2.72 Elements of Mechanical Design
Spring 2009

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Elements of Mechanical Design

Lecture 12: Belt, friction, gear drives

Schedule and reading assignment

Quiz

- Bolted joint qualifying Thursday March 19th

Topics

- Belts
- Friction drives
- Gear kinematics

Reading assignment

- *Read:*

  14.1 – 14.7

- *Skim:*

  Rest of Ch. 14

Topic 1:

Belt Drives

Belt Drives

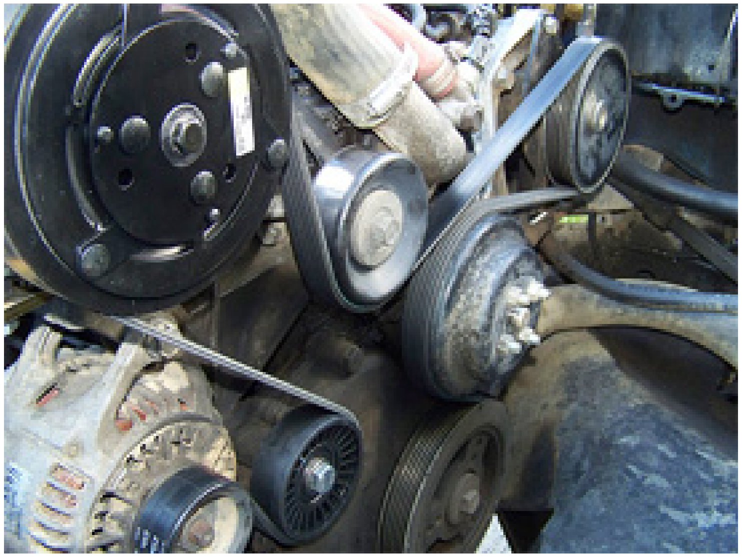
Why Belts?

- Torque/speed conversion
- Cheap, easy to design
- Easy maintenance
- Elasticity can provide damping, shock absorption

Image by dtwright on Flickr.

Keep in mind

- Speeds generally 2500-6500 ft/min
- Performance decreases with age



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Please see:

http://www.tejasthumpcycles.com/Parts/primaryclutch/3.35-inch-harley-Street-Belt-Drive.jpg

http://www.al-jazirah.com.sa/cars/topics/serpentine_belt.jpg

Belt Construction and Profiles

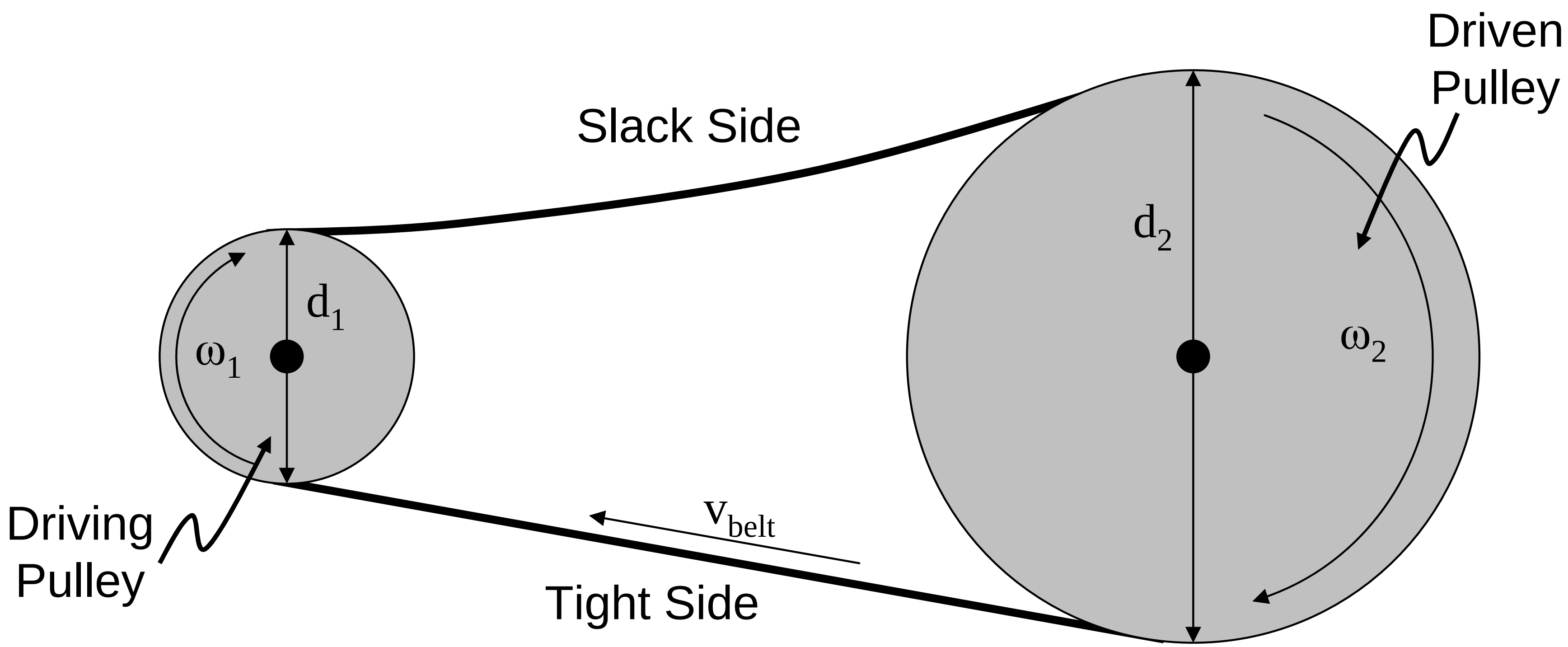
Many flavors

- Flat is cheapest, natural clutch
- Vee allows higher torques
- Synchronous for timing

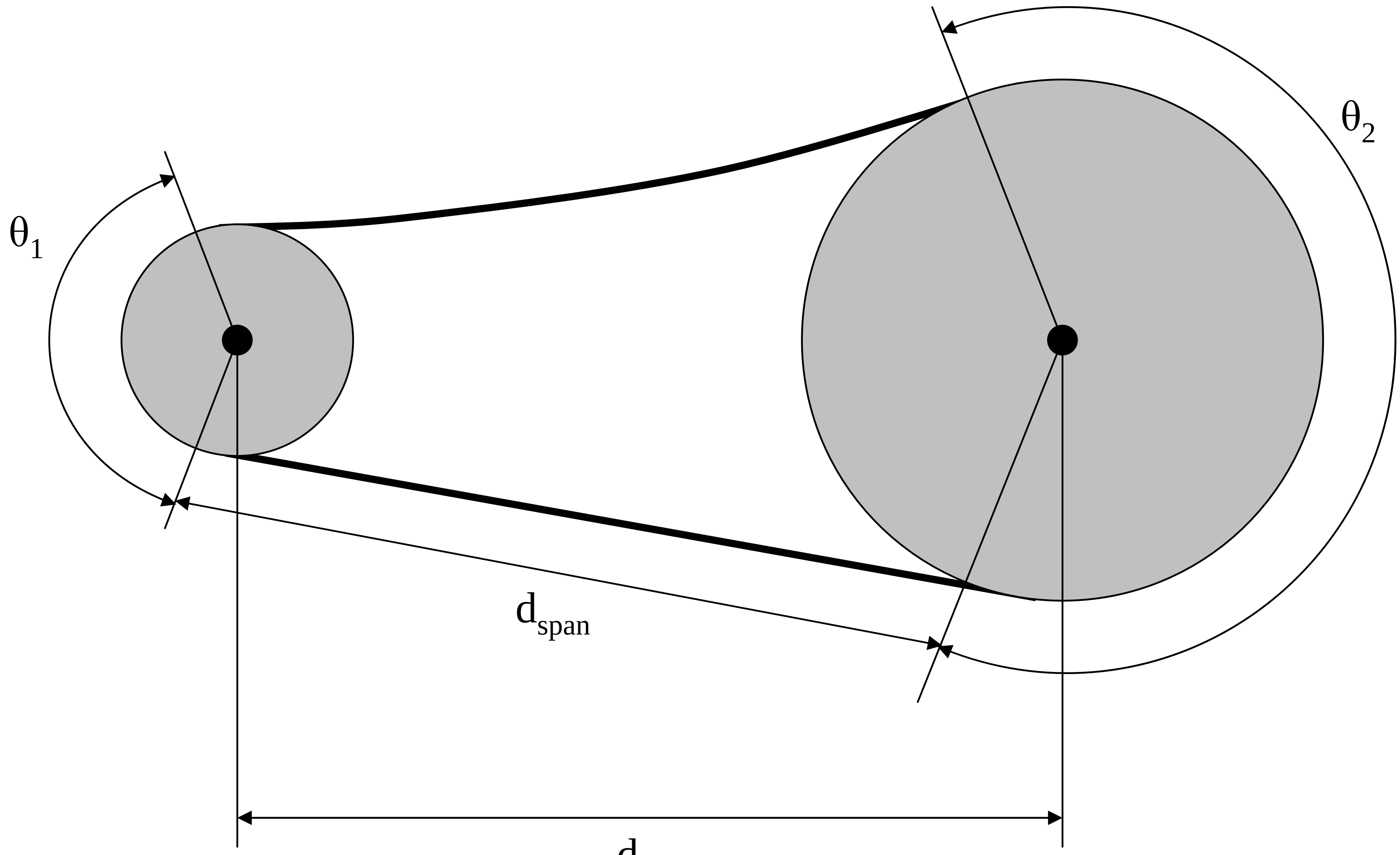
Usually composite structure

- Rubber/synthetic surface for friction
- Steel cords for tensile strength

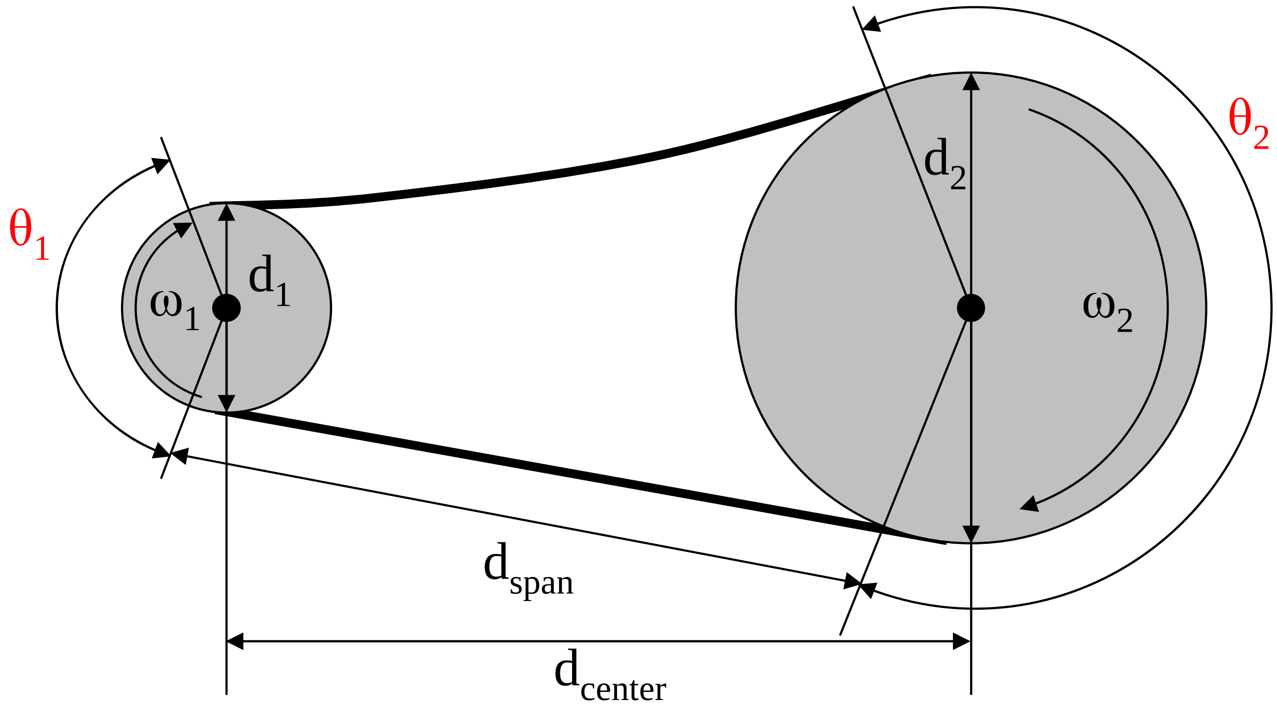
Belt Drive Geometry



Belt Drive Geometry

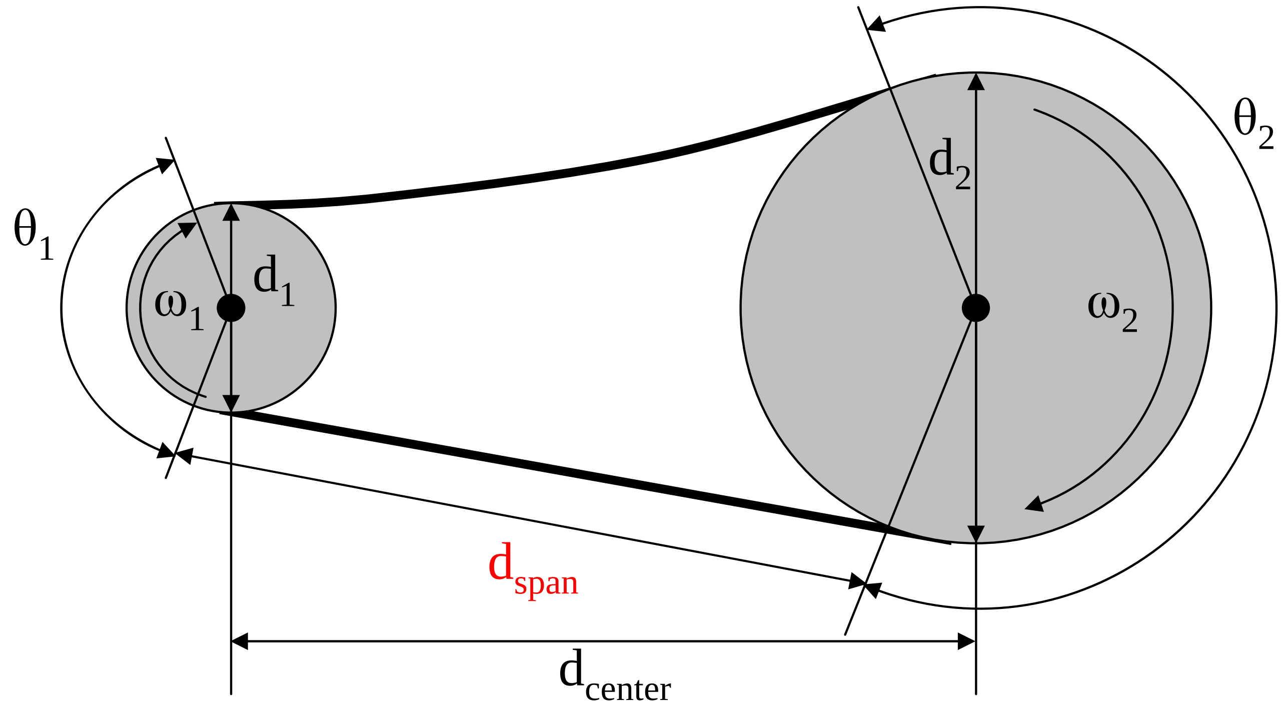


Contact Angle Geometry



$$\theta_1 = \pi - 2\sin^{-1}\left(\frac{d_2 - d_1}{2d_{center}}\right) \qquad \theta_2 = \pi + 2\sin^{-1}\left(\frac{d_2 - d_1}{2d_{center}}\right)$$

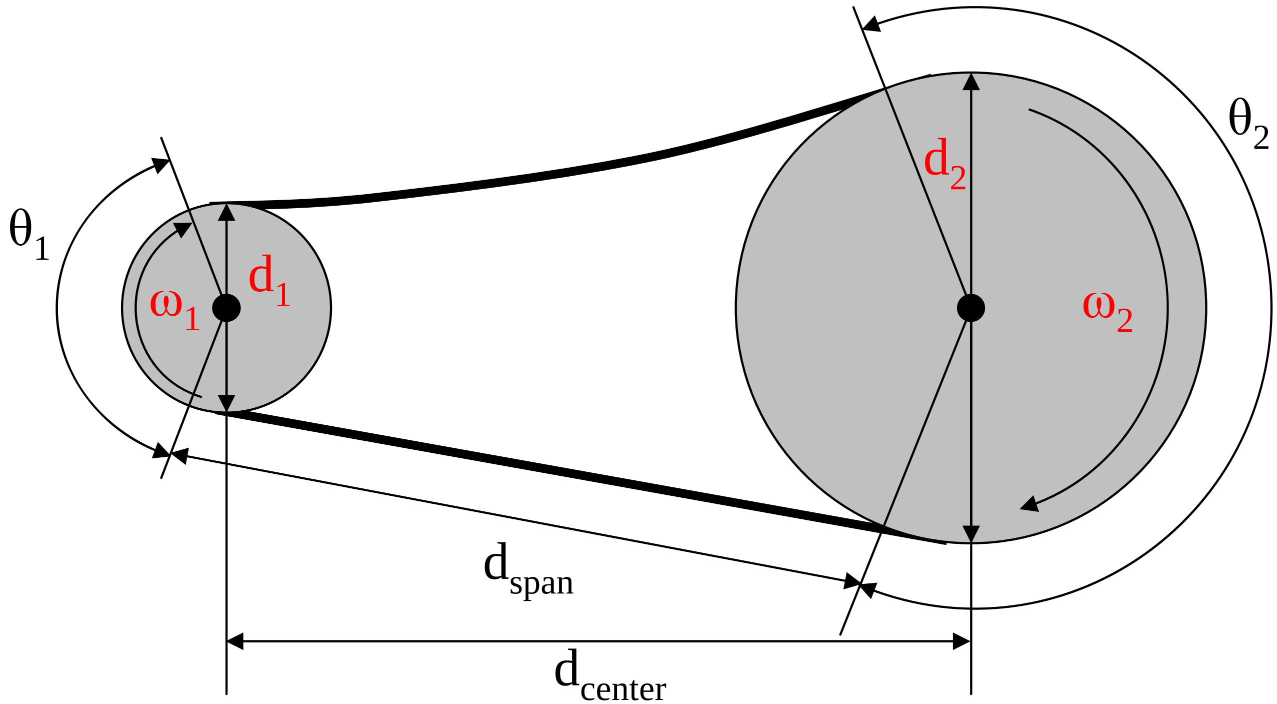
Belt Geometry



$$d_{span} = \sqrt{d_{center}^2 - \left(\frac{d_2 - d_1}{2}\right)^2}$$

$$L_{belt} = \sqrt{4d_{center}^2 - (d_2 - d_1)^2} + \frac{1}{2}(d_1\theta_1 + d_2\theta_2)$$

Drive Kinematics



$$v_b = \frac{d_1}{2}\omega_1 = \frac{d_2}{2}\omega_2$$

$$\frac{d_1}{d_2} = \frac{\omega_2}{\omega_1}$$

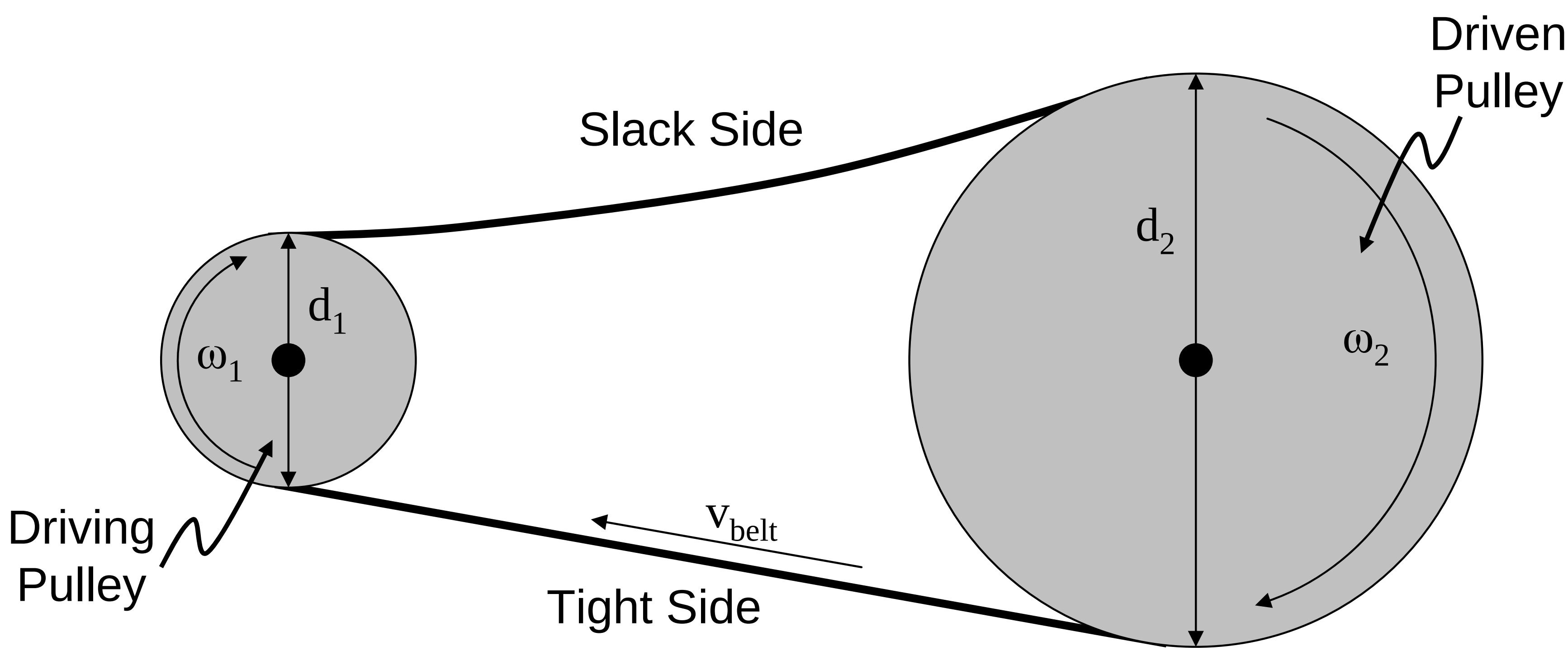
Elastomechanics

Elastomechanics → torque transmission

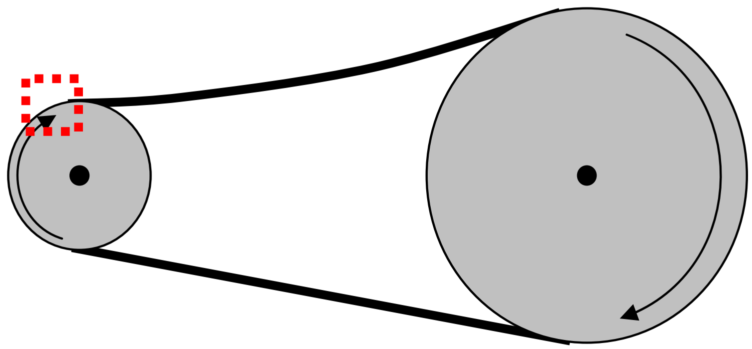
- Kinematics → speed transmission

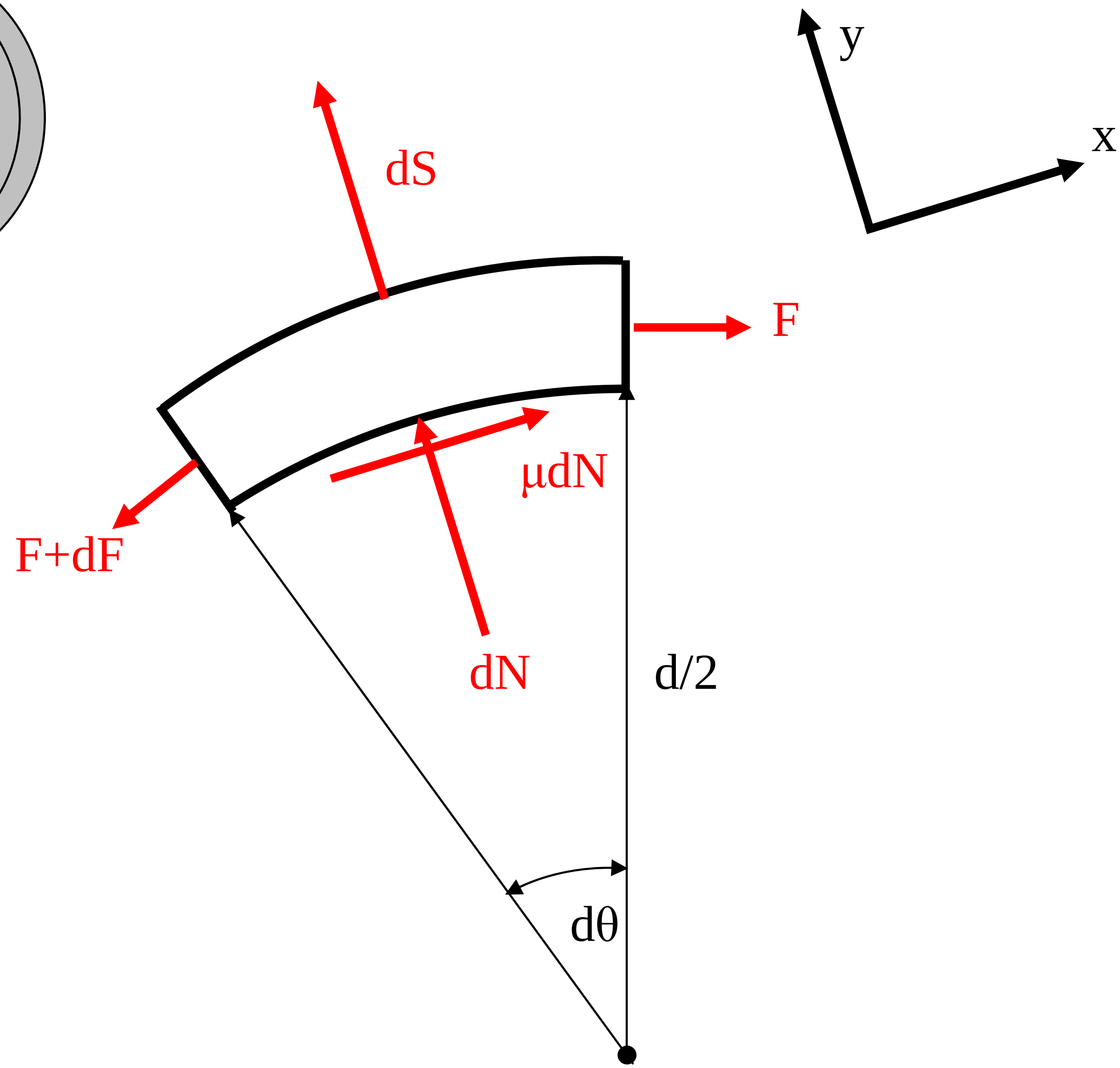
Link belt preload to torque transmission

- Proceeding analysis is for flat/round belt



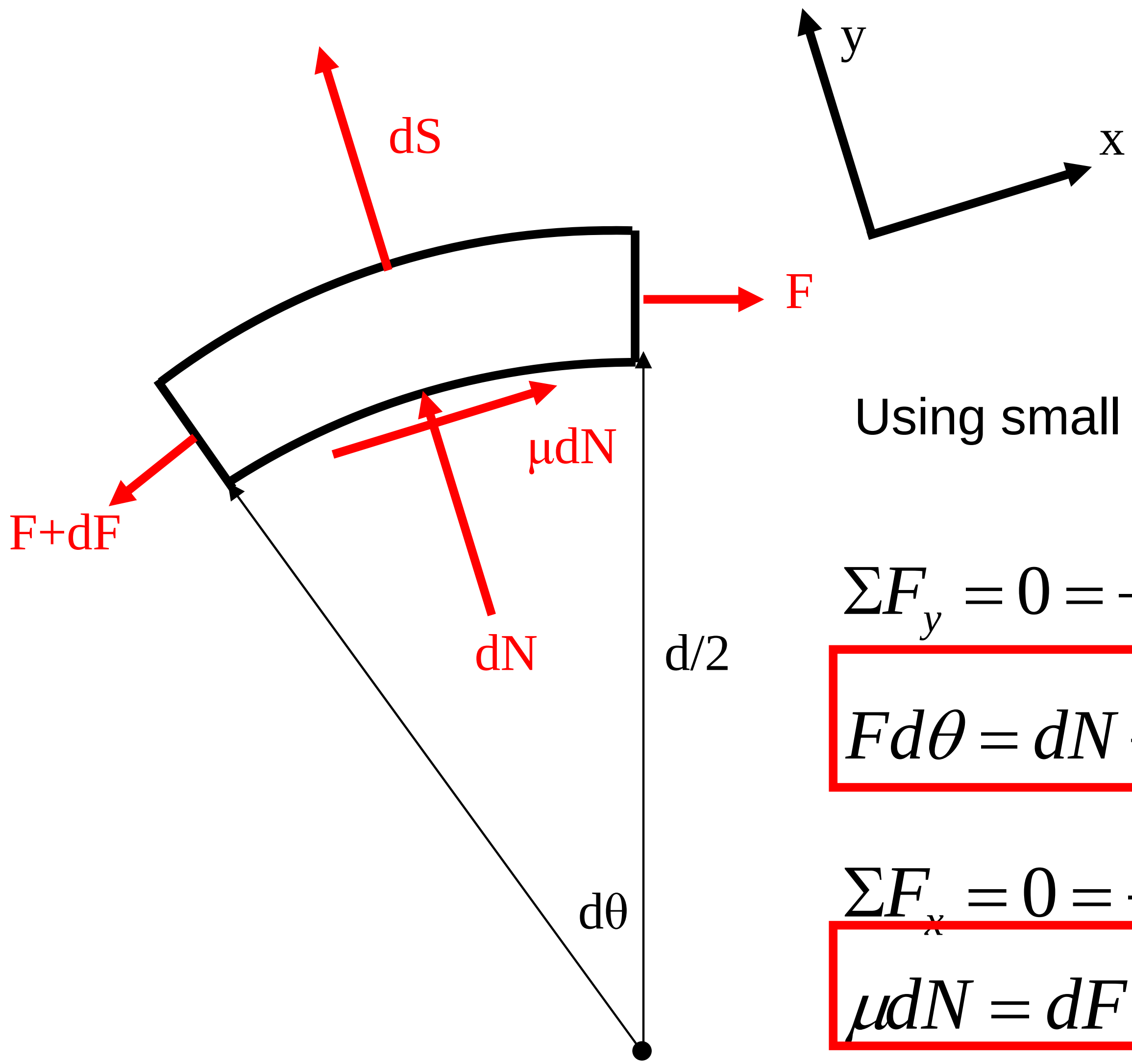
Free Body Diagram





- Tensile force (F)
- Normal force (N)
- Friction force (μN)
- Centrifugal force (S)

Force Balance



Using small angle approx:

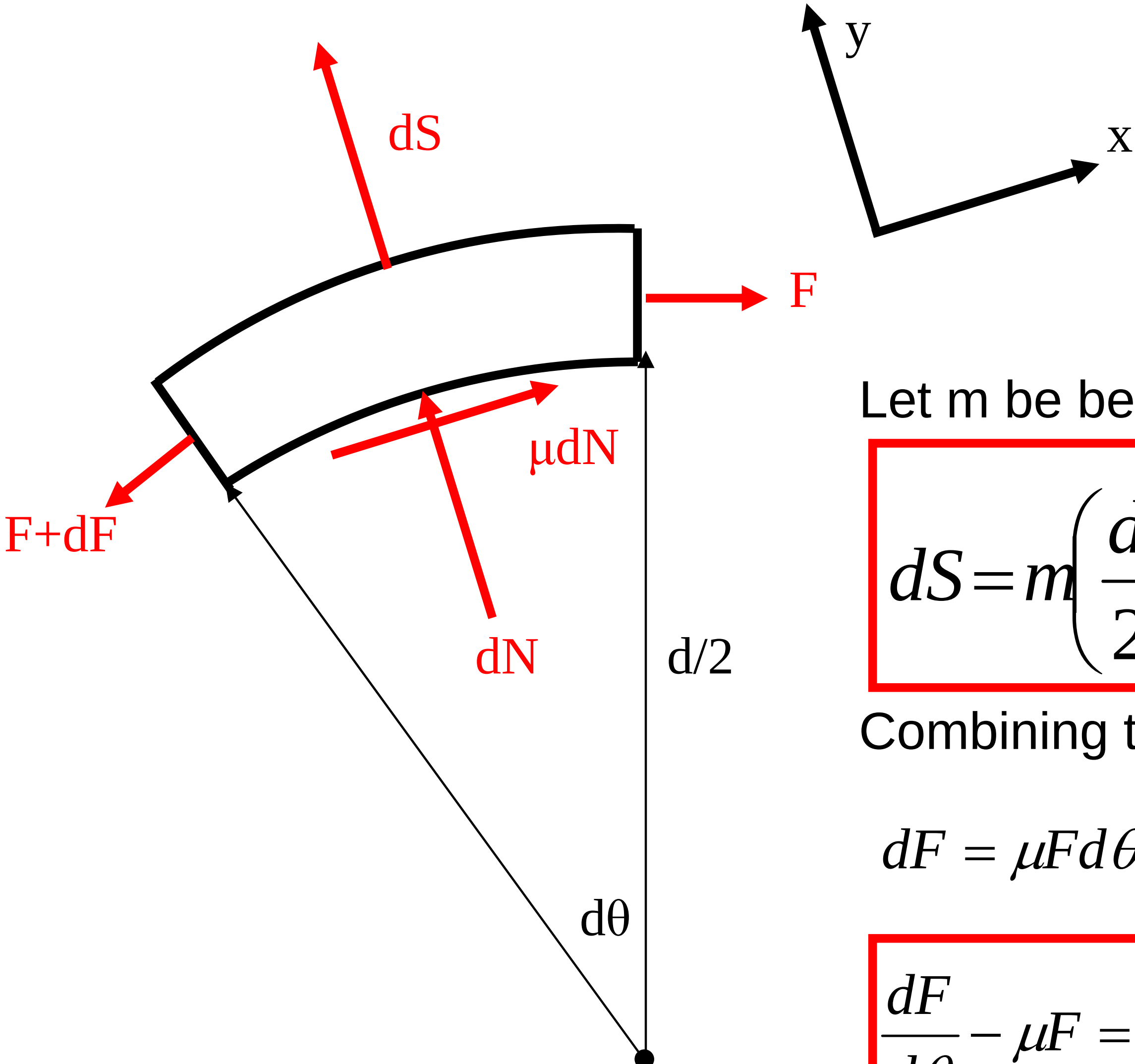
$$\Sigma F_y = 0 = -\left(F + dF\right)\frac{d\theta}{2} - F\frac{d\theta}{2} + dN + dS$$

$$F d\theta = dN + dS$$

$$\Sigma F_x = 0 = -\mu dN - F + \left(F + dF\right)$$

$$\mu dN = dF$$

Obtaining Differential Eq



Let m be belt mass/unit length

$$dS = m\left(\frac{d}{2}\right)^2 \omega^2 d\theta$$

Combining these red eqns:

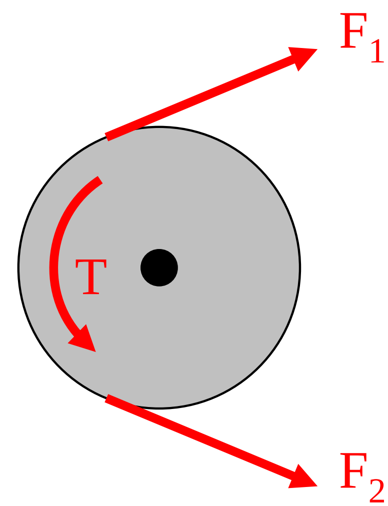
$$dF = \mu F d\theta - \mu m\left(\frac{d}{2}\right)^2 \omega^2 d\theta$$

$$\frac{dF}{d\theta} - \mu F = -\mu m\left(\frac{d}{2}\right)^2 \omega^2$$

Belt Tension to Torque

Let the difference in tension between the loose side (F_2) and the tight side (F_1) be related to torque (T)

$$F_1 - F_2 = \frac{T}{d/2}$$



Solve the previous integral over contact angle and apply F_1 and F_2 as b.c.'s and then do a page of algebra:

$$F_{tension} = \frac{T}{d}\frac{e^{\mu\theta_{contact}}+1}{e^{\mu\theta_{contact}}-1}$$

$$F_1 = m\left(\frac{d}{2}\right)^2\omega^2 + F_{tension}\frac{2e^{\mu\theta_{contact}}}{e^{\mu\theta_{contact}}+1}$$

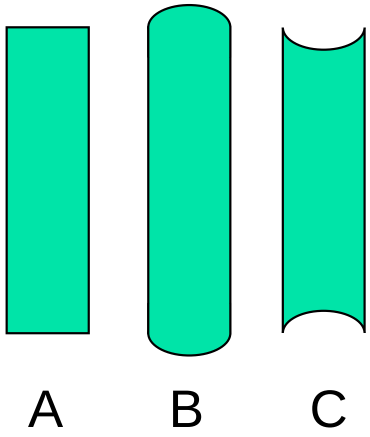
$$F_2 = m\left(\frac{d}{2}\right)^2\omega^2 + F_{tension}\frac{2}{e^{\mu\theta_{contact}}+1}$$

Used to find stresses in belt!!!

Practical Design Issues

Pulley/Sheave profile

- Which is right?

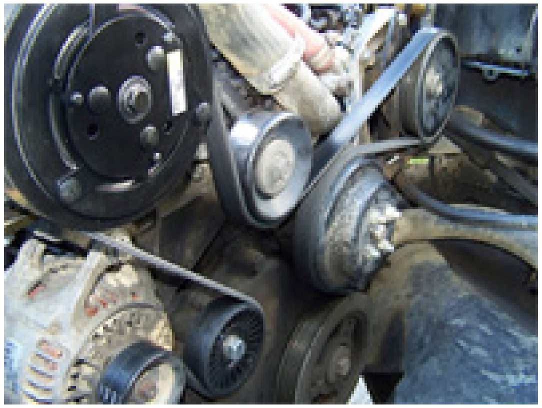


Manufacturer → lifetime eqs

- Belt Creep (loss of load capacity)
- Lifetime in cycles

Idler Pulley Design

- Catenary eqs → deflection to tension
- Large systems need more than 1



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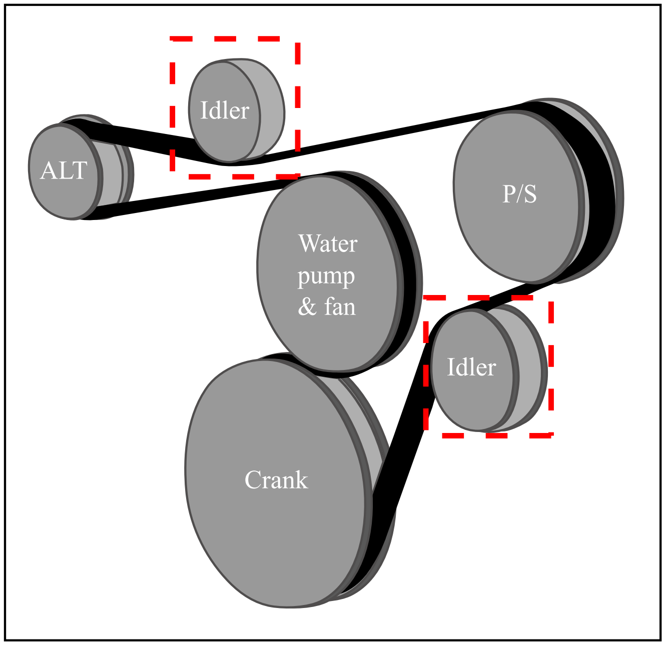

Figure by MIT OpenCourseWare.

Practice problem

Delta 15-231 Drill Press

- 1725 RPM Motor (3/4 hp)
- 450 to 4700 RPM operation
- Assume 0.3 m shaft separation
- What is max torque at drill bit?
- What size belt?
- Roughly what tension?

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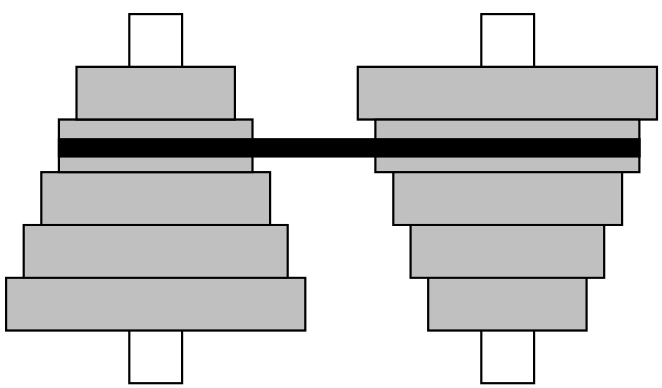

Topic 2:

Friction Drives

Friction Drives

Why Friction Drives?

- Linear ↔ Rotary Motion
- Low backlash/deadband
- Can be nm-resolution

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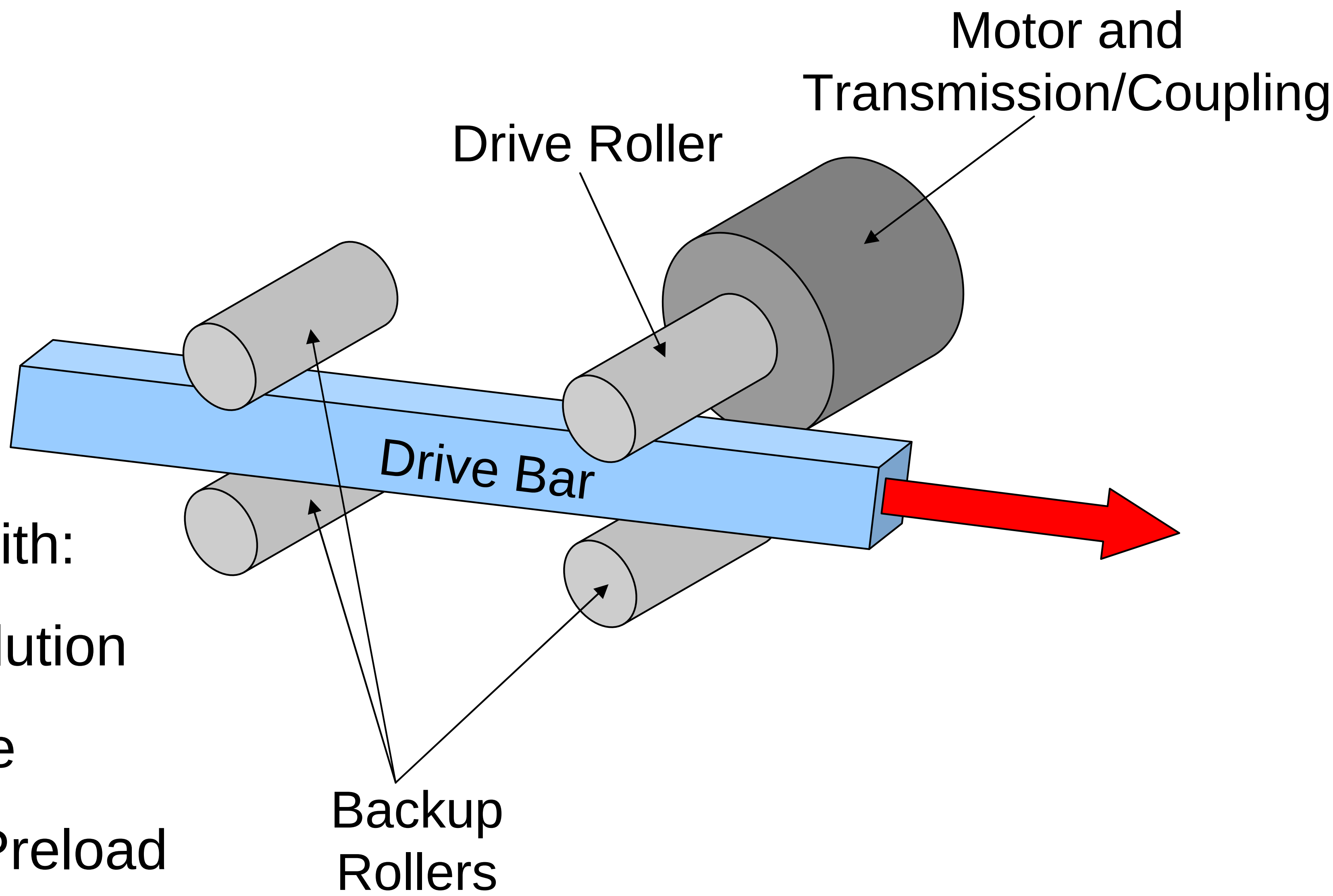
http://www.beachrobot.com/images/bata-football.jpg

http://www.borbollametrology.com/PRODUCTOS1/Wenzel/WENZELHorizontal-ArmCMMRSPlus-RSDPlus_files/rsplus.jpg

Keep in mind

- Preload → bearing selection
- Low stiffness and damping
- Needs to be clean
- Low drive force

Friction Drive Anatomy



Concerned with:

•Linear Resolution

•Output Force

•Max Roller Preload

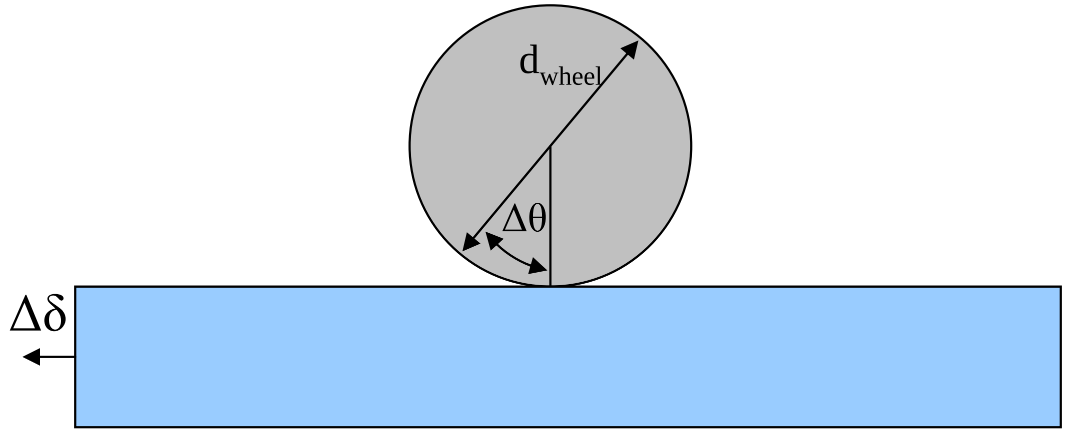
•Axial Stiffness

Drive Kinematics/Force Output

Kinematics found from no slip cylinder on flat

$$\Delta\delta_{bar} = \Delta\theta \cdot \frac{d_{wheel}}{2}$$

$$v_{bar} = \omega_{wheel} \frac{d_{wheel}}{2}$$



Force output found from static analysis

- Either motor or friction limited

$$F_{output} = \frac{2T_{wheel}}{d_{wheel}} \quad where\ F_{output} \le \mu F_{preload}$$

Maximum Preload

$$E_e = \left(\frac{1-\nu_{wheel}^2}{E_{wheel}} + \frac{1-\nu_{bar}^2}{E_{bar}} \right)^{-1}$$

$$R_e = \left(\frac{1}{d_{wheel}/2} + \frac{1}{r_{crown}} \right)^{-1}$$

Variable Definitions

$$a_{contact} = \left(\frac{3F_{preload}R_e}{2E_e} \right)^{\frac{1}{3}}$$

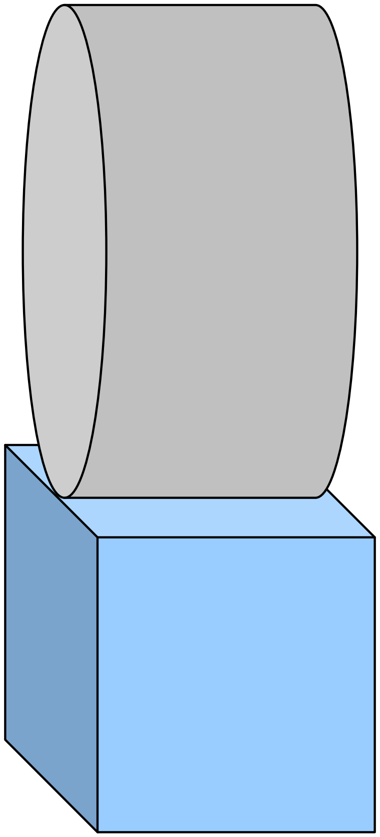
For metals:

$$\tau_{\max} = \frac{3\sigma_y}{2}$$

Shear Stress Equation

$$\tau_{wheel} = \frac{a_{contact}E_e}{2\pi R_e} \left(\frac{1+2\nu_{wheel}}{2} + \frac{2}{9} \cdot (1+\nu_{wheel}) \cdot \sqrt{2(1+\nu_{wheel})} \right)$$

$$F_{preload,\max} = \frac{16\pi^3 \tau_{\max}^3 R_e^2}{3E_e^2 \left(\frac{1+2\nu_{wheel}}{2} + \frac{2}{9} \cdot (1+\nu_{wheel}) \cdot \sqrt{2(1+\nu_{wheel})} \right)^3}$$



Axial Stiffness

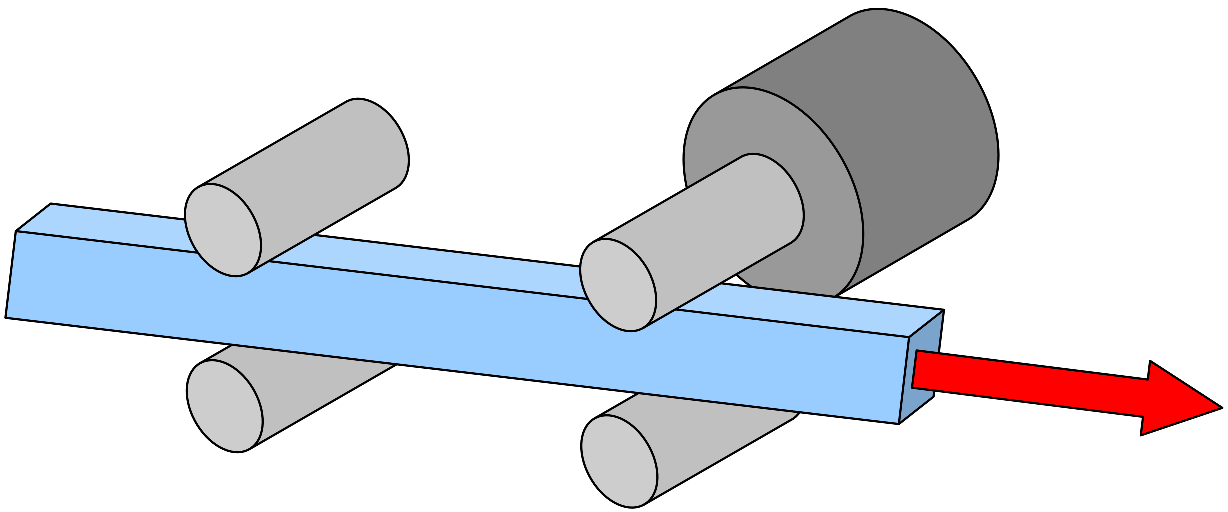
$$k_{axial} = \left(\frac{1}{k_{shaft}} + \frac{1}{\frac{k_{torsion}}{d_{wheel}^2}} + \frac{1}{k_{tangential}} + \frac{1}{k_{bar}} \right)^{-1}$$

$$k_{tangential} = \frac{4 a_e E_e}{(2-\nu)(1+\nu)}$$

$$k_{shaft} = \frac{3 \pi E d_{shaft}^4}{4 L^3}$$

$$k_{torsion} = \frac{\pi G d_{wheel}^4}{32 L}$$

$$k_{bar} = \frac{E A_{c,\,bar}}{L}$$



Friction Drives

Proper Design leads to

- Pure radial bearing loads
- Axial drive bar motion only

Drive performance linked to motor/transmission

- Torque ripple
- Angular resolution

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Topic 3:

Gear Kinematics

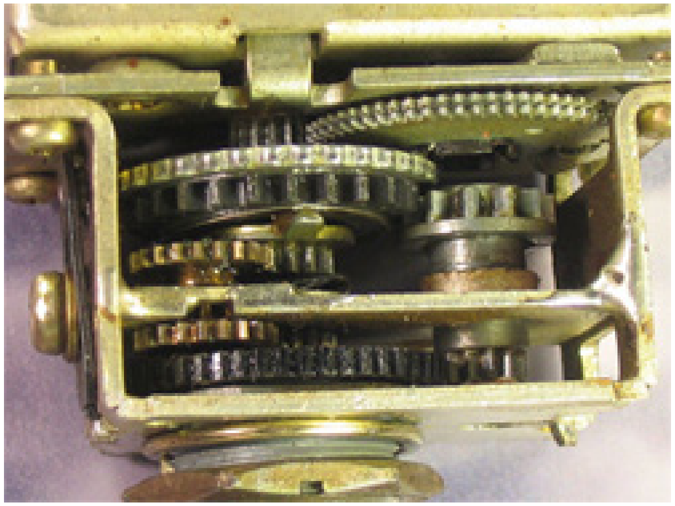
Gear Drives

Why Gears?

- Torque/speed conversion
- Can transfer large torques
- Can run at low speeds
- Large reductions in small package

Keep in mind

- Requires careful design
- Attention to tooth loads, profile



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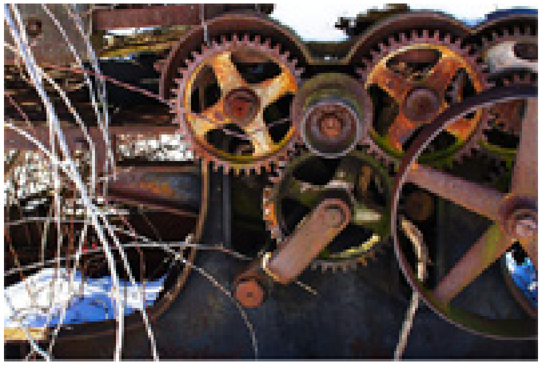

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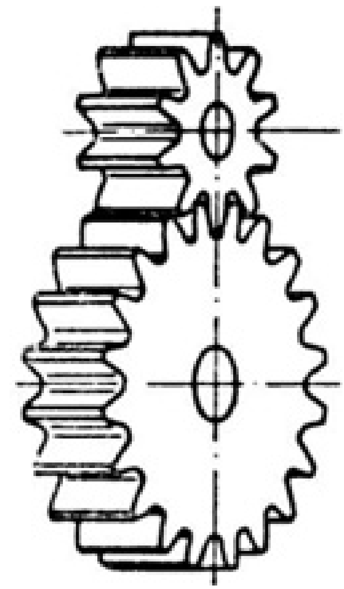
http://elecon.nlihost.com/img/gear-train-backlash-and-contact-pattern-checking.jpg

http://www.cydgears.com.cn/products/Planetarygeartrain/planetarygeartrain.jpg

Gear Types and Purposes

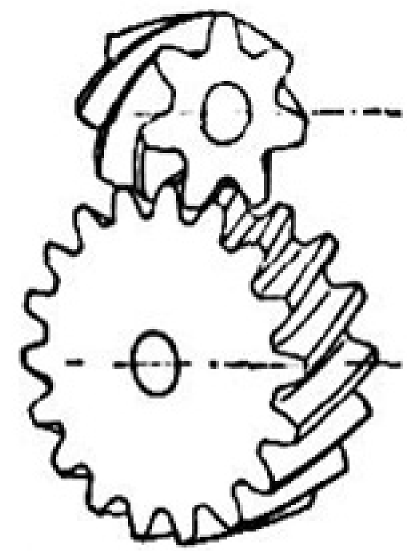
Spur Gears

- Parallel shafts
- Simple shape → easy design, low $$$
- Tooth shape errors → noise
- No thrust loads from tooth engagement



Helical Gears

- Gradual tooth engagement → low noise
- Shafts may or may not be parallel
- Thrust loads from teeth reaction forces
- Tooth-tooth contact pushes gears apart



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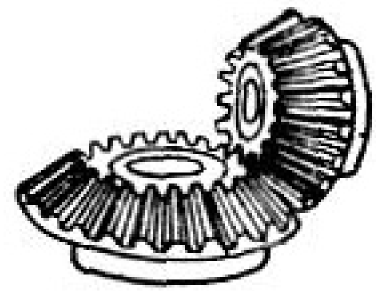
Gear Types and Purposes

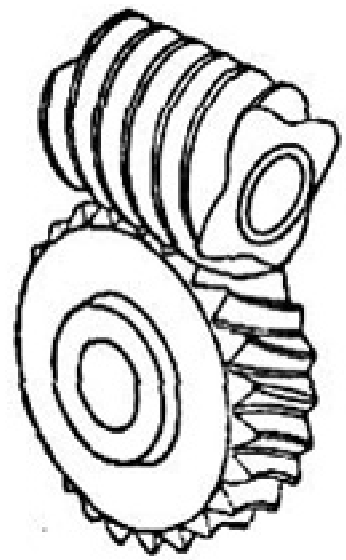
Bevel Gears

- Connect two intersecting shafts
- Straight or helical teeth



Worm Gears

- Low transmission ratios
- Pinion is typically input (Why?)
- Teeth sliding → high friction losses



Rack and Pinion

- Rotary ↔ Linear motion
- Helical or straight rack teeth

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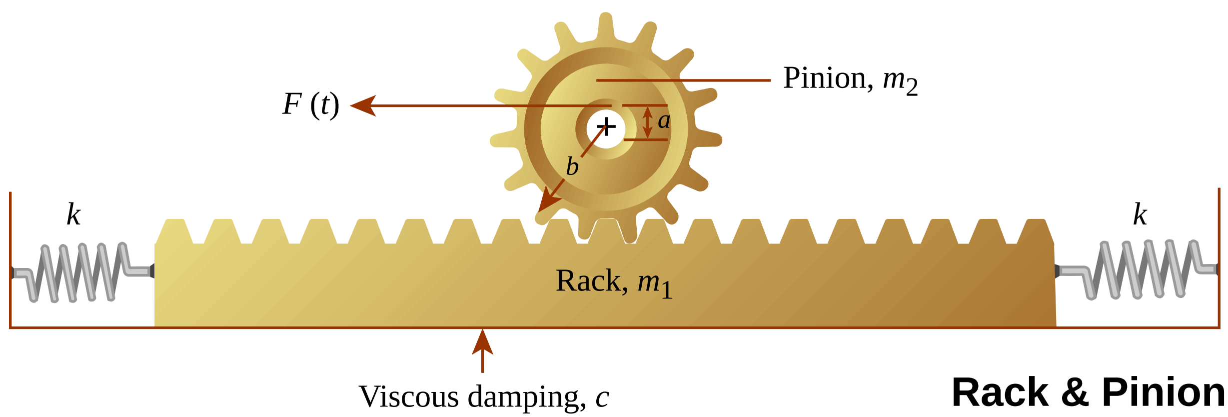

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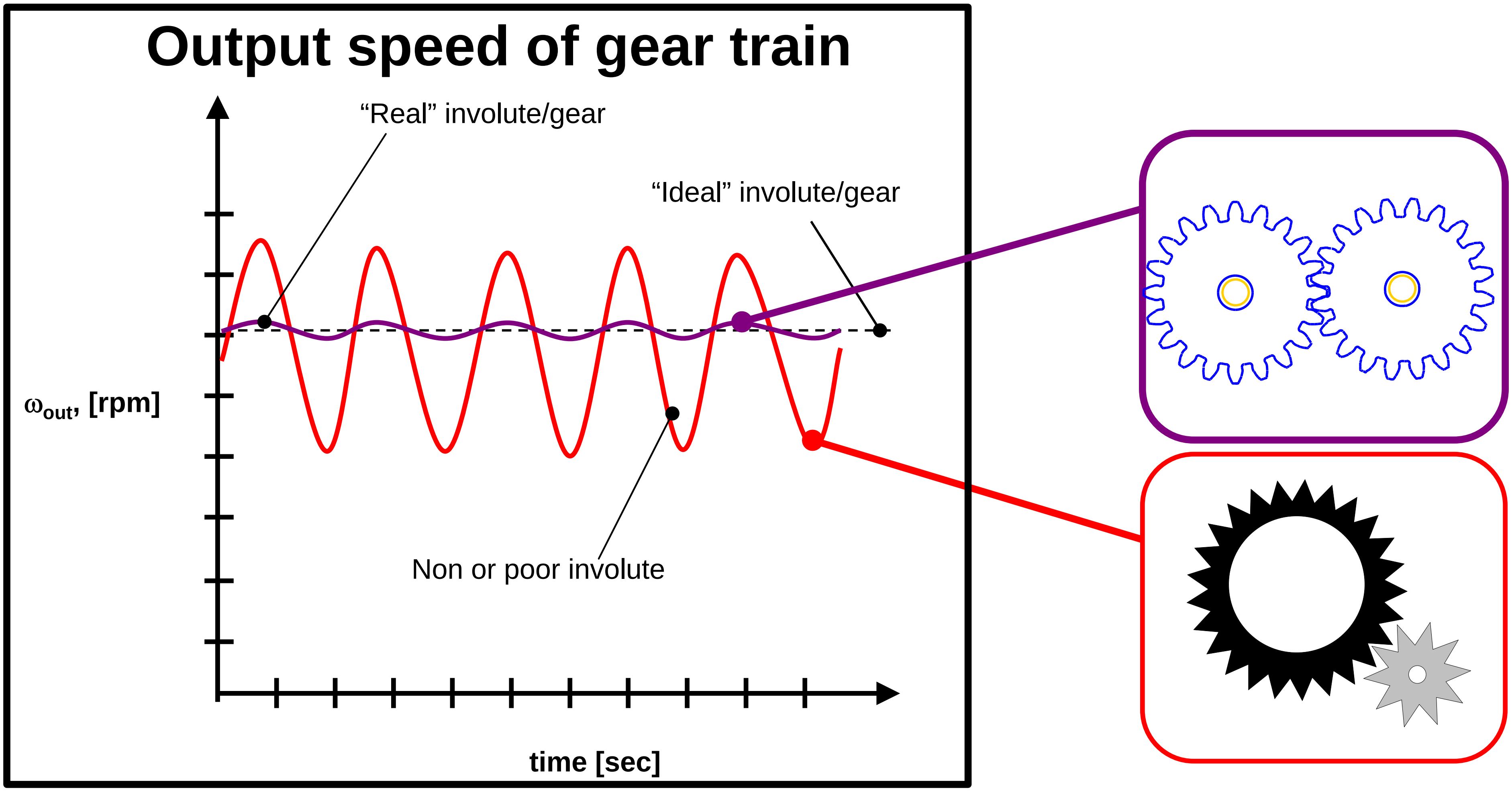
Tooth Profile Impacts Kinematics

Want constant speed output

- ❑ Conjugate action = constant angular velocity ratio
- ❑ Key to conjugate action is to use an involute tooth profile



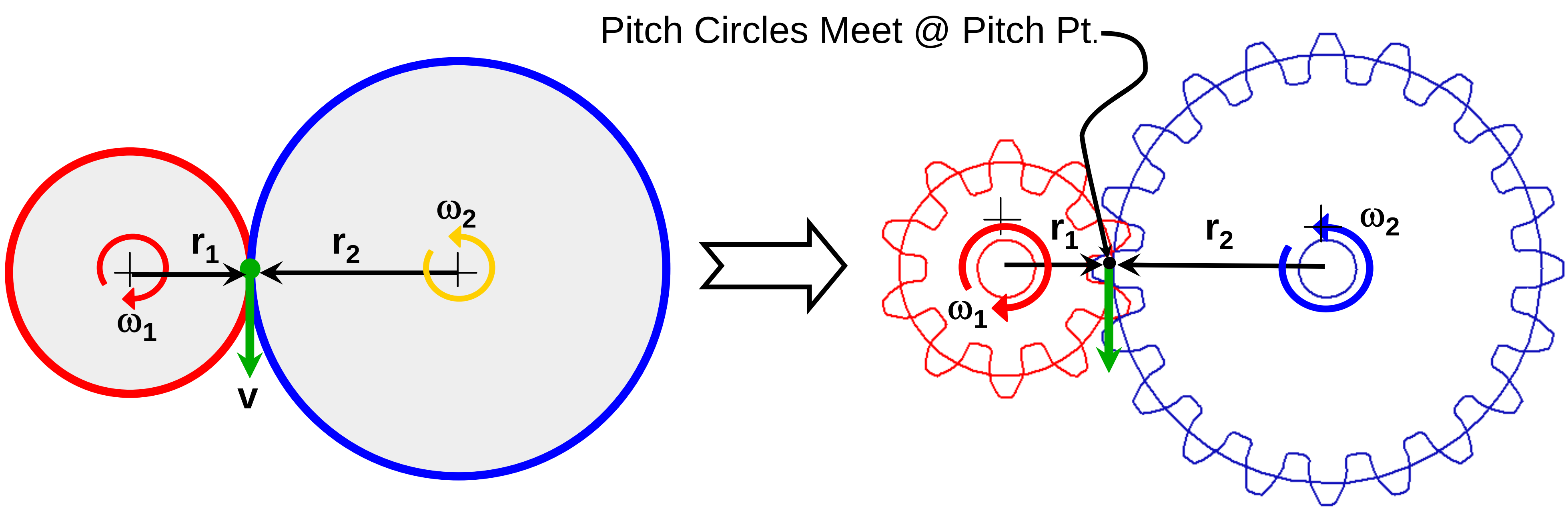
Instantaneous Velocity and Pitch

Model as rolling cylinders (no slip condition):

$$\vec{v} = \vec{\omega}_1 \times \vec{r}_1 = \vec{\omega}_2 \times \vec{r}_2 \quad \rightarrow \quad \boxed{\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}}$$

Model gears as two pitch circles

- Contact at pitch point



Instantaneous Velocity and Pitch

Meshing gears must have same pitch

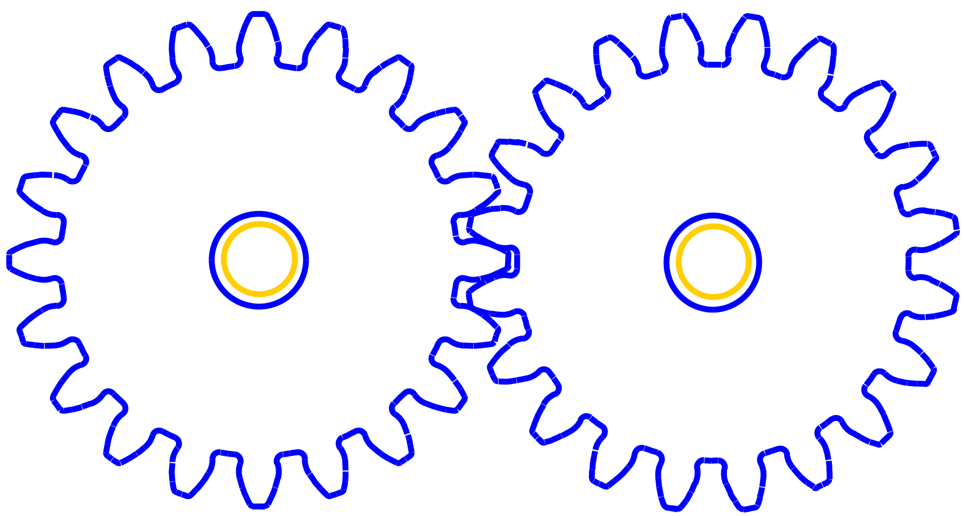
- N_g = # of teeth, D_p = Pitch circle diameter

Diametral pitch, P_D:

$$P_D = \frac{N_g}{D_p}$$

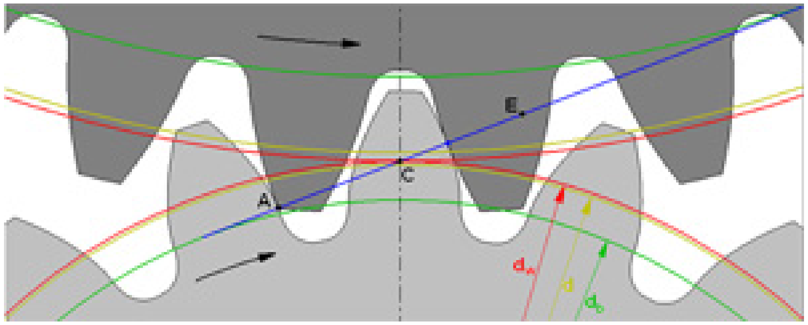
Circular pitch, P_C:

$$P_C = \frac{\pi D_p}{N_g} = \frac{\pi}{P_D}$$



Drawing the Involute Profile

•Gear is specified by diametral pitch and pressure angle, Φ



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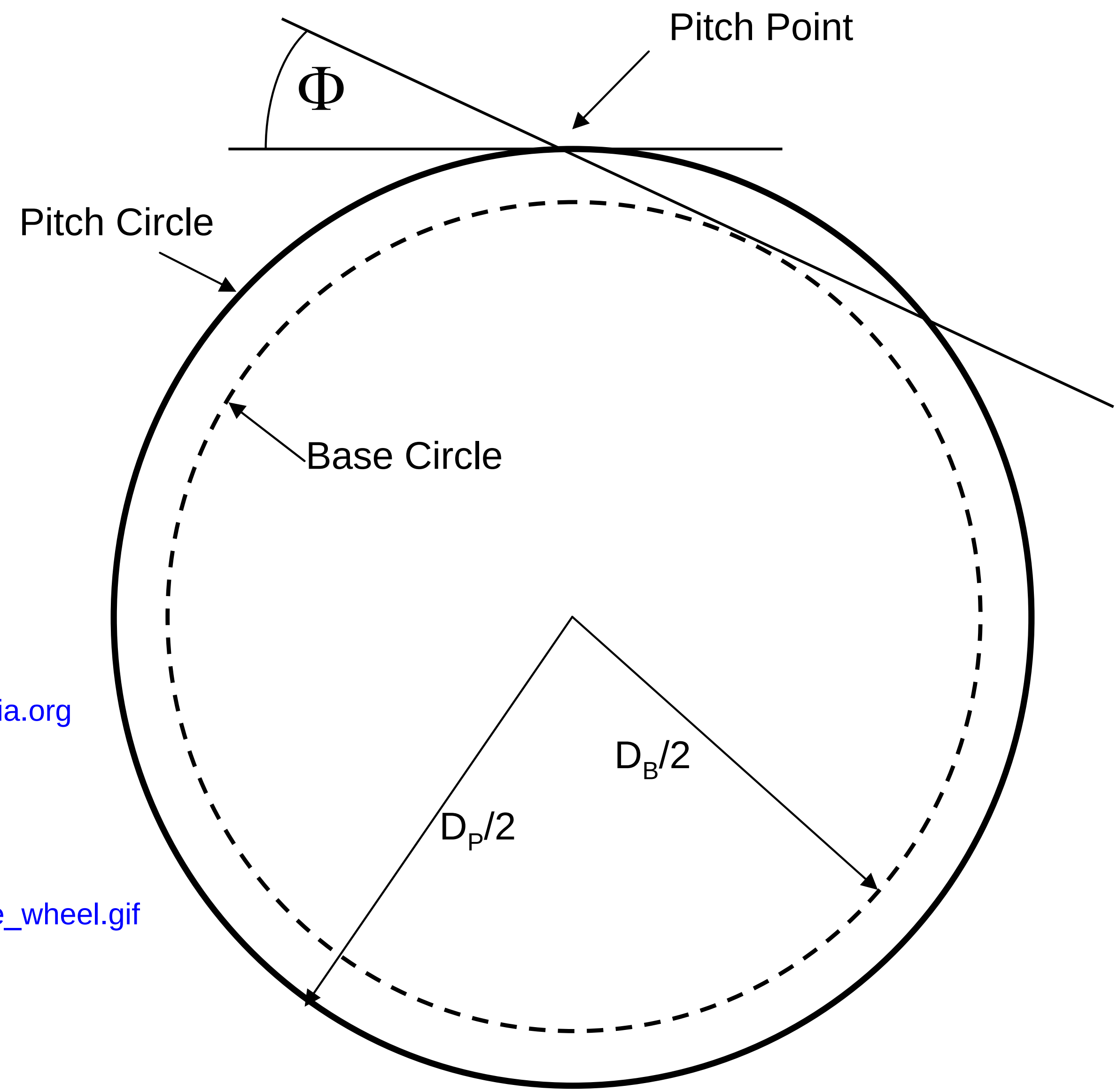
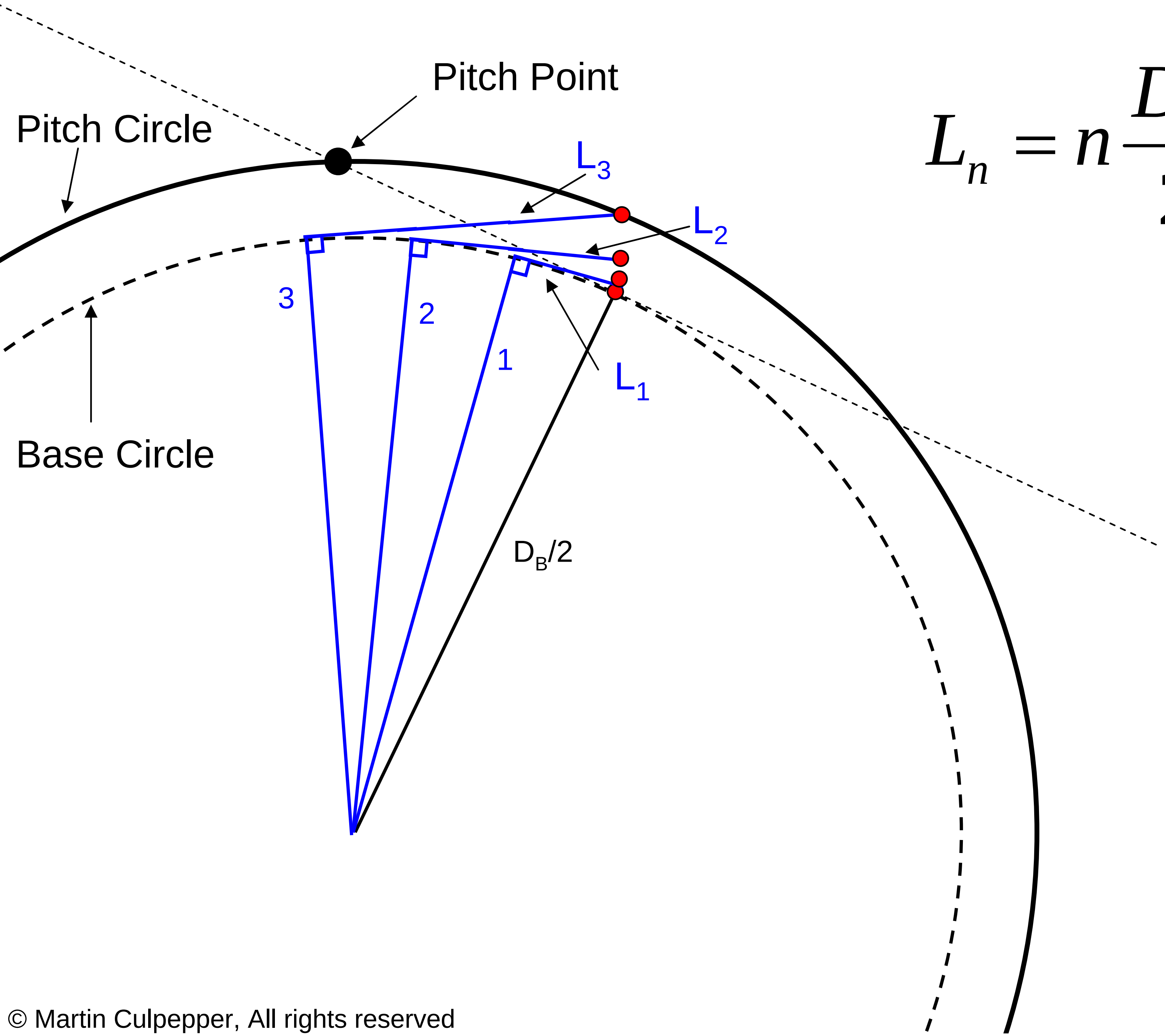
$$D_B = D_P \cos\Phi$$



Drawing the Involute Profile

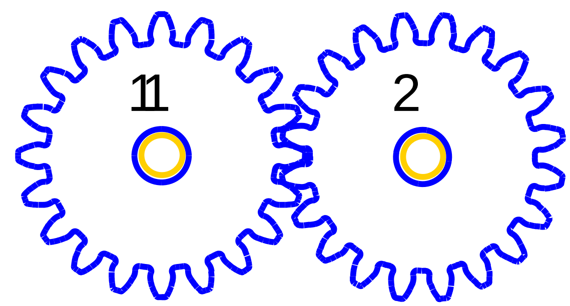


$$L_n = n\frac{D_B}{2}\Delta\theta$$

Transmission Ratio for Serial Gears

Transmission ratio for elements in series: $$\mathrm{TR} = (proper\ sign) \cdot \frac{\omega_{out}}{\omega_{in}}$$

From pitch equation: $$P_1 = \frac{N_1}{D_1} = \frac{N_2}{D_2} = P_2 \longrightarrow \frac{D_1}{D_2} = \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1}$$



For Large Serial Drive Trains:

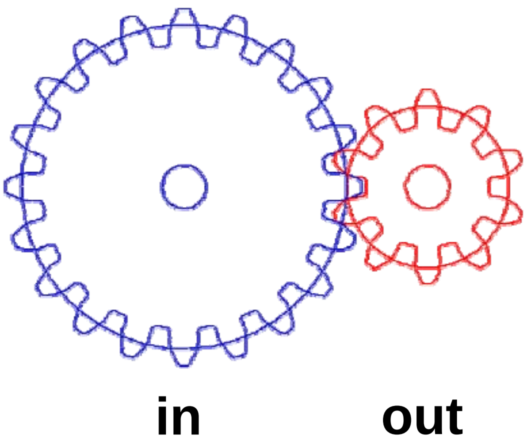
$$TR = (proper\ sign) \cdot \frac{\text{Product of driving teeth}}{\text{Product of driven teeth}}$$

Transmission Ratio for Serial Gears

Serial trains:

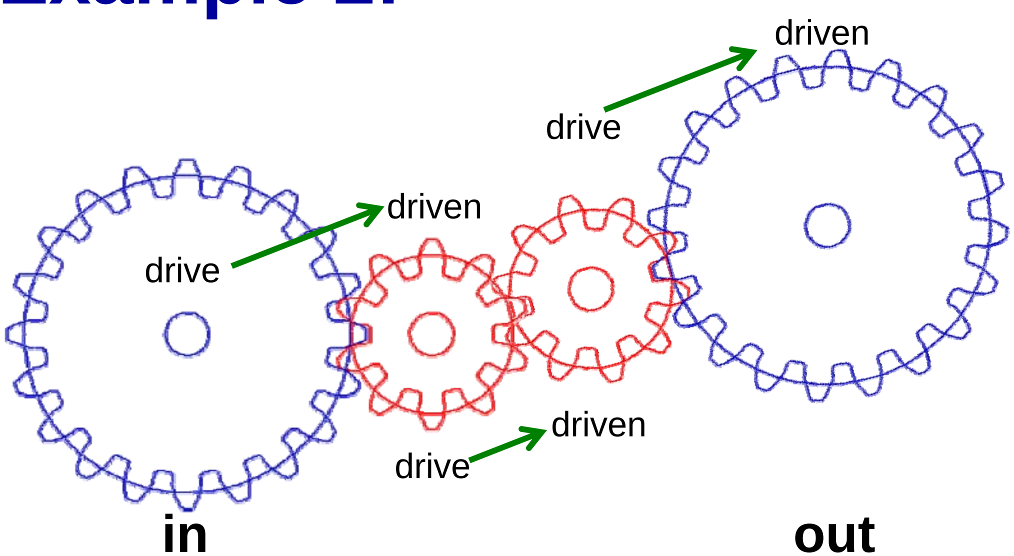
$$TR = (proper\ \ sign) \cdot \frac{\text{Product of driving teeth}}{\text{Product of driven teeth}}$$

Example 1:



$$TR = ?$$

Example 2:



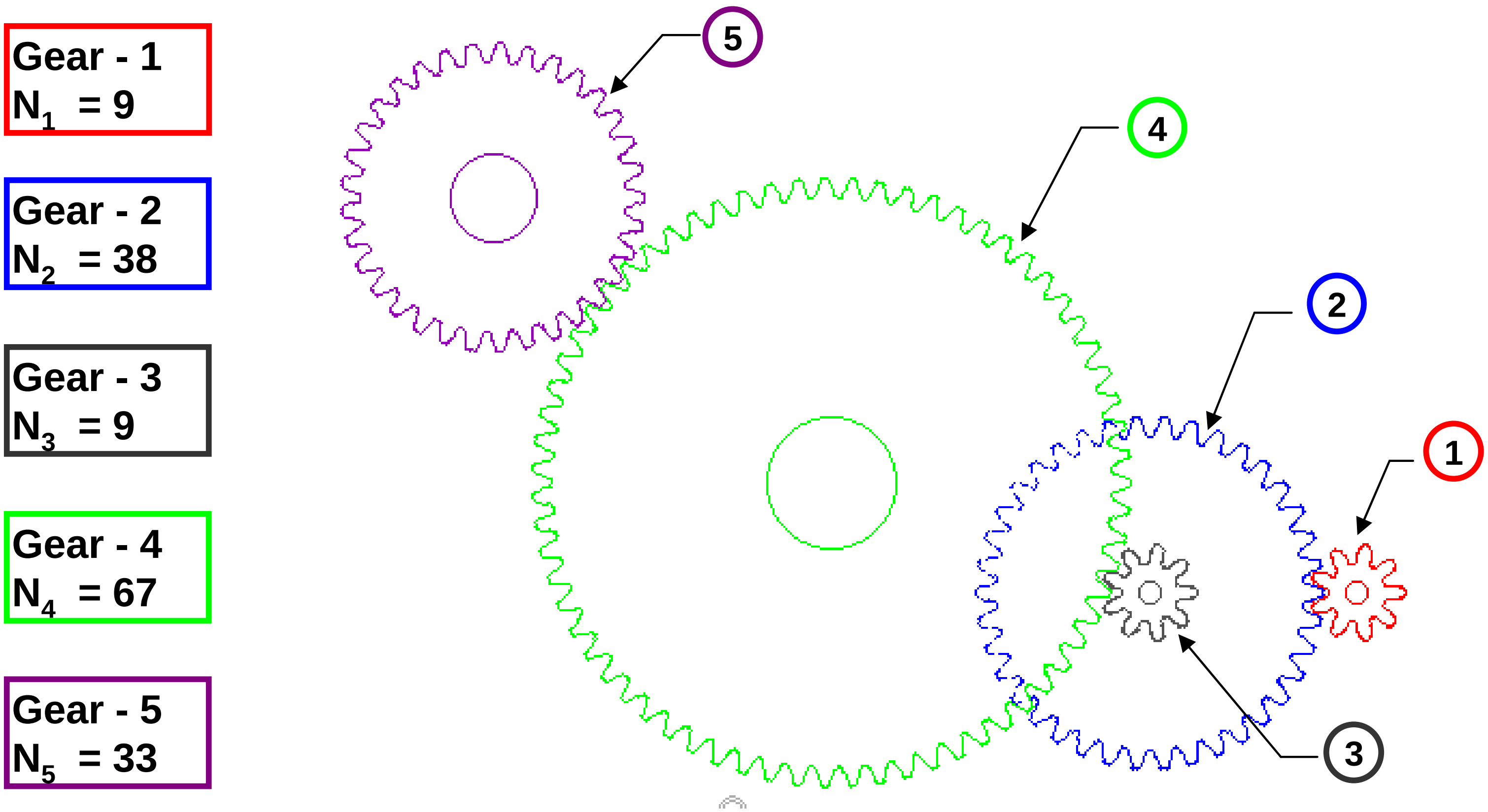
$$TR = ?$$

Transmission Ratio for Serial Gears

Example 3: Integral gears in serial gear trains

- What is TR? Gear 1 = input and 5 = output

$$TR = (proper\ \ sign) \cdot \frac{\text{Product of driving teeth}}{\text{Product of driven teeth}}$$

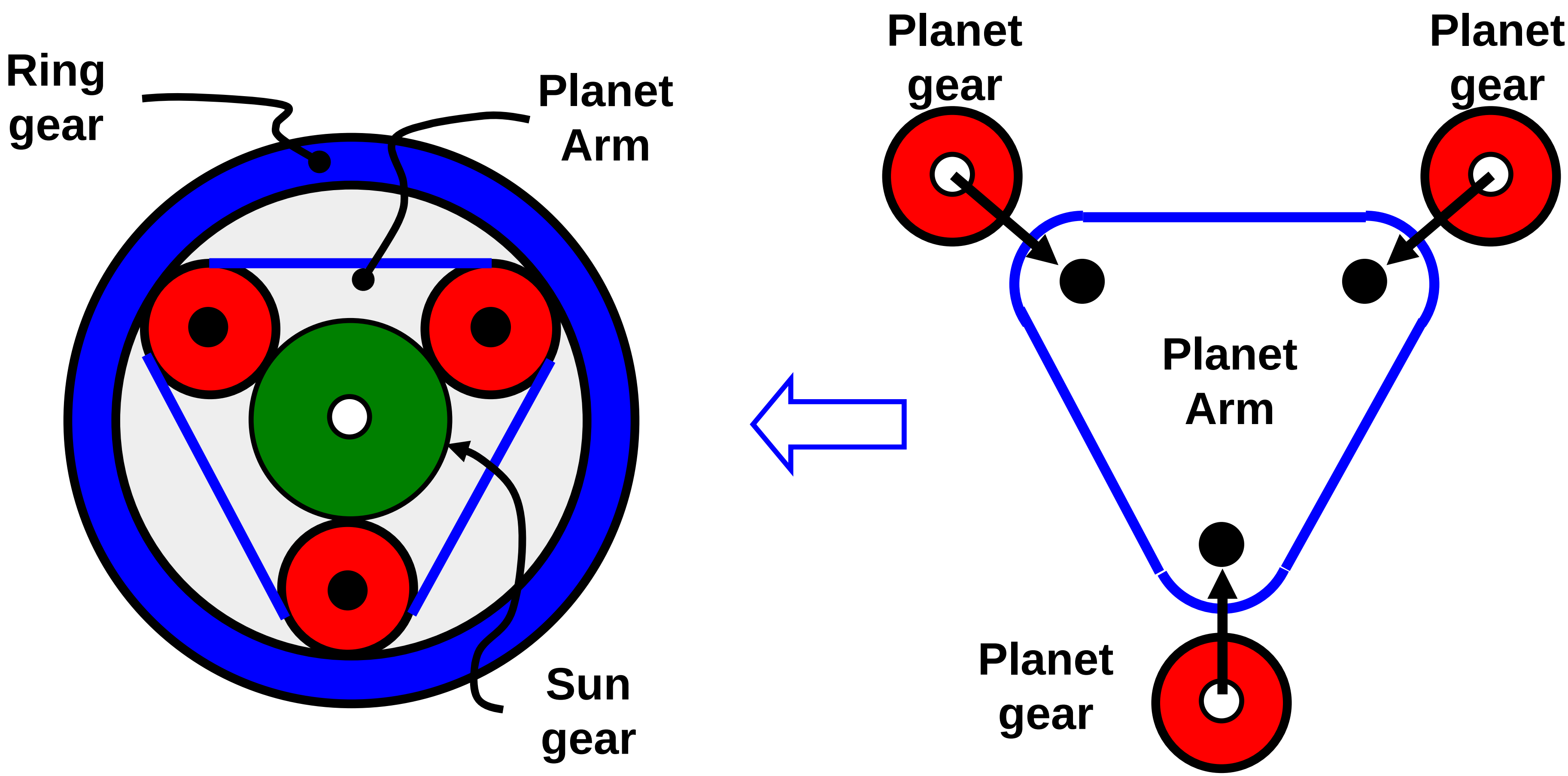


Planetary Gear Trains

Planetary gear trains are very common

- Very small/large TRs in a compact mechanism

Terminology:



Planetary Gear Train Animation

How do we find the transmission ratio?

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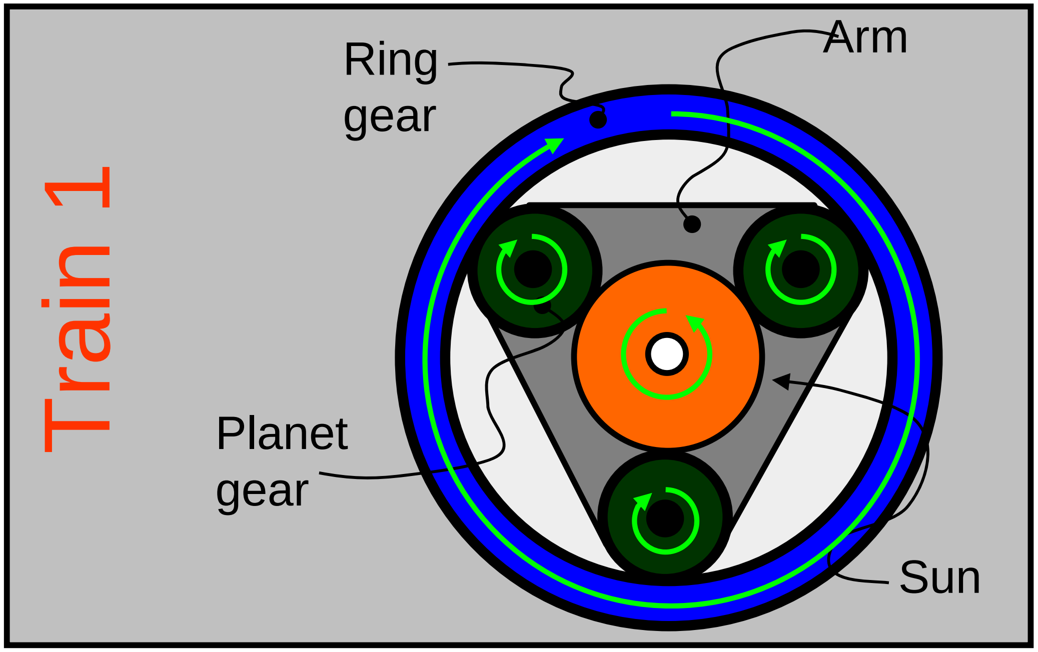
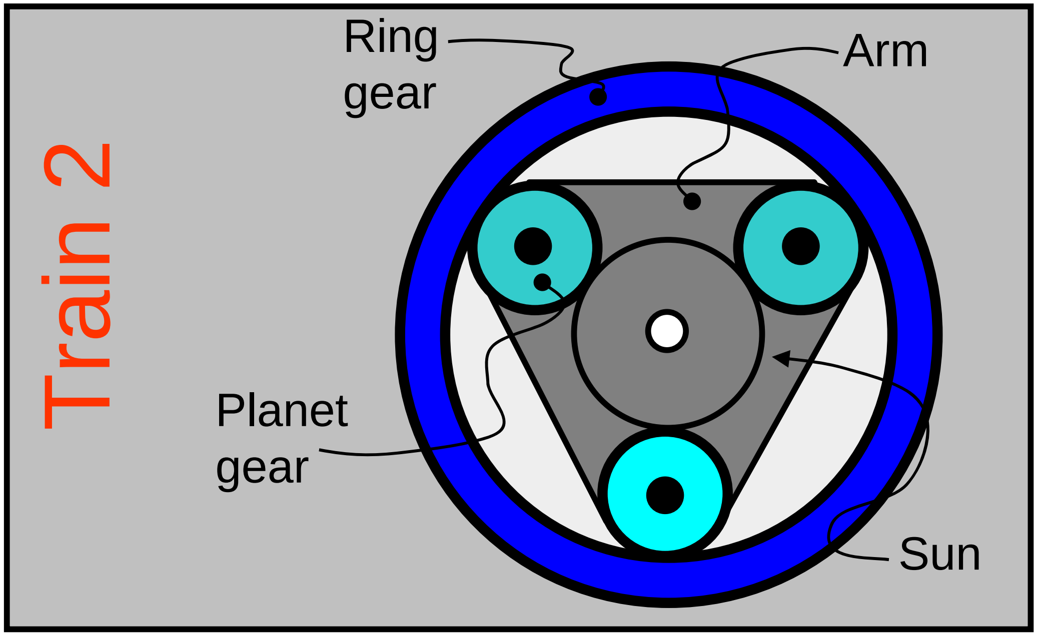
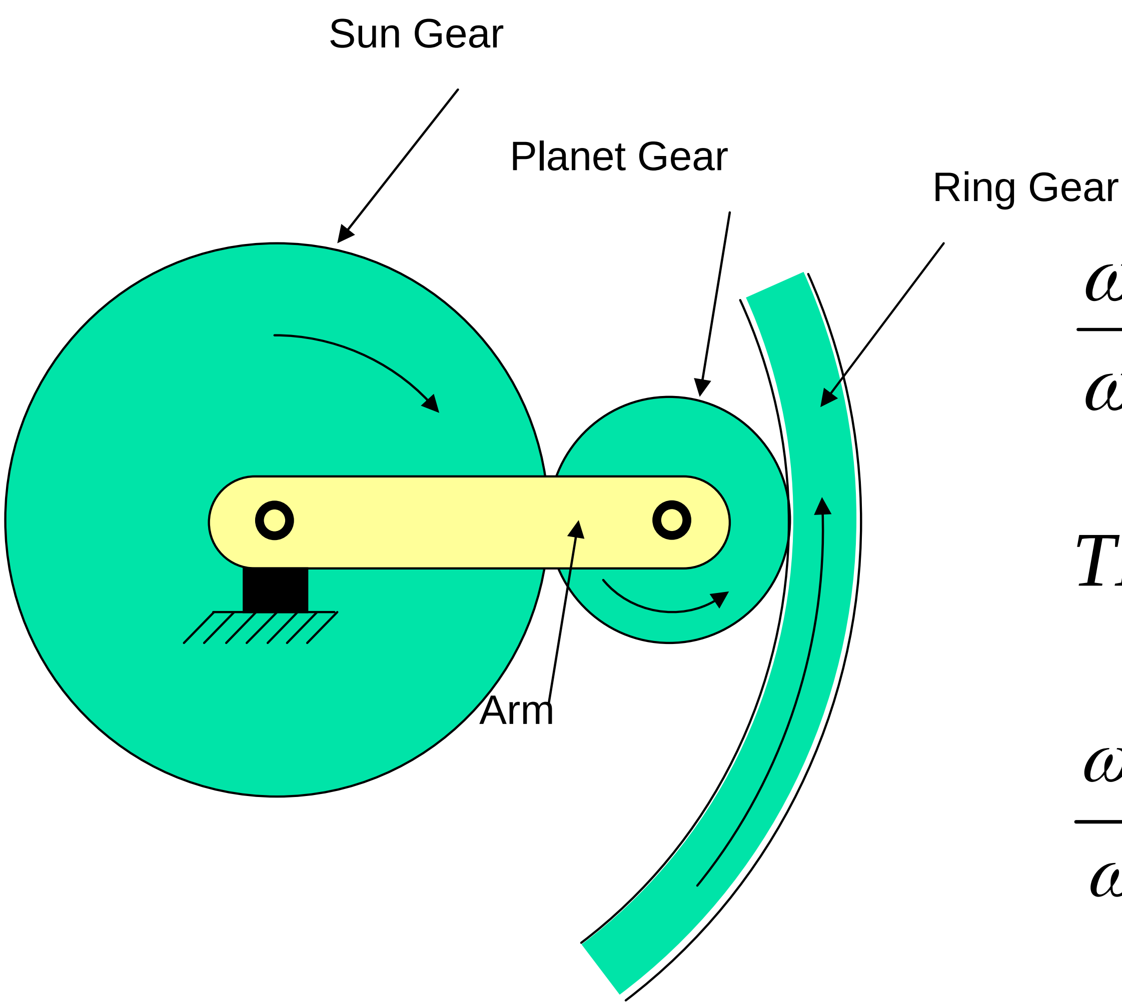




Planetary Gear Train TR



If we make the arm stationary, than this is a serial gear train:

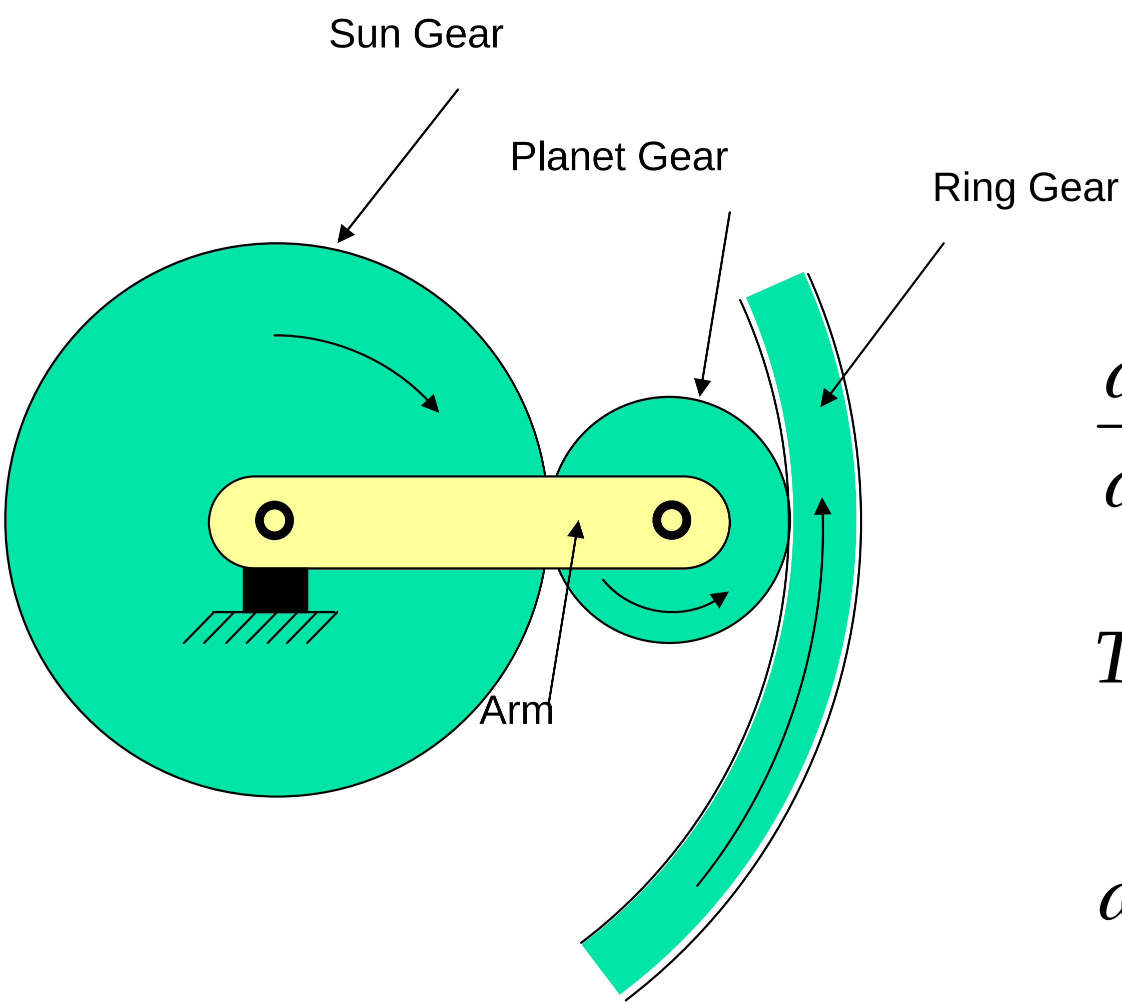
$$\frac{\omega_{ra}}{\omega_{sa}} = \frac{\omega_{ring} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} = TR$$

$$TR = -\frac{N_{sun}}{N_{planet}} \cdot \frac{N_{planet}}{N_{ring}} = -\frac{N_{sun}}{N_{ring}}$$

$$\frac{\omega_{pa}}{\omega_{sa}} = \frac{\omega_{planet} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} = TR$$

$$TR = -\frac{N_{sun}}{N_{planet}}$$

Planetary Gear Train Example



If the sun gear is the input, and the ring gear is held fixed:

$$\frac{\omega_{ra}}{\omega_{sa}} = \frac{0-\omega_{arm}}{\omega_{sun}-\omega_{arm}} = TR$$

$$TR = -\frac{N_{sun}}{N_{planet}} \cdot \frac{N_{planet}}{N_{ring}} = -\frac{N_{sun}}{N_{ring}}$$

$$\omega_{output} = \omega_{arm} = \frac{TR}{TR-1}\omega_{sun}$$

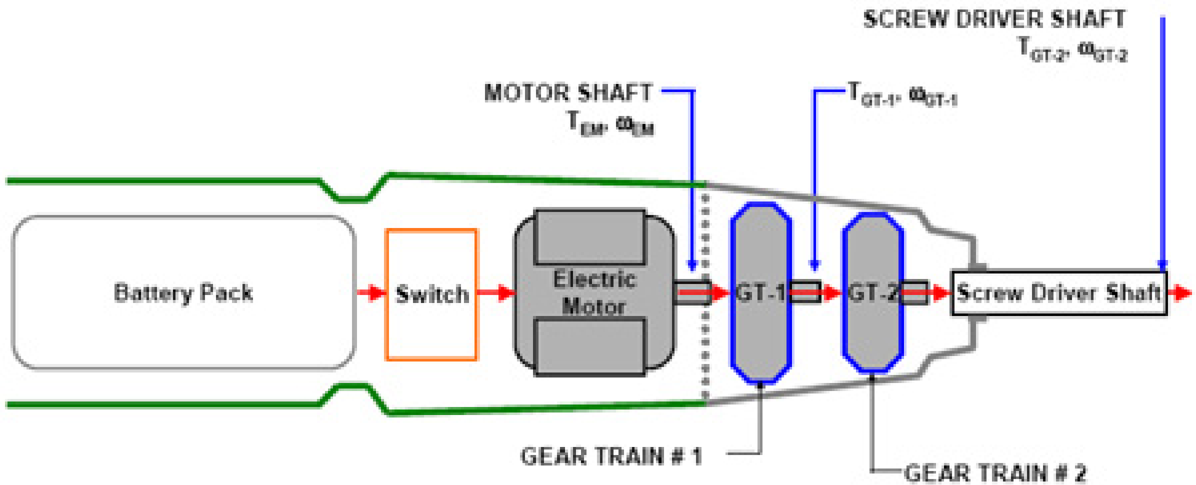
Case Study: Cordless Screwdriver

Given: Shaft T_{SH} (ω_{SH}) find motor T_M (ω_{SH})

- Geometry dominates relative speed (Relationship due to TR)

2 Unknowns: T_M and ω_M with 2 Equations:

- Transmission ratio links input and output speeds
- Energy balance links speeds and torques



Example: DC Motor shaft

T(ω):

$$T(\omega) = T_S \cdot \left(1 - \frac{\omega}{\omega_{NL}}\right)$$

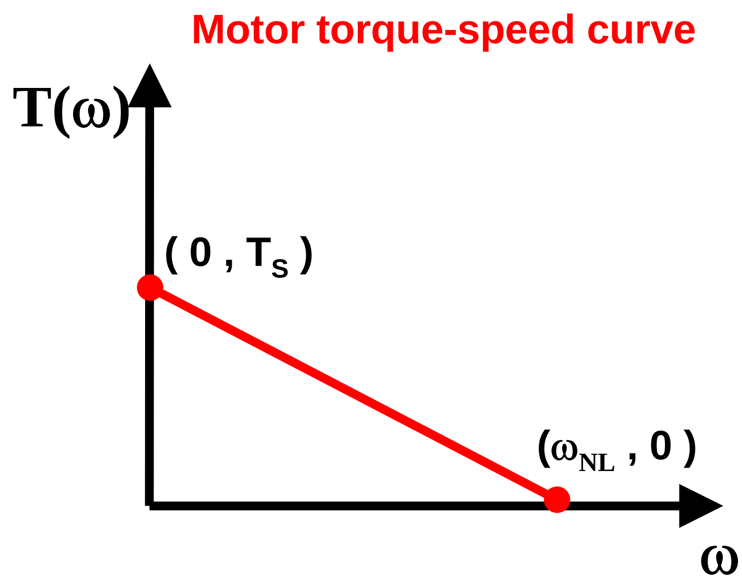
P(ω) obtained from P(ω) = T(ω) • ω

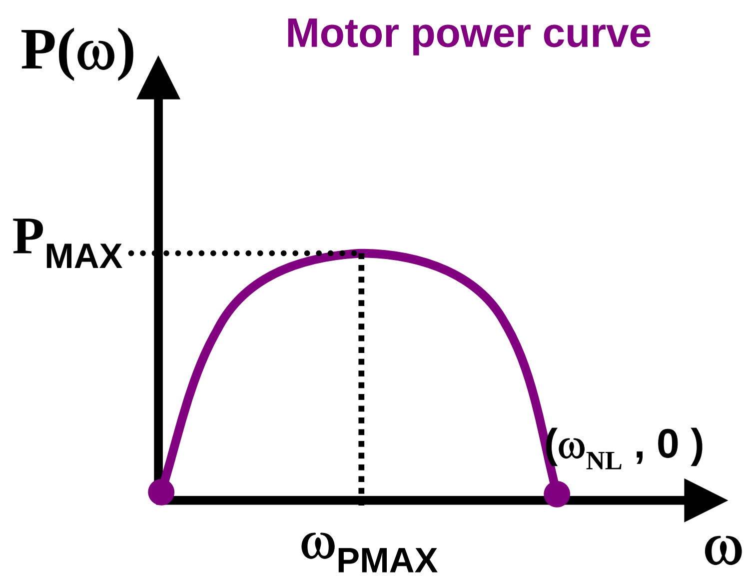
Speed at maximum power output:

$$P(\omega) = T(\omega) \cdot \omega = T_S \cdot \left(\omega - \frac{\omega^2}{\omega_{NL}}\right)$$

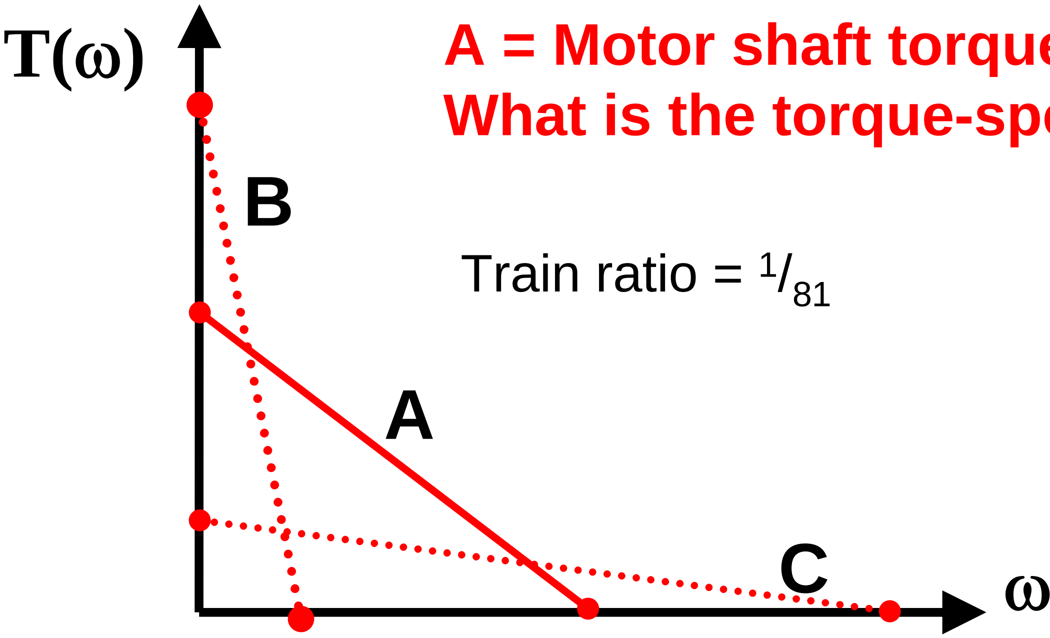
$$\omega_{PMAX} = \frac{\omega_{NL}}{2}$$

$$P_{MAX} = T_S \cdot \left(\frac{\omega_{NL}}{4}\right)$$



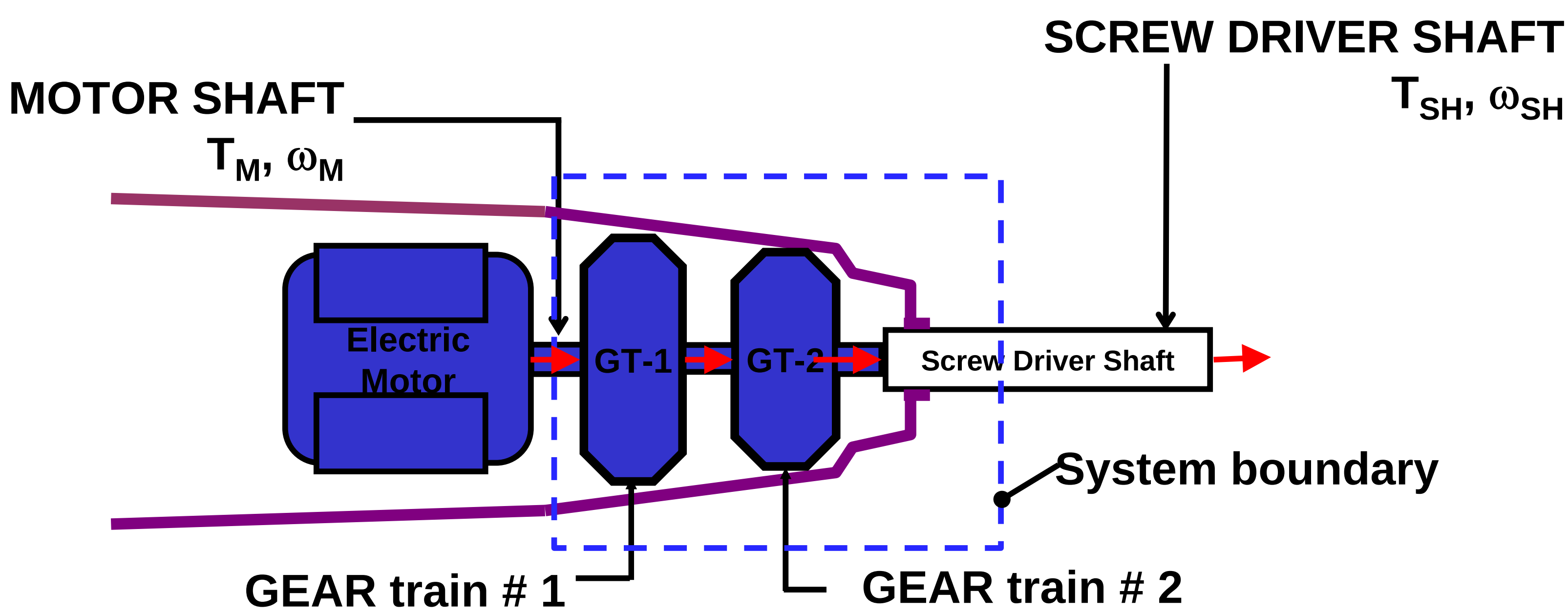


Example: Screw driver shaft

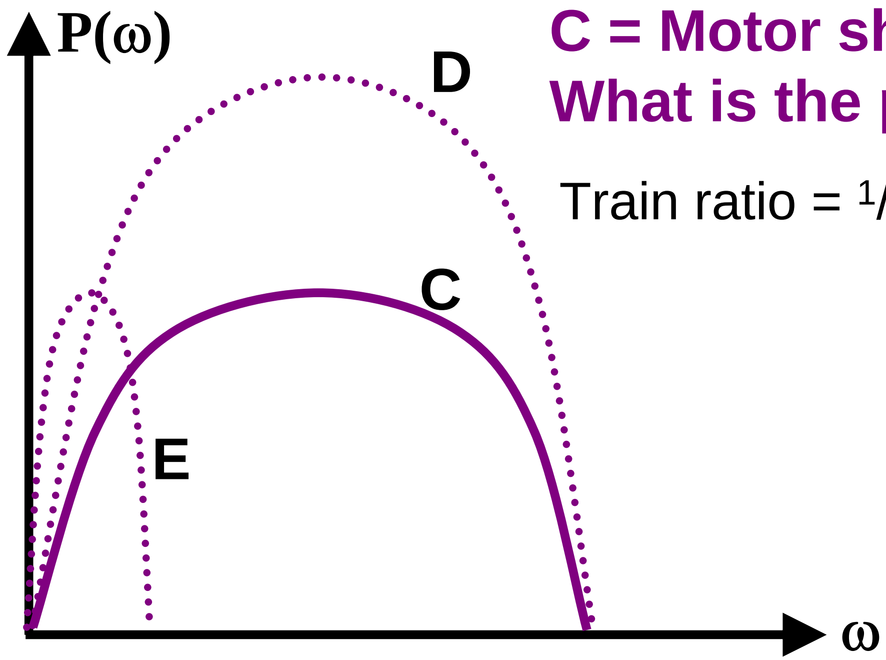


A = Motor shaft torque-speed curve
What is the torque-speed curve for the screw driver?

Train ratio = $^1/_{81}$



Example: Screw driver shaft



C = Motor shaft power curve

What is the power-speed curve for the screw driver?

Train ratio = $^{1}/_{81}$