2.72 Elements of Mechanical Design
Spring 2009

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Elements of Mechanical Design

Lecture 13: Gear failure prevention

Schedule and reading assignment

Quiz

- Activity at end: Optional/Extra credit

Images removed due to copyright restrictions.

Please see images of very large and very small gears, such as:

http://mems.sandia.gov/gallery/images_gears_and_transmissions.html

http://www.cage-gear.com/large_gear_cutting.htm

Topics

- Gear lifetime/selection

Reading assignment

- None!

Selection vs. design of gears

It is rare to custom DESIGN a gear.

Many gear selection programs…

Anybody can read S/M and plug in #s

BASIC considerations to select gears:

- Ensure geometric compatibility (e.g. equal pitch and same type)
- Avoid low-cycle failure (e.g. root stress)
- Avoid high-cycle failure (e.g. pitting)

Focus on what is important

A failure…

How to model the gear teeth…

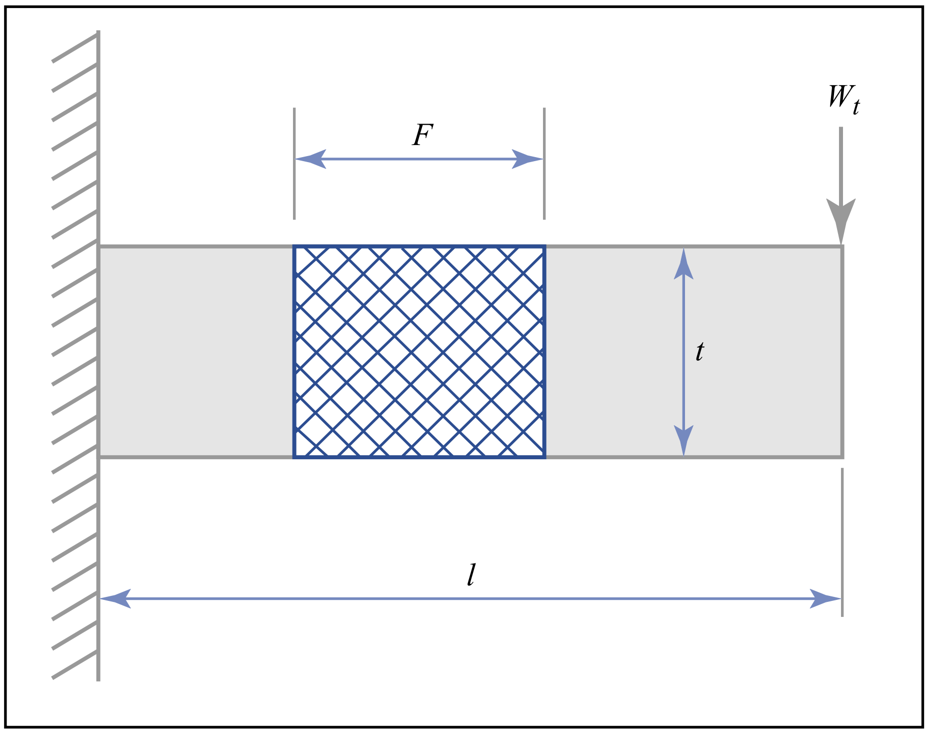


Figure by MIT OpenCourseWare.

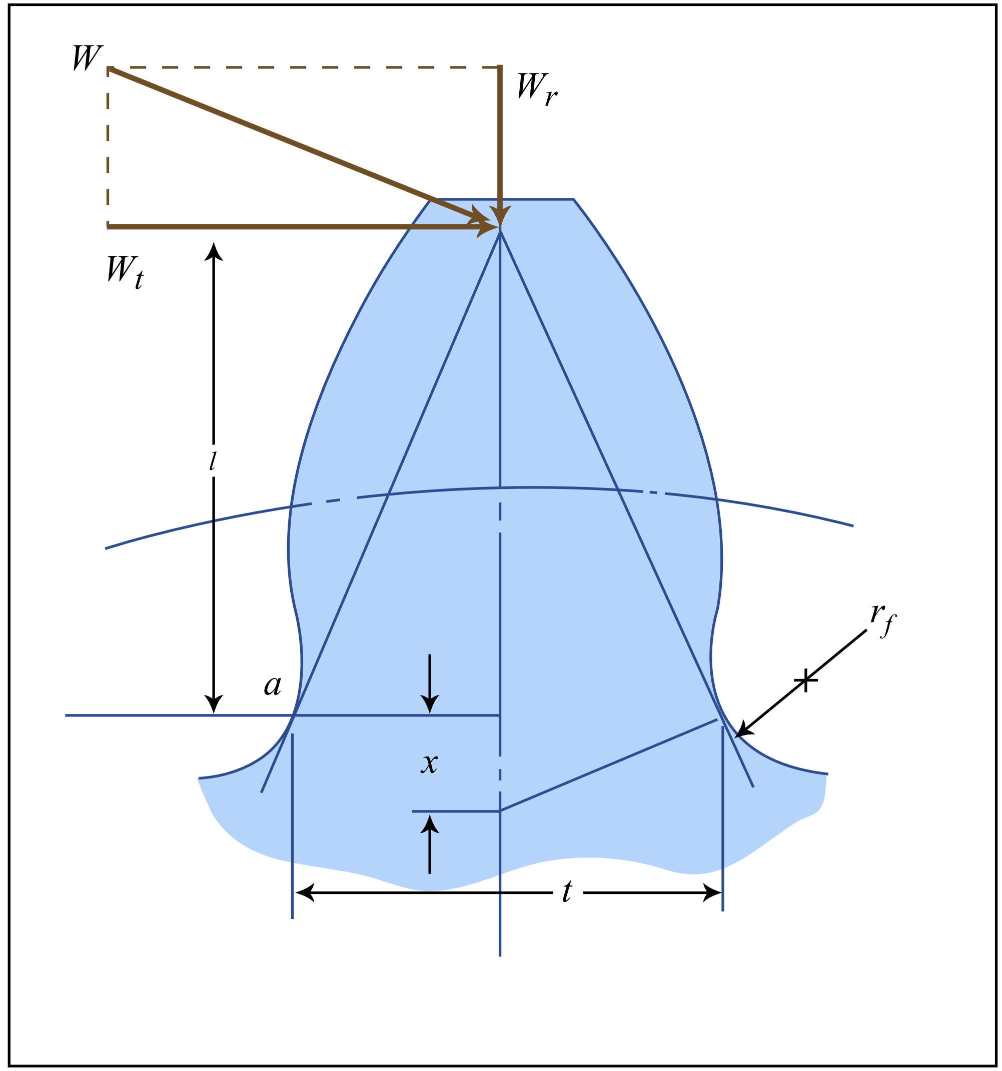
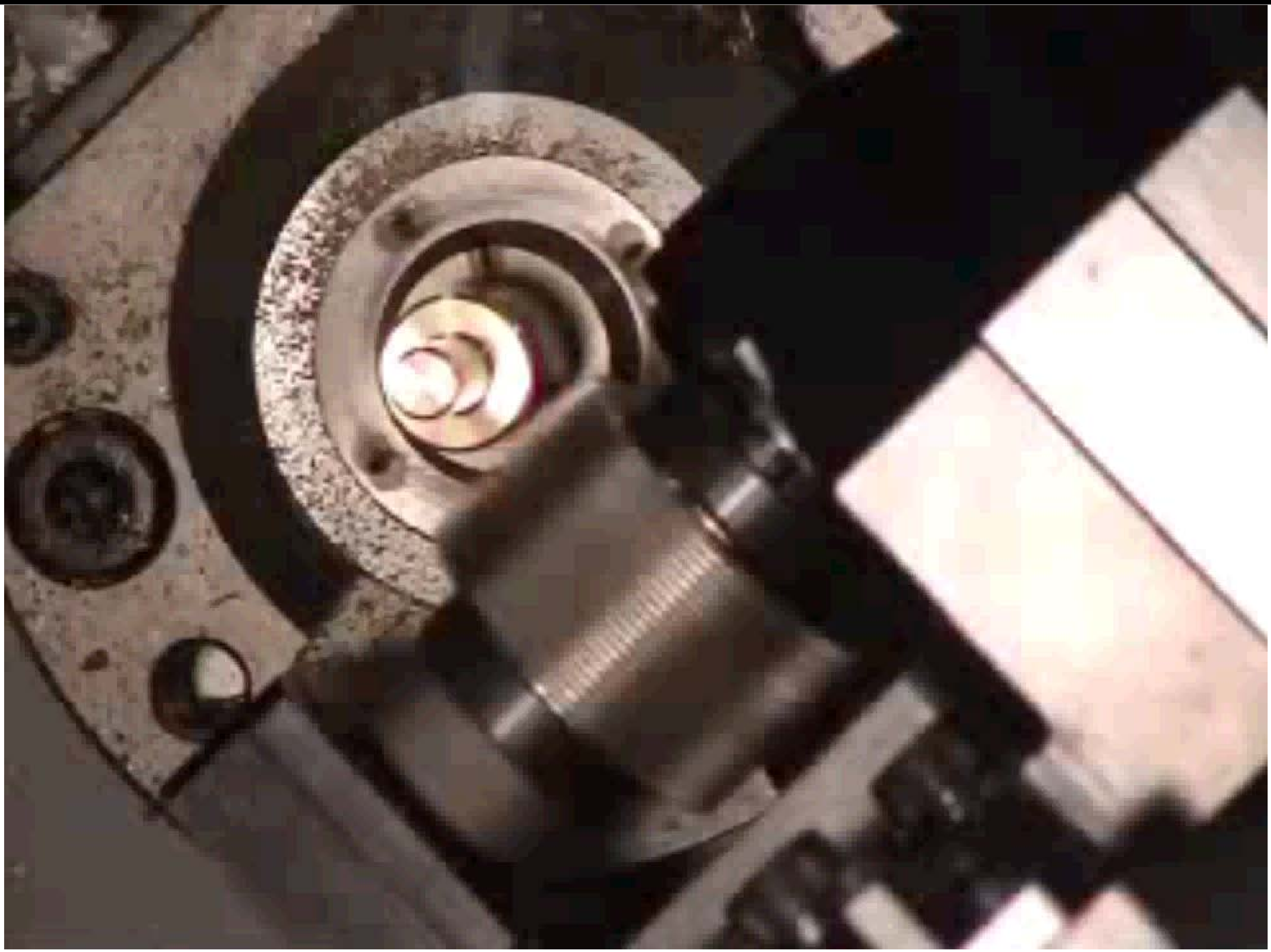


Figure by MIT OpenCourseWare.

Gear manufacturing

Gear manufacturing - Hobbing



Please see digtos. "GEAR HOBBING M20/32 CITIZEN CINCOM." February 17, 2008. YouTube. Accessed October 26, 2009. http://www.youtube.com/watch?v=fR2duvm3lPo

Gear manufacturing - Shaping

Please see rolvon. "Gear Cutting." May 16, 2008. YouTube. Accessed October 26, 2009. http://www.youtube.com/watch?v=xF9CjluRFJ4

Selection vs. design of gears

Why do we care about gear tooth surface finish

- What affects the finish on the gear surfaces?
- How good could it be?
- How much would it cost?

Why do we care about the tooth geometry at the root

- What affects the quality of the fillet at the root?
- How good could it be?
- How much would it cost?

Perspective

Failure modes

- Tooth bending/shear
- Contact failure

Science modeling → Engineering modeling

American Gear Manufacturers Association (AGMA)

- Example
- Single pressure angle
- Full-depth teeth
- Others

Calculating stresses

$$\sigma_{bending} = W_t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \quad (U.S.\ units)$$

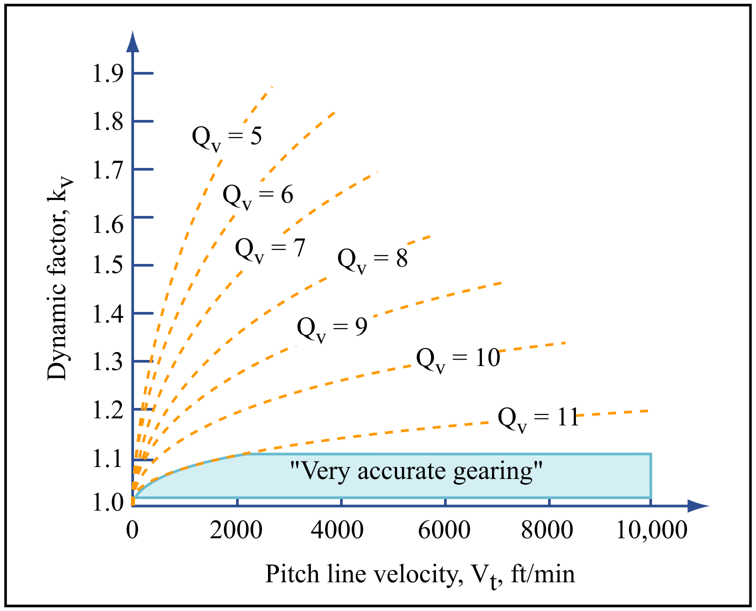


Figure by MIT OpenCourseWare. Adapted from Fig. 14-9 in Shigley & Mischke.

$$\sigma_{contact} = C_p \sqrt{\left(W_t K_o K_v K_s\right) \frac{K_m}{d_p F} \frac{C_f}{I}} \quad (U.S.\ units)$$

Incredibly uninteresting, plug-chug & ‘non-scientific’

Gear failure at the root

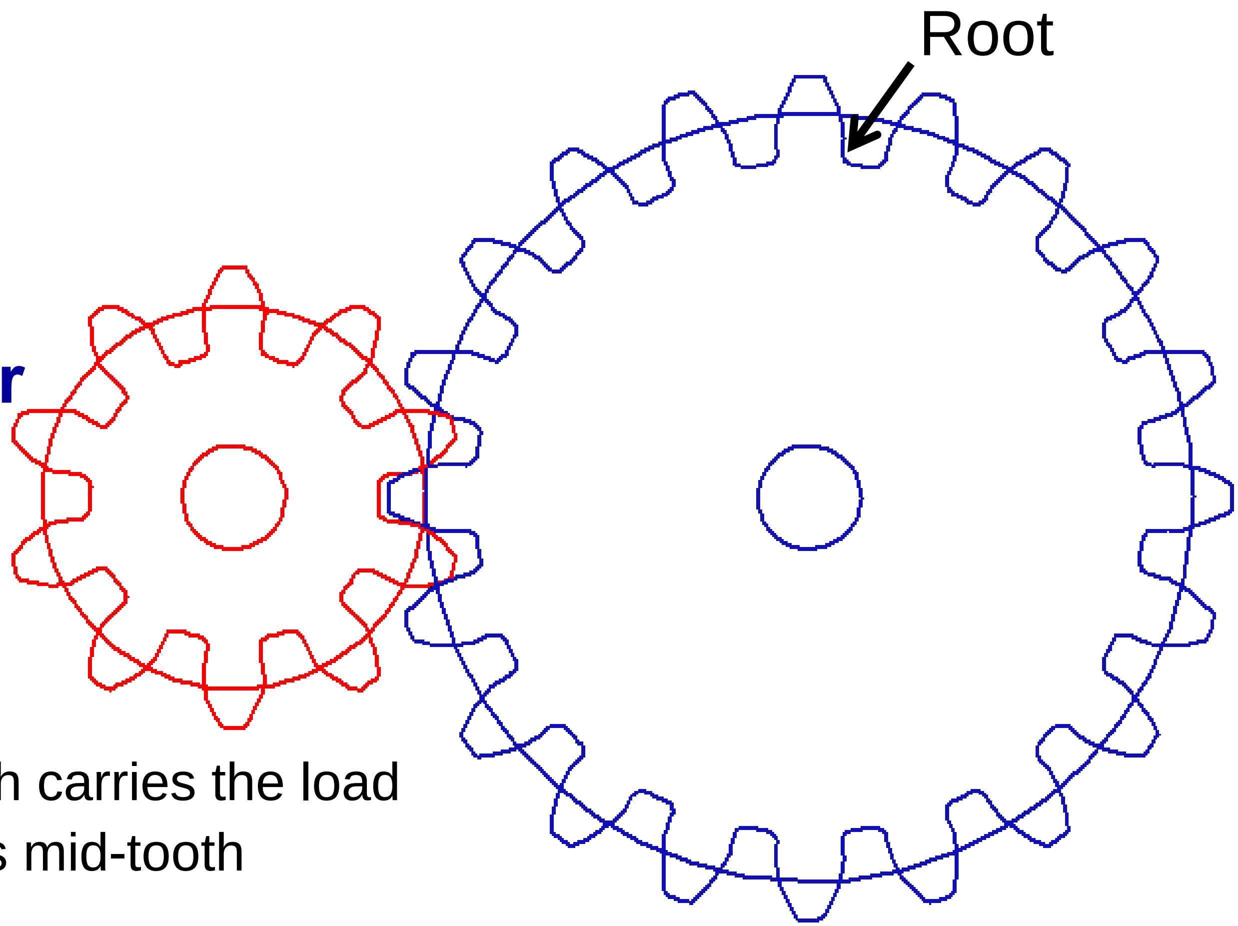
Bending

Basic stress calculation

Stress near the tooth root, model tooth as a cantilever

$$\sigma = \frac{M\,c}{I} \qquad \sigma = \frac{6\,W_t\,L}{F\,t^2}$$

$$\sigma = \frac{W_t\,P}{F\;Y}$$



P: Diametral pitch

Y: Lewis form factor

- ~¼ to ½ for $\phi = 20^o$
- f(# of teeth)

Conservative:

- Implies that one tooth carries the load
- Heaviest load occurs mid-tooth

Basic stress calculation

Stress near the tooth root, model tooth as a cantilever

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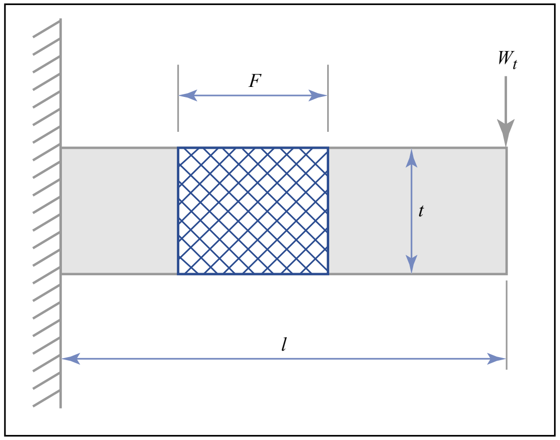

Figure by MIT OpenCourseWare.

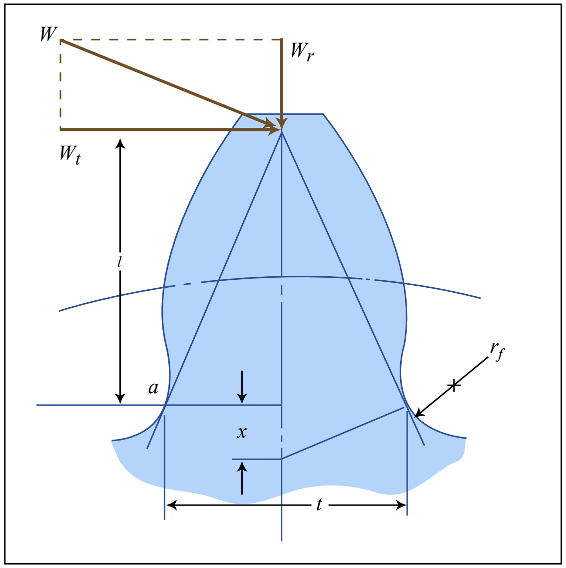

Figure by MIT OpenCourseWare.

P: Diametral pitch

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- ~¼ to ½ for ϕ = 20º
- *f*(# of teeth)

Image removed due to copyright restrictions. Please see http://www.oilanalysis.com/Backup/200101/Gear3.jpg

Conservative:

- Implies that one tooth carries the load
- Heaviest load occurs mid-tooth

Dynamic effects

How to incorporate dynamic effects

- One way of addressing

$$K_v = \left(\frac{a + V^b}{a} \right)^c$$

- V = pitch line velocity
- K_v depends on fab

For rough estimates

$$\sigma = K_V \frac{W_t P}{F Y}$$

- This is for English units, for SI is different

Allowable bending stress

These types of plots are associated with conditions

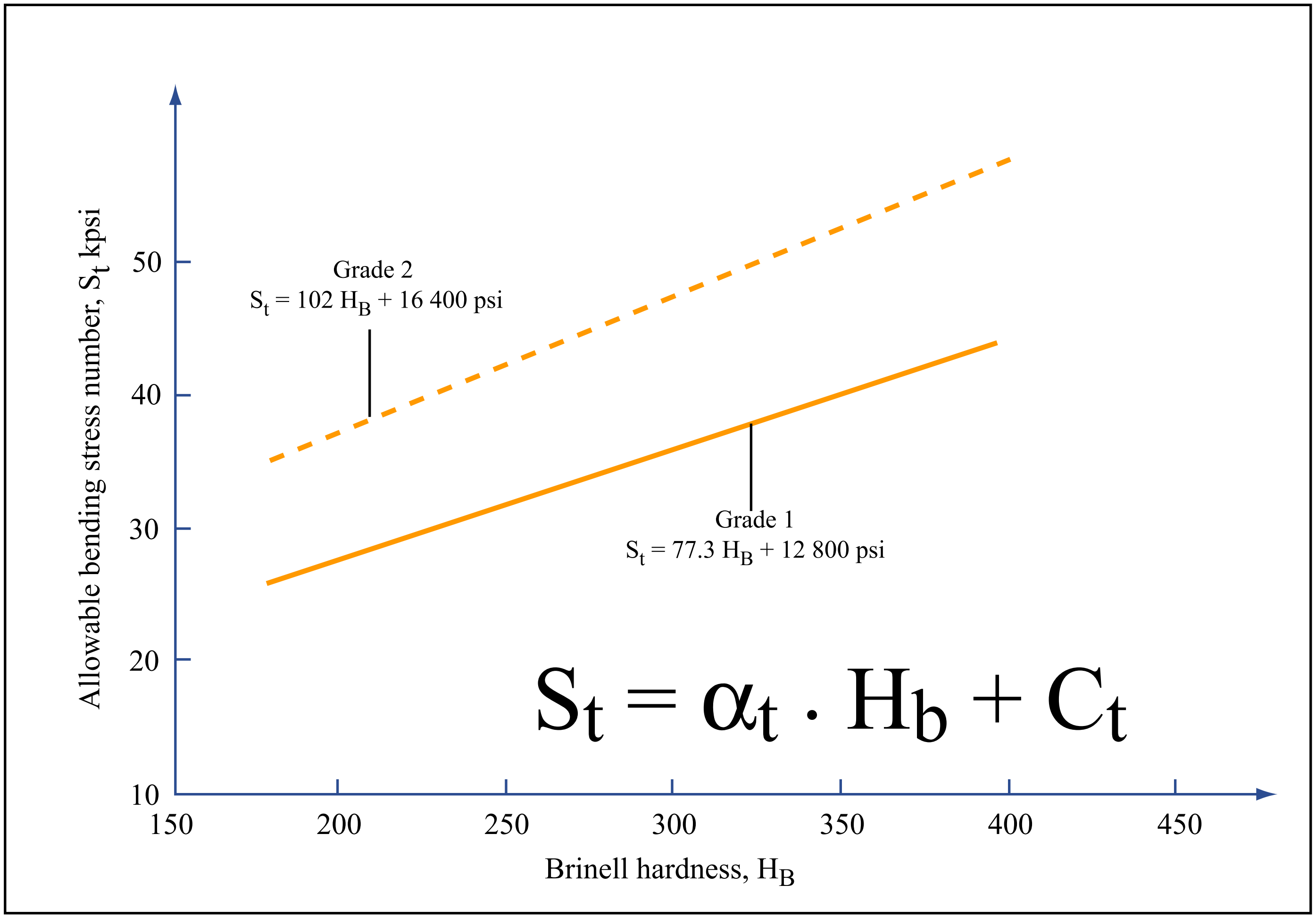

Figure by MIT OpenCourseWare. Adapted from Fig. 14-2 in Shigley & Mischke.

Allowable bending stress

$$\sigma_{all} = \frac{S_t}{S_F}\frac{Y_N}{K_T K_R} \quad (U.S.\ units) \qquad \sigma_{all} = \frac{S_t}{S_F}\frac{Y_N}{Y_\theta Y_Z} \quad (SI\ units)$$

Elements of the equations:

- S_t Allowable bending stress
- Y_N Stress cycle life factor
- K_T Temperature factors
- K_R Reliability factors
- S_F AGMA factor of safety

Allowable stresses for:

- Unidirectional loading
- 10 million stress cycles
- 99 percent reliability

Gear failure
at the surface

Fatigue

High cycle failure: Pitting

Images removed due to copyright restrictions. Please see any photos of surface pitting in gears, such as:

http://2.bp.blogspot.com/_tBh5ORa6LOk/R8UaDR3pgVI/AAAAAAAAAGE/DikmlvWPS84/s1600-h/pitting.gif

http://commons.wikimedia.org/wiki/File:Roue_creuse_03.jpg

Avoiding high cycle failure: Stress variables

Equivalent modulus

$$E_e = \frac{1}{\frac{1-v_1^{\;2}}{E_1} + \frac{1-v_{21}^{\;2}}{E_2}}$$

$$\upsilon = 0.333$$

Half contact width

$$b = \frac{2 W_t \, d_1 \, d_2}{\pi \, L \, E_e \left(d_1 + d_2\right)}$$

Watch out! The book switches meaning of F here…

Maximum contact pressure

$$q = \frac{2 W_t}{\pi \, b L}$$

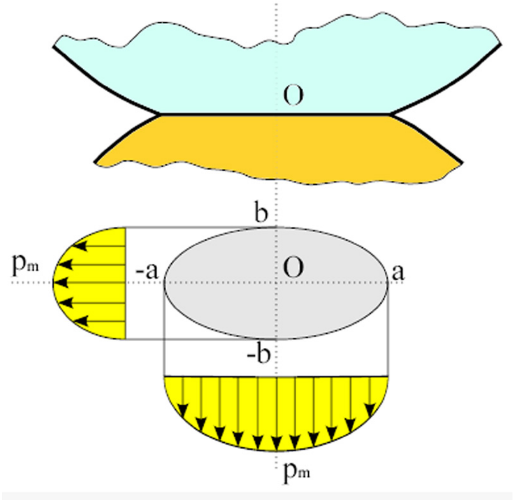

Image from Wikimedia Commons, http://commons.wikimedia.org

Allowable contact stress [ANSI/AGMA 2001-D04 and 2101-D04]

$$S_c = \alpha_c \cdot H_b + C_c$$

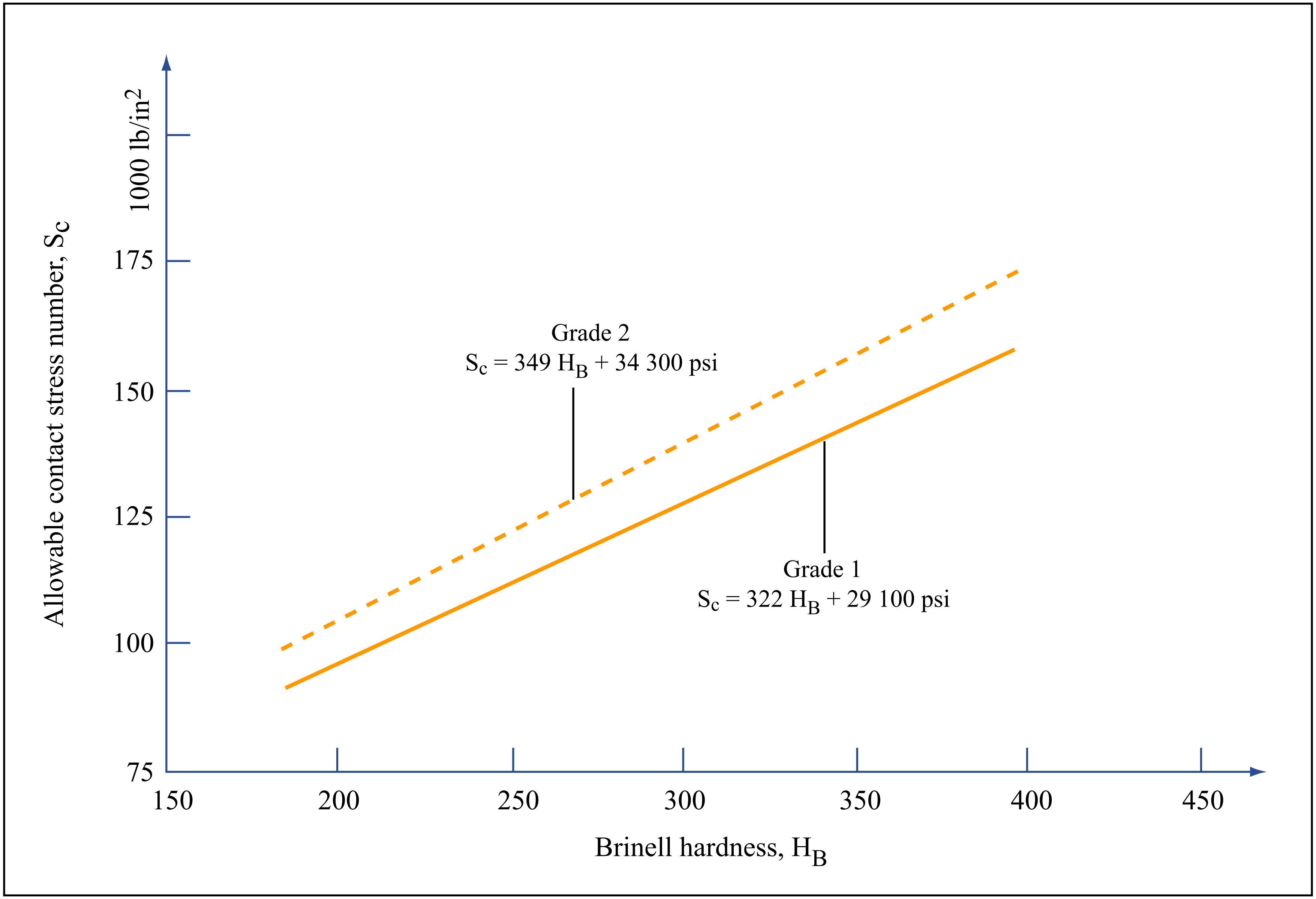

Figure by MIT OpenCourseWare. Adapted from Fig. 14-5 in Shigley & Mischke.

Allowable contact stress

$$\sigma_{c,all} = \frac{S_C}{S_H}\frac{Z_N C_H}{K_T K_R} \quad (U.S.\ units) \qquad \sigma_{c,all} = \frac{S_C}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z} \quad (SI\ units)$$

Elements of the equations:

- S_C Allowable contact stress
- Z_N Stress cycle life factor
- C_H Hardness ratio factors for pitting resistance
- K_T Temperature factors
- K_R Reliability factors
- S_H AGMA factor of safety

Allowable stresses for:

- Unidirectional loading
- 10 million stress cycles
- 99 percent reliability

Exercise

Gears

General machine design

Activity: Refit lathes for the mfg. shop

Study the lathes in the shop…

1. What types of failures do we have?

2. Calcs/sims/tests need to augment Shigley/Mischke:

- Gearing
- Belts
- Friction elements

3. Worst case consequence of these kinds of failures:

- Pitting failure
- Tool break/failure
- Fatigue

Exercise
Windmill gear boxes

Windmill failures: Catastrophic

Please see mrturbodk. "windmill failure." February 28, 2008. YouTube. Accessed October 26, 2009.

http://www.youtube.com/watch?v=TmM3KQnFmXs

Wind energy overview: Lakawona

Images removed due to copyright restrictions. Please see

http://i124.photobucket.com/albums/p7/NBBooks/WTGTurbinesGettingLargerSM.jpg

http://i124.photobucket.com/albums/p7/NBBooks/WTGUSWindResources.jpg

Images removed due to copyright restrictions. Please see p. 19 in

http://www.clipperwind.com/pdf/liberty_brochure.pdf

Activity: Extra credit - As a group

You are tasked to build a windfarm off Cape Cod

1. **Shigley/Mischke is not perfectly suited to cover gear needs in this application. Why/how?**

2. **What calculations/simulations/tests would you do to augment Shigley/Mischke?**

3. **What happens if you have pitting failure and what would you do about it?**

4. **What happens if you have failure at a tooth root and what would you do about it?**