2.72 Elements of Mechanical Design
Spring 2009

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Elements of Mechanical Design

Lecture 16: Dynamics and damping

Schedule and reading assignment

Quiz

- None

Topics

- Vibration physics
- Connection to real world
- Activity

Reading assignment

- Skim last gear reading assignment… (gear selection)

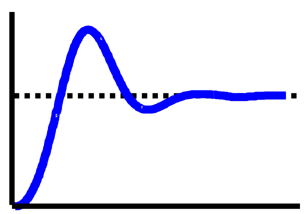
Resonance

Basic Physics

- Exchange potential-kinetic energy
- Energy transfer with loss

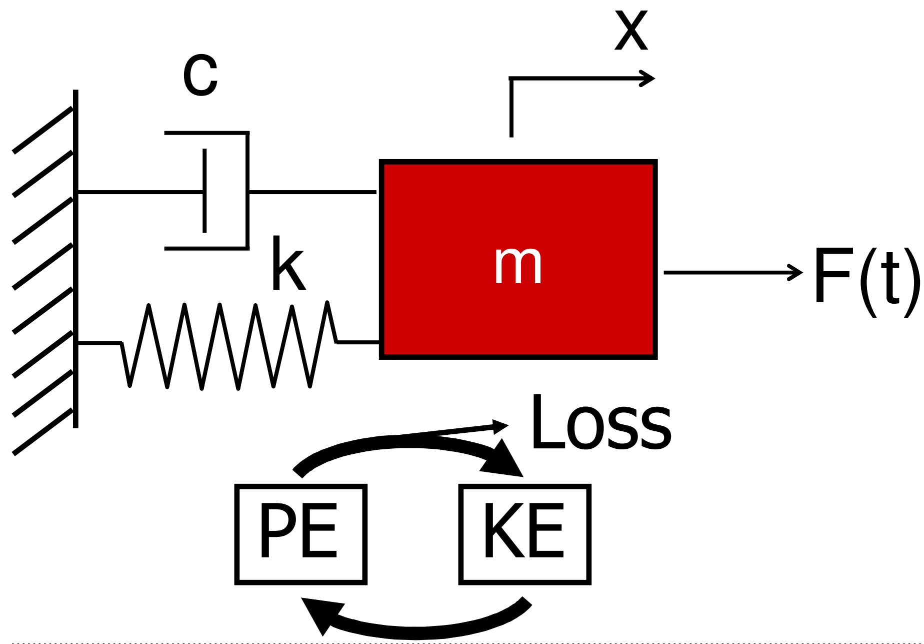
Modeling

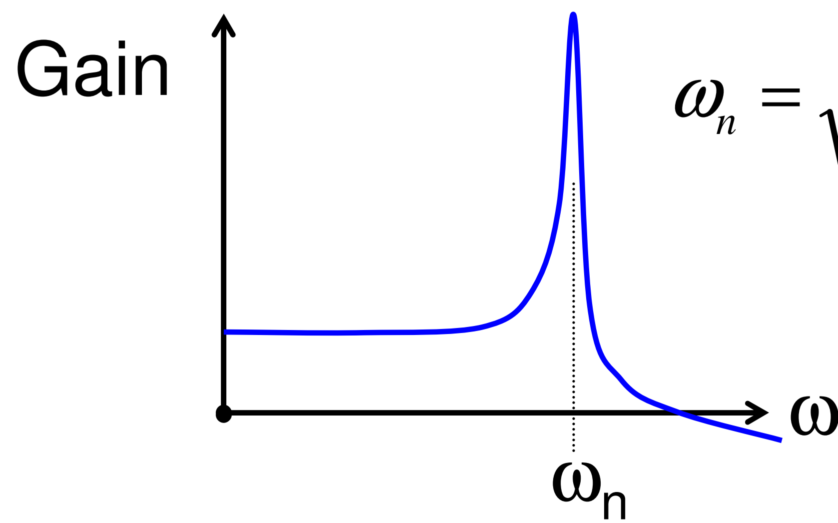
- 2nd order system model
- Spring mass damper
- Differential equations, Laplace Transforms



Why Do We Care

- Critical to understanding motion of structures- desired and undesired
- Generally not steady state
- Location error
- Large forces, high fatigue





$$\omega_n = \sqrt{\frac{k}{m}}$$

Vibrations - Input

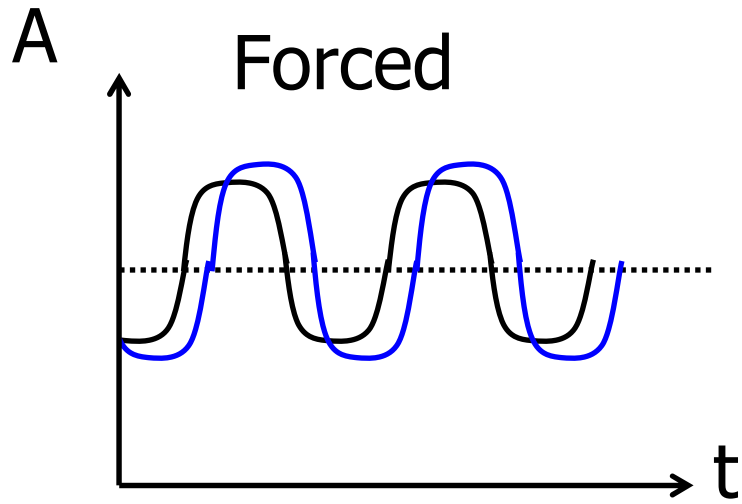
Oscillation of System

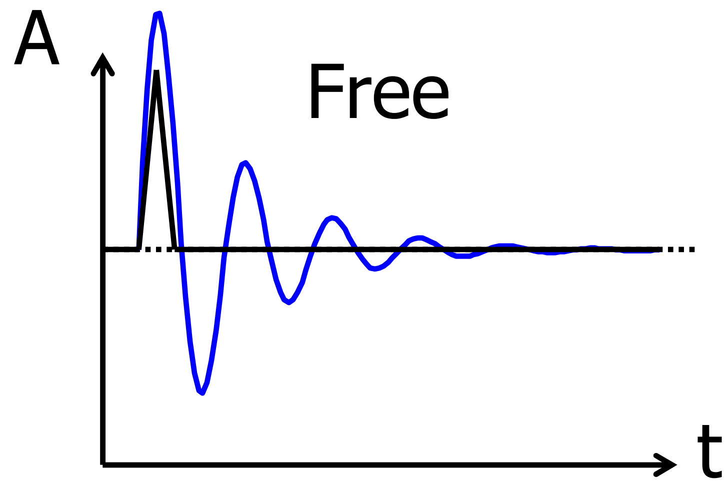
Why Categorize

- Different causes
- Different solutions

Input Form

- Forced – Steady State
  - *Command Signal*
  - *Electrical (60 Hz)*
- Free – Transient
  - *Impacts*

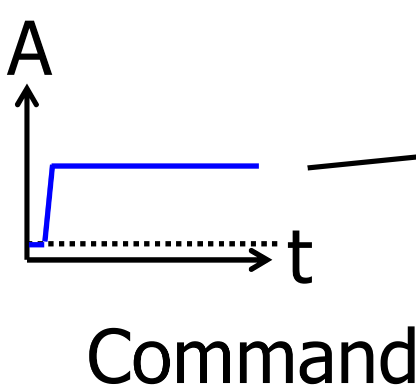


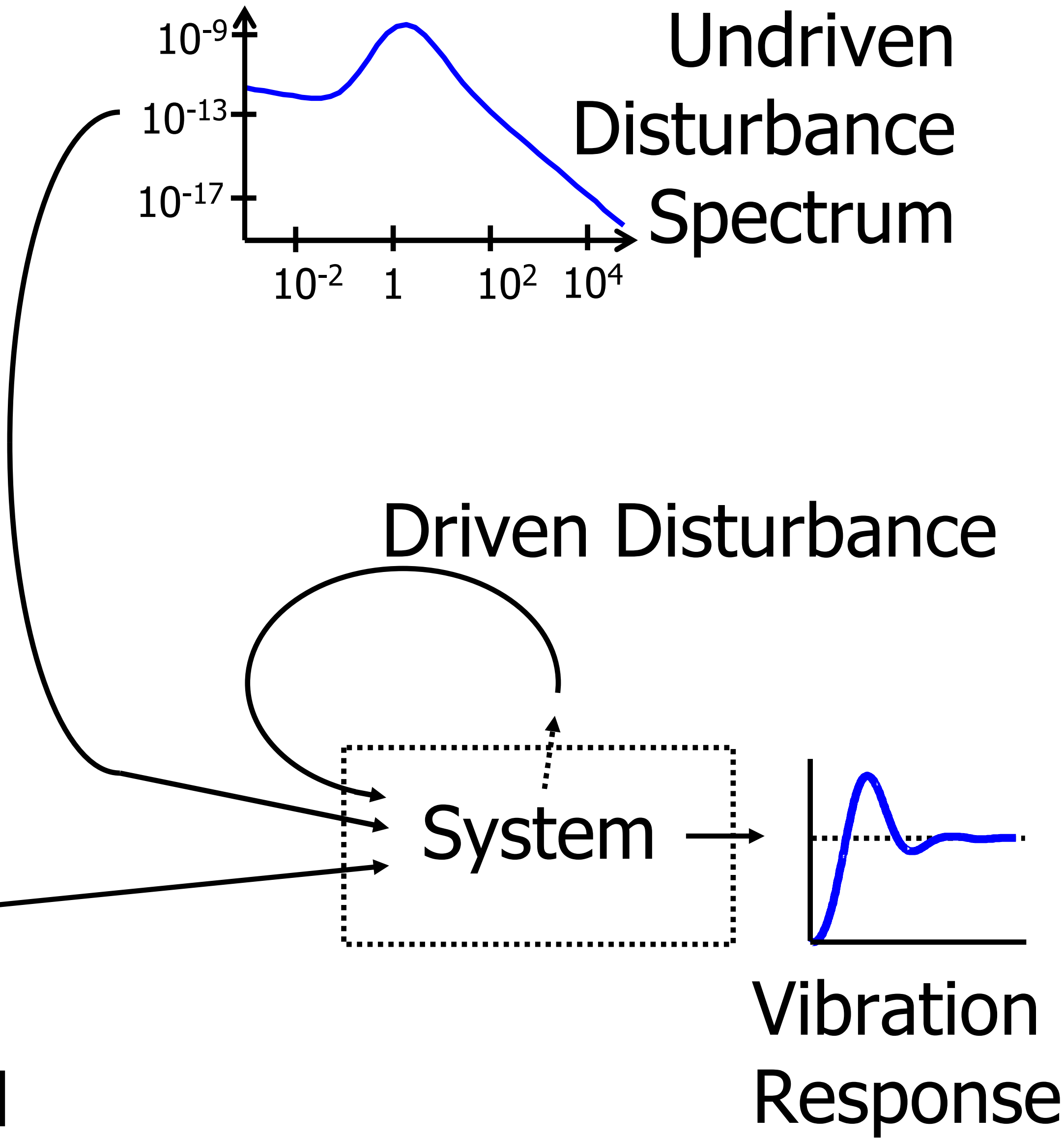


Vibrations - Source

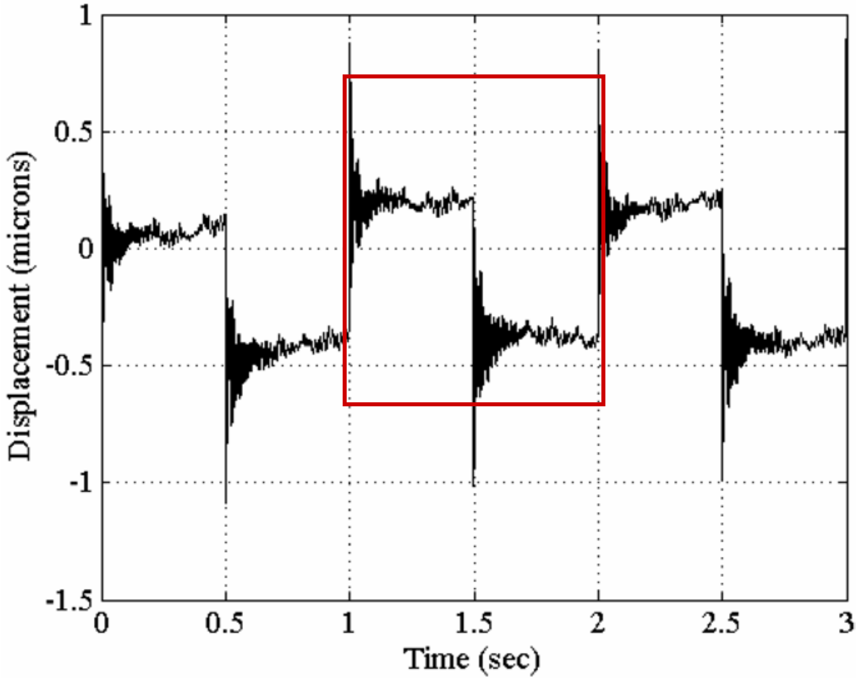
Source

- Disturbance
  - *Driven*
    - Device Motors
    - Rotating Components
  - *Undriven*
    - Electrical (60 Hz)
    - People (2 Hz)
    - Cars (10 Hz)
    - Nearby Equipment
- Command
  - *Step Response*

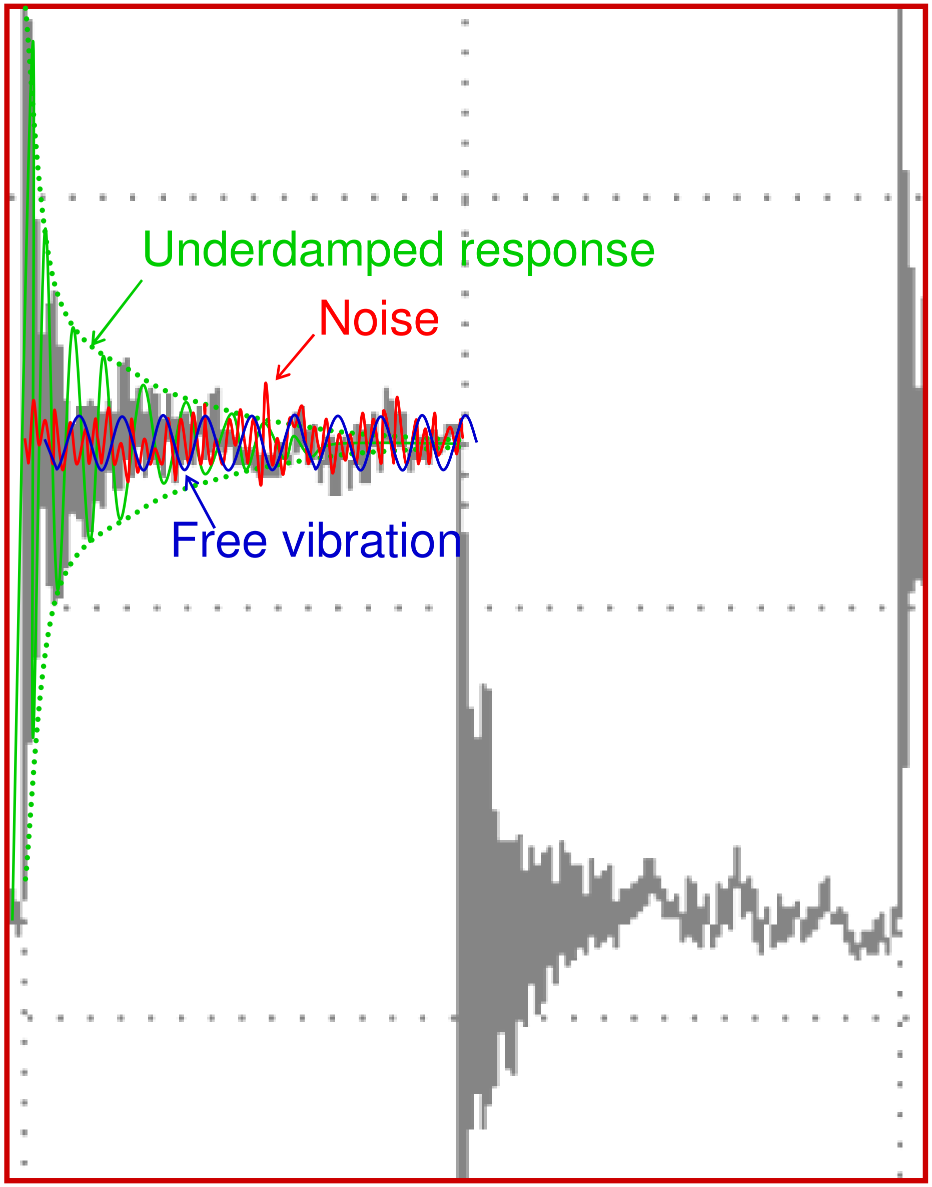




Vibrations: Command vs. Disturbance



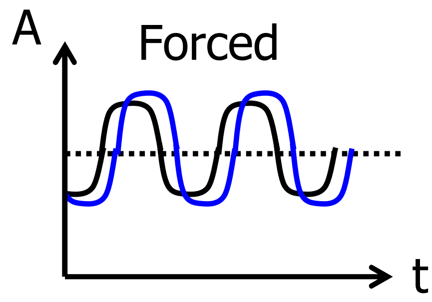
Golda, D. S., "Design of High-Speed, Meso-Scale Nanopositioners Driven by Electromagnetic Actuators," Ph.D. Thesis, Massachusetts Institute of Technology, ***2008****.*

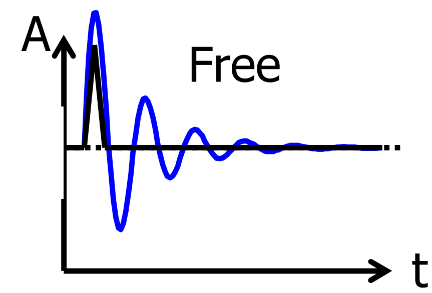


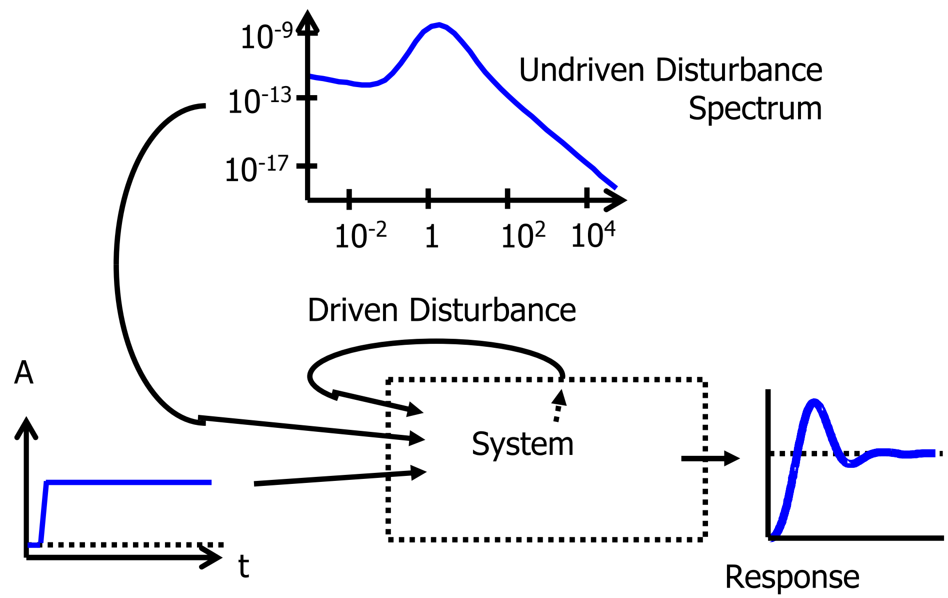
Vibrations - Example

Example

- Chinook
- Identify:
  - *Mode*
  - *Form*
  - *Source*
  - *Response*







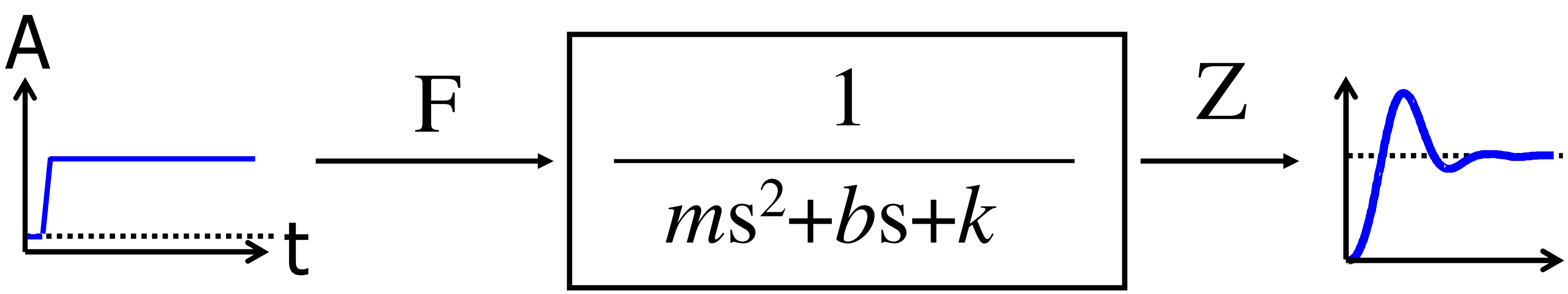
Attenuating Vibrations

Change System

- Mass, stiffness, **damping**
- Adjust mode shapes

Change Inputs

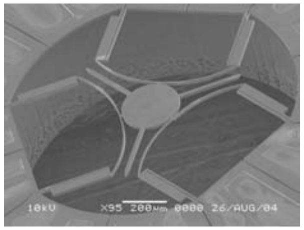
- Command: Input Modulation, Feedback
- Disturbance, reduce:
  - *undriven vibrations, e.g. optical table*
  - *driven vibrations – alter device structure (damping on motors, etc.)*

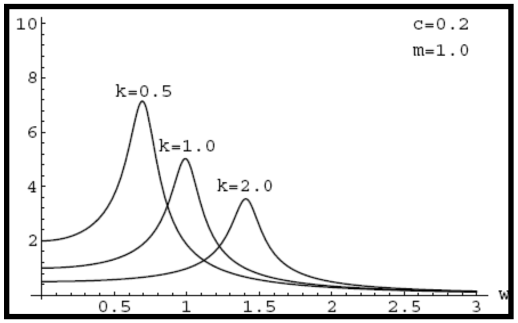


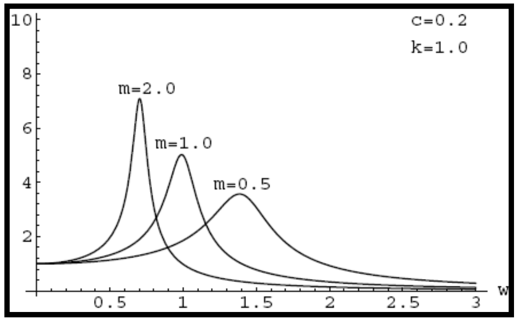
Modulating Command Vibrations

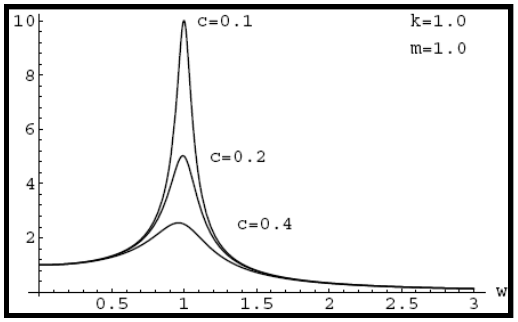
Change
m, k, c

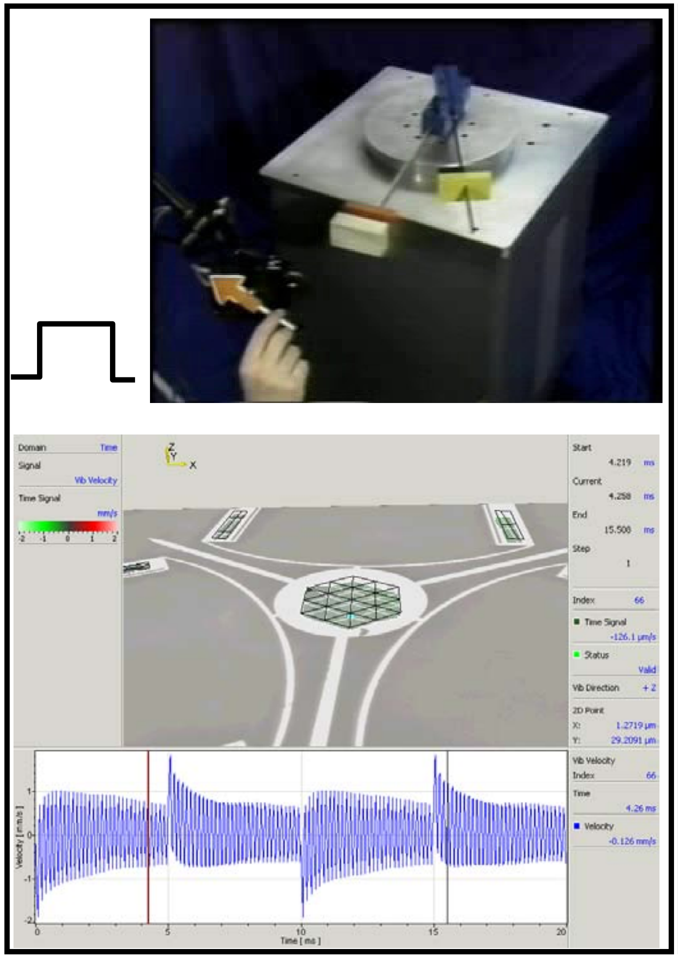
Input shaping

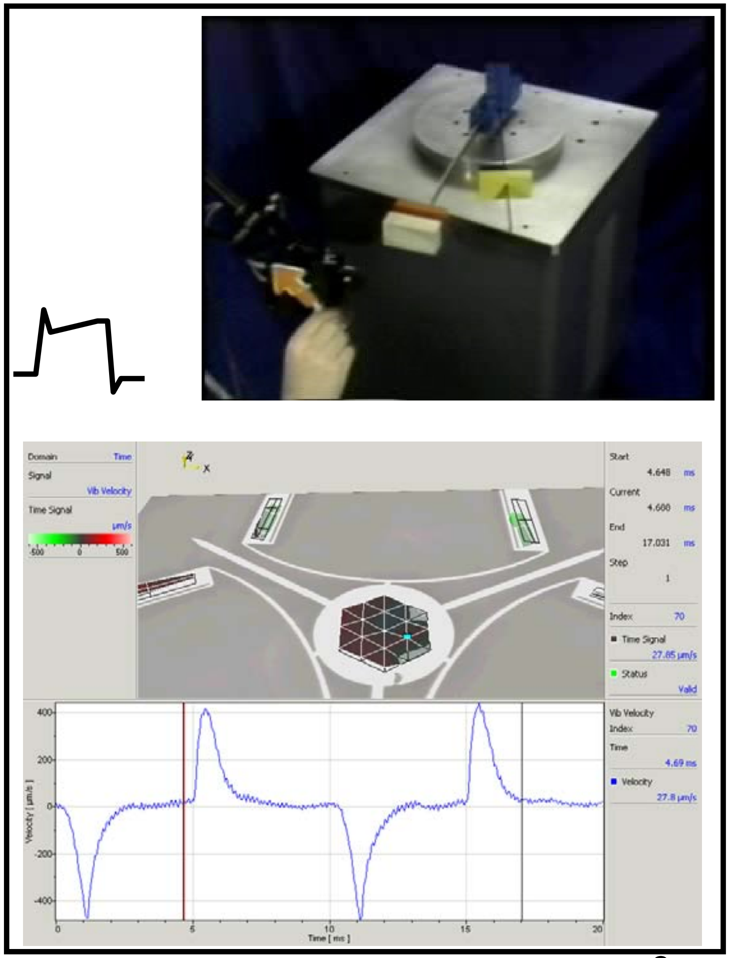












Behavior

Regimes

- (1) Low Frequency ($\omega<\omega_n$)
- (2) Resonance ($\omega\approx\omega_n$)
- (3) High Frequency ($\omega>\omega_n$)
- Example – Spring/Mass demo

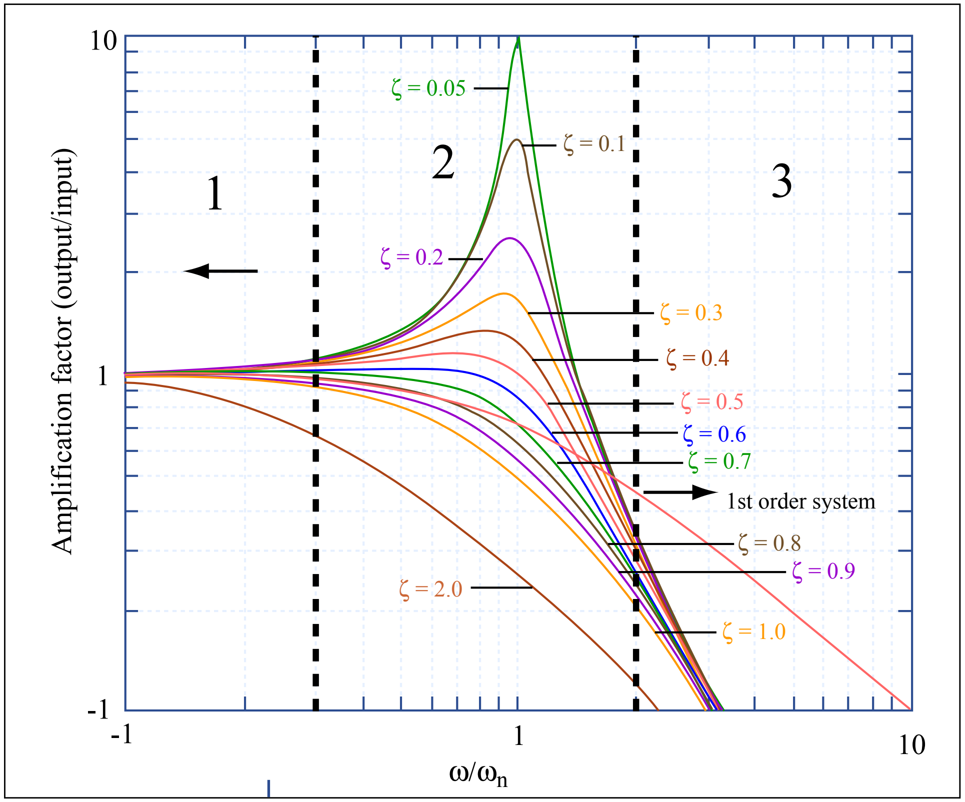


Figure by MIT OpenCourseWare.

$$\frac{x}{F}=\frac{1}{ms^2+bs+k}=\frac{1}{k(\omega)}$$

Behavior – Low Frequency

Frequency Response Regimes

- (1) Low Frequency ($\omega<\omega_n$)
- (2) Resonance ($\omega\approx\omega_n$)
- (3) High Frequency ($\omega>\omega_n$)
- Example – Spring/Mass demo

Low Frequency ($\omega<\omega_n$)

- System tracks commands
- Ideal operating range
- High disturbance rejection

$$\frac{x}{F}\approx\frac{1}{k}$$

Resonance Frequency ($\omega\approx\omega_n$)

- System Response >> command
- $k_{eff}\downarrow$
- Disturbances will cause very large response
- Quality factor = magnitude of peak
- Damping ↑ = Q ↓

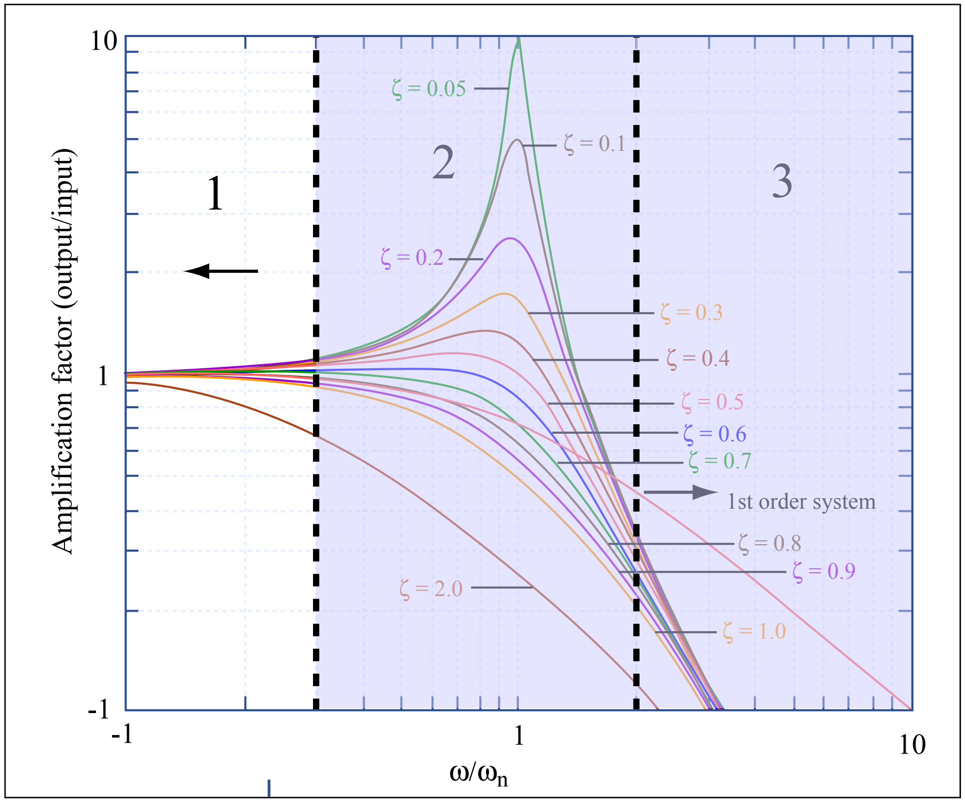


Figure by MIT OpenCourseWare.

$$\frac{x}{F}=\frac{1}{ms^2+bs+k}$$

High Frequency ($\omega\approx\omega_n$)

- System Response << command
- High disturbance rejection

Behavior - Resonance

Frequency Response Regimes

- (1) Low Frequency ($\omega<\omega_n$)
- (2) Resonance ($\omega\approx\omega_n$)
- (3) High Frequency ($\omega>\omega_n$)
- Example – Spring/Mass demo

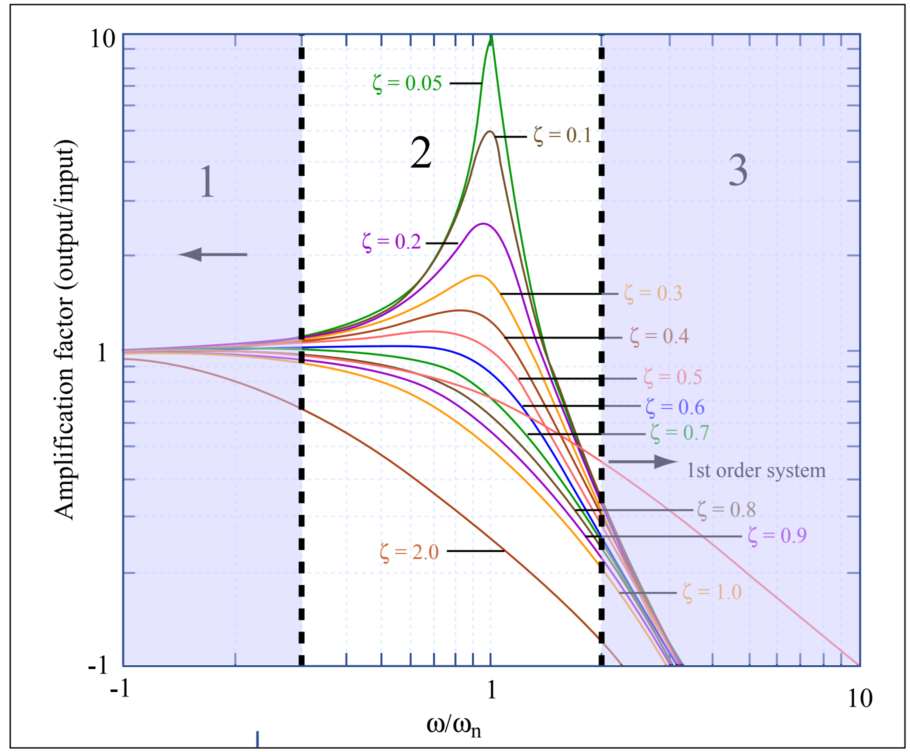


Figure by MIT OpenCourseWare.

$$\frac{x}{F} = \frac{1}{ms^2 + bs + k}$$

Low Frequency ($\omega<\omega_n$)

- System tracks commands
- Ideal operating range
- High disturbance rejection

Resonance Frequency ($\omega\approx\omega_n$) $\frac{1}{k} < \frac{x}{F} \le \frac{Q}{k}$

- System Response >> command
- k_{eff} ↓
- Disturbances will cause very large response
- Quality factor = magnitude of peak
- Damping ↑ = Q ↓

High Frequency ($\omega\approx\omega_n$)

- System Response << command
- High disturbance rejection

Behavior – High Frequency

Frequency Response Regimes

- (1) Low Frequency ($\omega<\omega_n$)
- (2) Resonance ($\omega\approx\omega_n$)
- (3) High Frequency ($\omega>\omega_n$)
- Example – Spring/Mass demo

Low Frequency ($\omega<\omega_n$)

- System tracks commands
- Ideal operating range
- High disturbance rejection

Resonance Frequency ($\omega\approx\omega_n$)

- System Response >> command
- $k_{eff}\downarrow$
- Disturbances will cause very large response
- Quality factor = magnitude of peak
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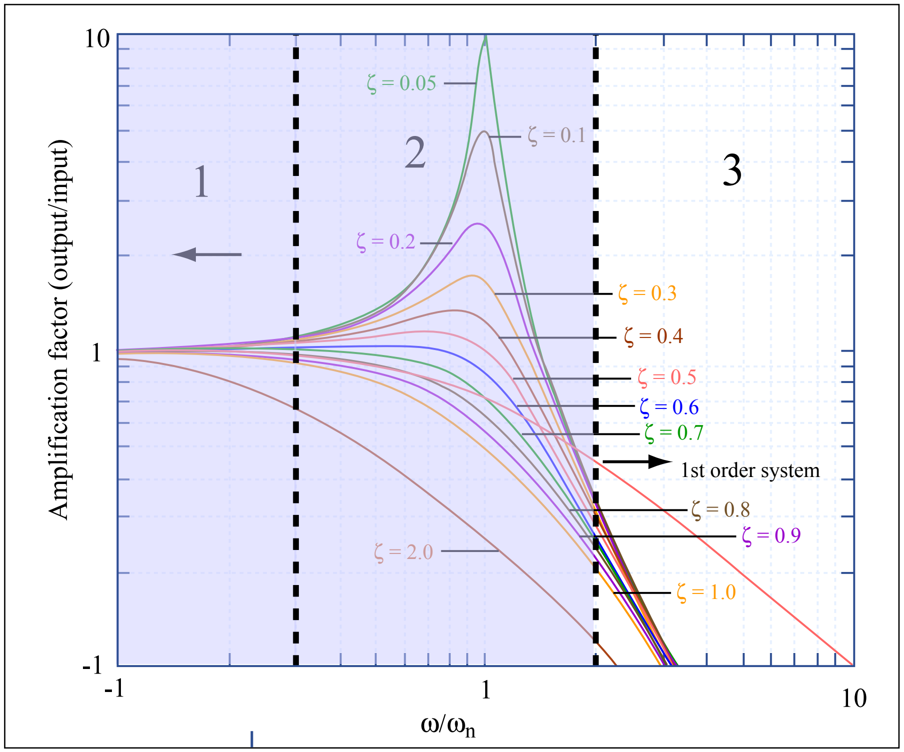


Figure by MIT OpenCourseWare.

$$\frac{x}{F}=\frac{1}{ms^2+bs+k}$$

High Frequency ($\omega\approx\omega_n$)

- System Response << command
- Poor disturbance rejection

$$\frac{x}{F}\approx\frac{1}{m\omega^2}$$

Constitutive Relations

Relevant equations

- Damping ratio → $\zeta = \dfrac{c}{2\sqrt{km}}$
- ω_n → $\omega_n = \sqrt{\dfrac{k}{m}}$ $\quad \omega_d = \omega_n\sqrt{1-\zeta^2}$
- Gain → $G_P = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$ $\quad \zeta < \dfrac{1}{\sqrt{2}}$
- Quality factor → $Q = \dfrac{1}{2\zeta\left(1-\zeta^2\right)}$

Application of Theory

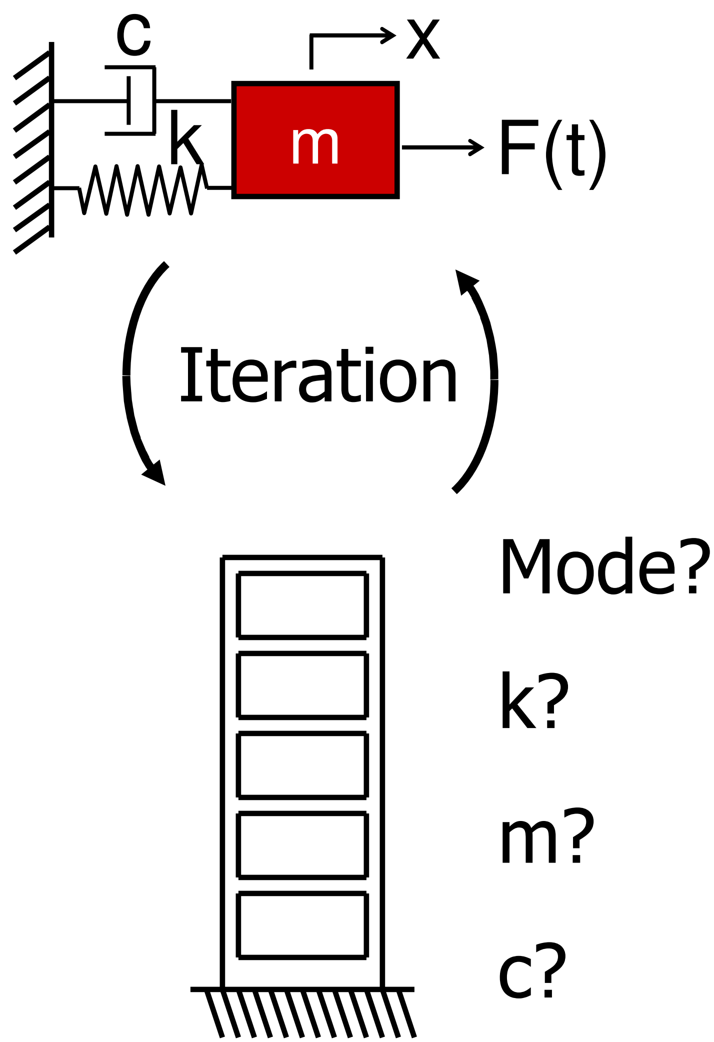
Relate Variables to Actual Parameters

- Vibrational Mode
- Mass
- Stiffness
- Damping

Transfer between Model and Reality

- Iterative
- Start simple (1 mass, 1 spring)
- Add complexity
- Limits

Example – building (video)



Strategies for damping

Material

- Grain boundary
- Internal lattice
- Viscoelastic (elastomers/goo)

Pros and cons of each

Viscous

- Air
- Fluid

Electromagnetic

Friction

Active

Combinations

- Sponge

Example: Couette flow relationships

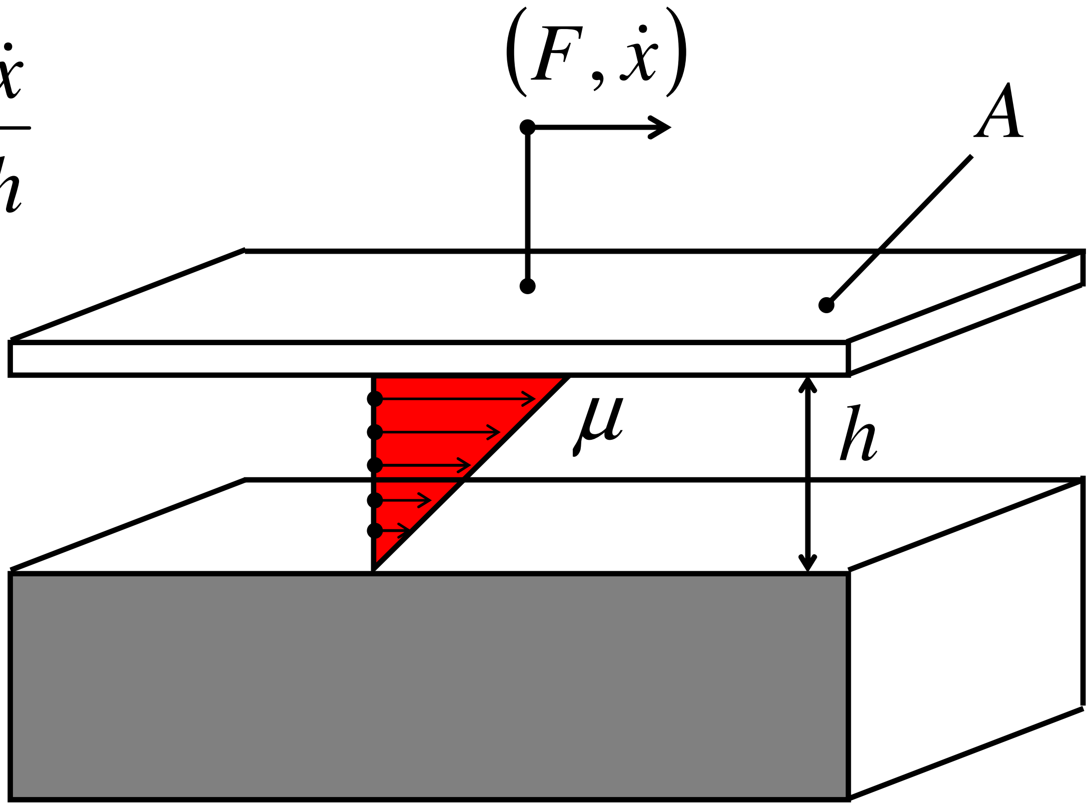
Relevant equations

$$\tau = \mu \frac{d\dot{x}}{dy}$$

$$F = \tau A = A\mu \frac{d\dot{x}}{dy} = A\mu \frac{\dot{x}}{h}$$

$$F = c\dot{x}$$

$$c = \frac{A\mu}{h}$$



Exercise (see next page too)

Perform a frequency analysis of the part

- Develop & prove (FEA) how to increase nat. freq. via geometry change
- Any constraints you might have? Geometry changes can't be unbounded
- Explain effect of your change on vibration amplitude (relative to outer base) at given ω, via sketches & plots

Xtra credit, assume:

- Flexure is contained between two parallel plates (on top and bottom)
- Viscous air damping in the gaps on both sides
- 1 micron gap between the flexure sides and plates
- Elaborate on how well flexure is damped (don't just use intuition)

Useful equations (c = damping coefficient, k = stiffness, m = mass)

Damping ratio

$$\zeta = \frac{c}{2\sqrt{km}}$$

Frequency at peak with max gain

$$\omega_p = \omega_n \sqrt{1-2\zeta^2}$$

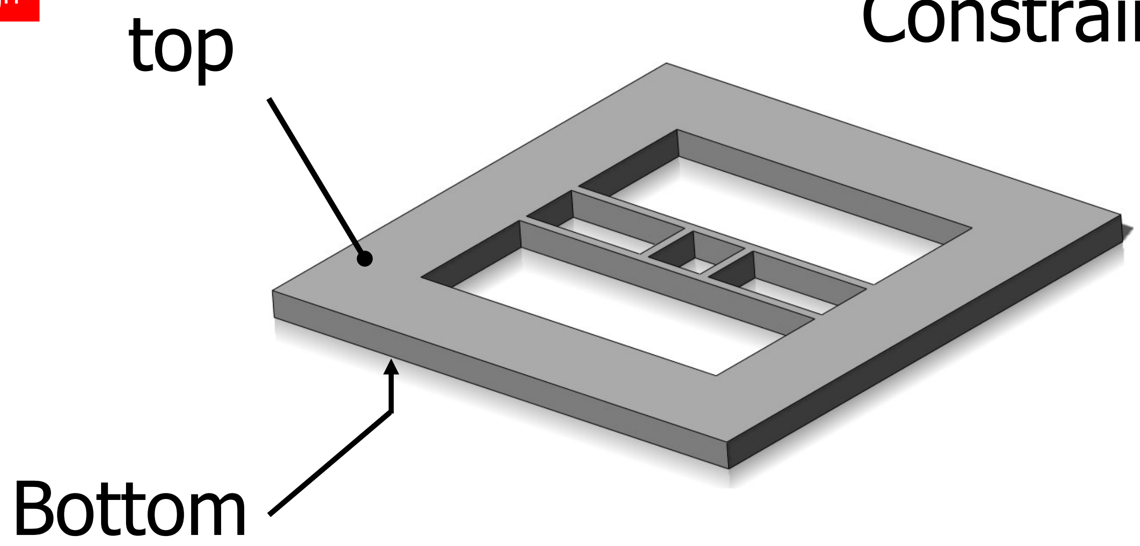
Gain at peak amplitude

$$G\Big|_p = \frac{1}{\sqrt{4\zeta^2 - 4\zeta^4}}$$

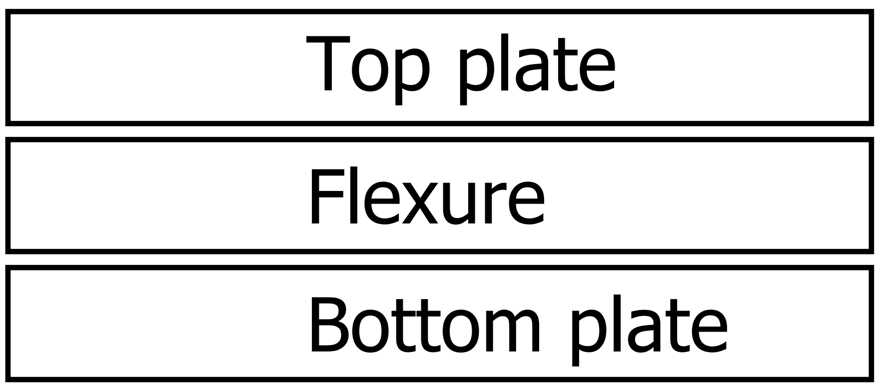
Flexure

Flexure design

Constrained on 4 sides



See this diagram for extra credit:



Multiple Resonances