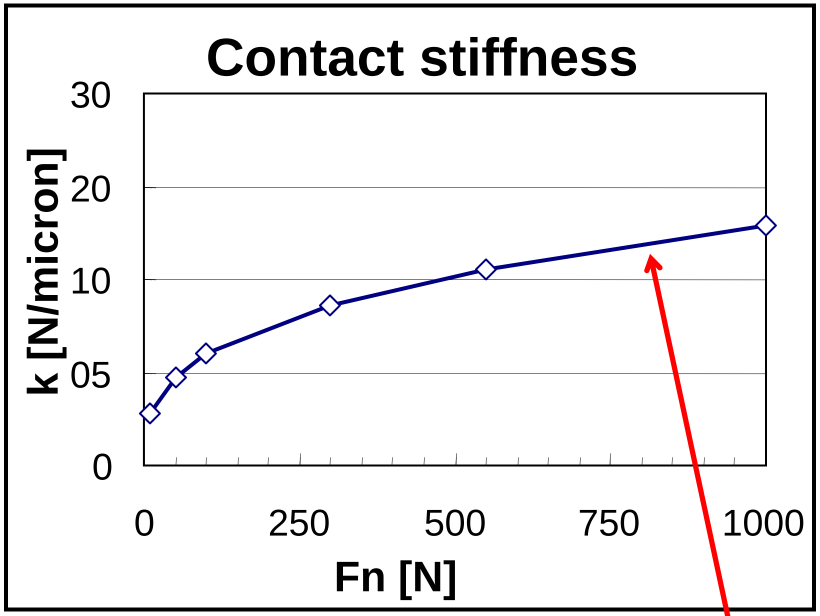
$$R_e = \frac{1}{\frac{1}{R_{1major}} + \frac{1}{R_{1minor}} + \frac{1}{R_{2major}} + \frac{1}{R_{2minor}}}$$

Equivalent modulus

$$E_e = \frac{1}{\frac{1-\eta_1^2}{E_1} + \frac{1-\eta_2^2}{E_2}}$$

Poisson's ratio

Young's modulus



Degree of nonlinearity is reduced as preload is increased

$$\delta_n = \left(\frac{9}{16} \cdot \frac{F_n^{\ 2}}{R_e \cdot E_e^{\ 2}} \right)^{1/3}$$

Scaling with Mat'l properties and geometry

Preload should be repeatable in magnitude & direction

Important scaling law

$$k_n(\delta_n) = \left(2 \cdot R_e^{\ 0.5} \cdot E_e\right) \cdot \delta_n^{\ 0.5} \longrightarrow k_n(F_n) = \text{Constant} \cdot \left(R_e^{\ 1/3} \cdot E_e^{\ 2/3}\right) \cdot F_n^{\ 1/3}$$

Friction and lubrication

The trend of the data is important

Wear in vs. "snow balling"

Magnitude depends on coupling design and test conditions

Slocum, A. H., Precision Engineering, 1988: Kinematic couplings for precision fixturing– Experimental determination of repeatability and stiffness