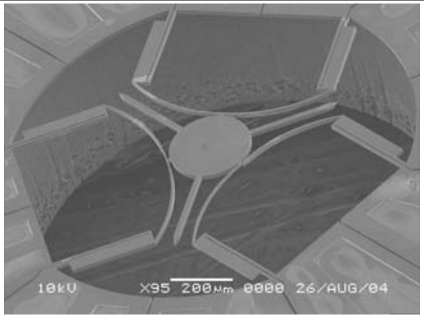
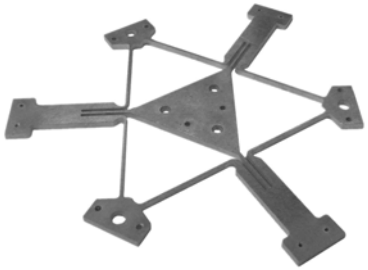
2.76 / 2.760 Lecture 3: Large scale

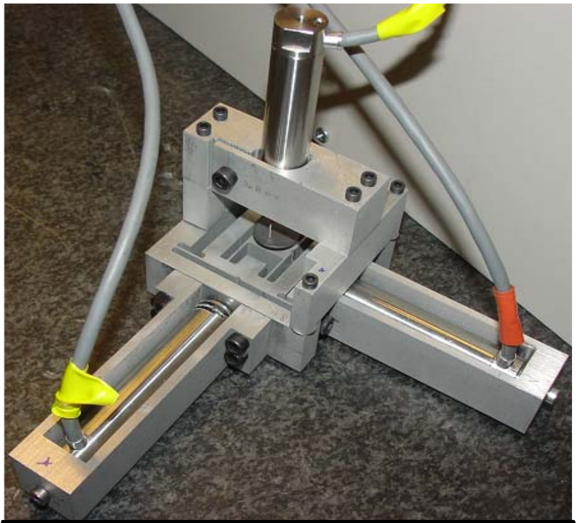


Big-small intuition

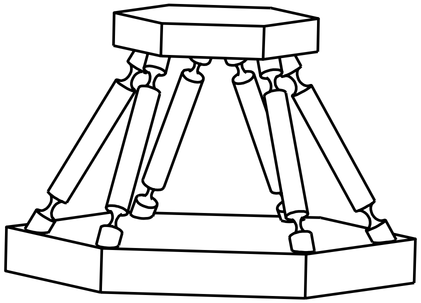


System modeling

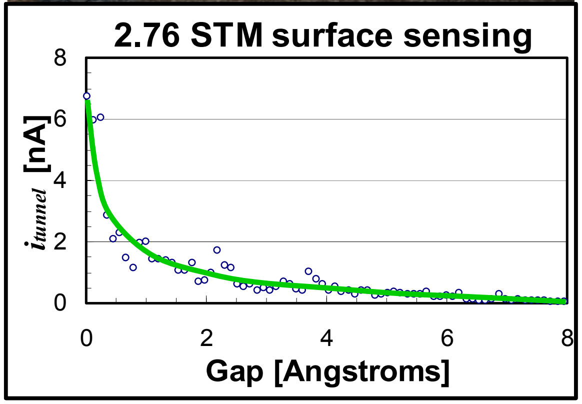
Big history



Big problems

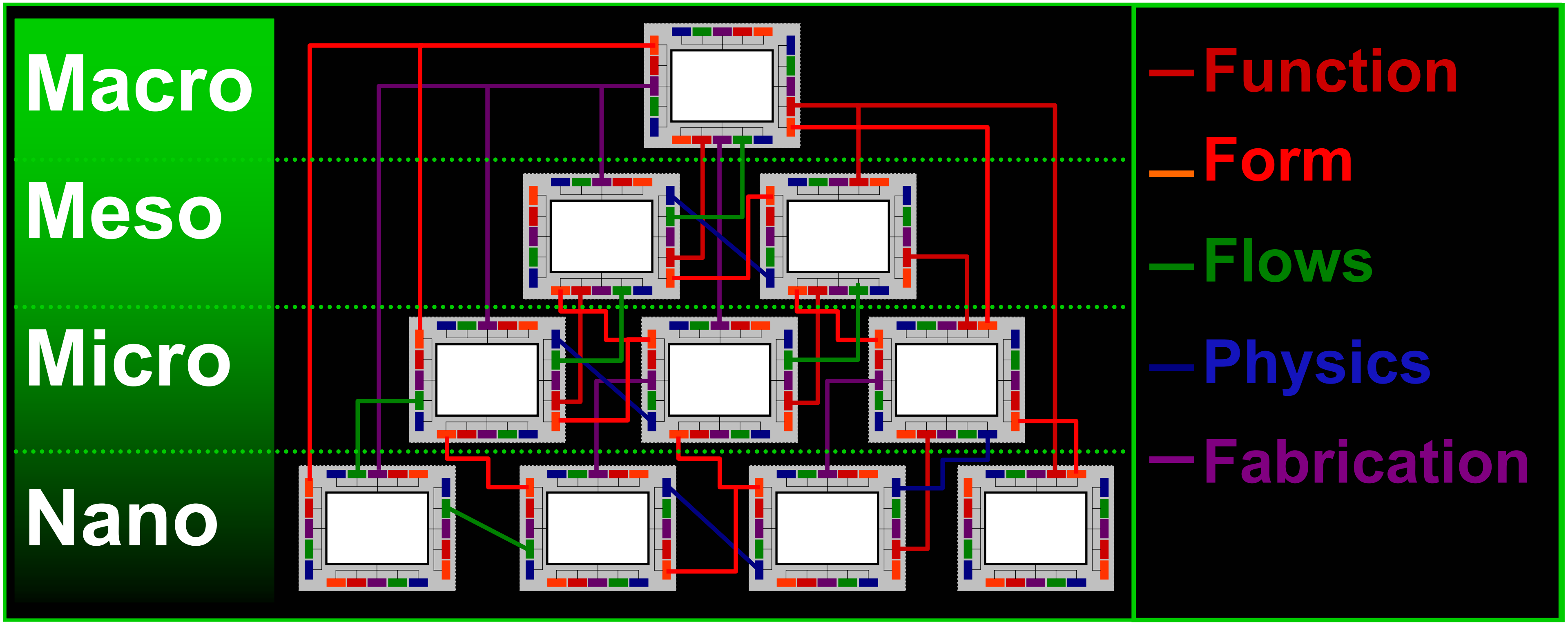


Flexures



Design experiment

Cross-scale coupling



Function	Form	Flow	Physics	Fabrication
What	Geometry	Mass	Application	Compatibility
Who	Motion	Momentum	Modeling	Quality
Why	Interfaces	Energy	Limiting	Rate
Where	Constraints	Information	Dominant	Cost
Etc…	Etc…	Etc…	Etc…	Etc…

Short experiment

(1) What cross-scale incompatibilities (5Fs) do you notice?

(2) What obstacles must be overcome to enable interaction between large/small?

Time Limit: 5 minutes
Email results to me when time is called
Bulleted points please

Discussion

What was the nature of the trouble?

Comment on

- ❑ Strain
- ❑ Control/sensing
- ❑ Momentum
- ❑ Noise

What does this tell you about sensitivity and resolution / discretization?

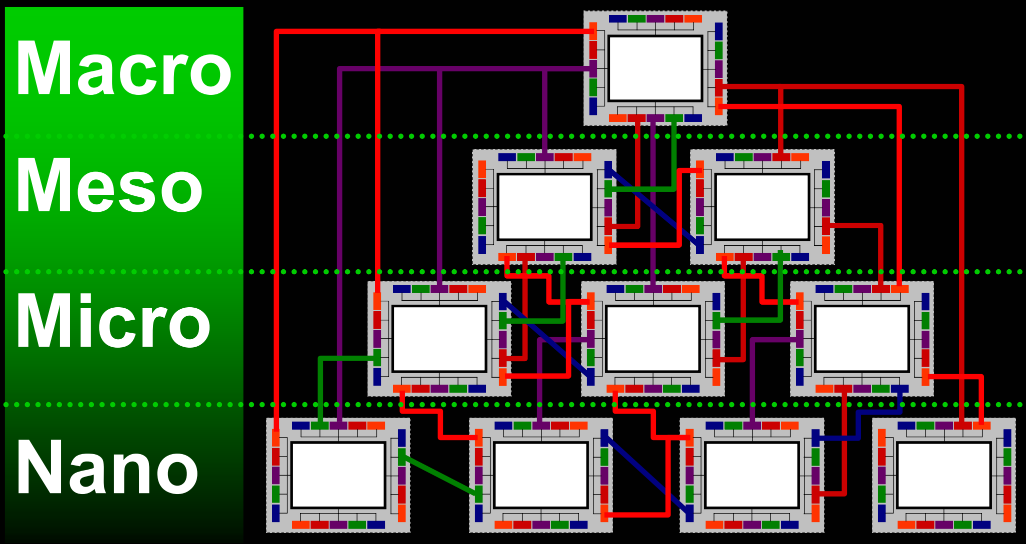
Stage 1: Synthesis & selection

- ❑ Big issues
- ❑ Selection

Stage 2: Detailed design

- ❑ Analysis
- ❑ Optimization

– Function
– Form
– Flows
– Physics
– Fabrication



Cake
Or
Death ?

Is this a difficult decision?

- ❑Decision making is differential
- ❑Difference is indicated by model
- ❑Model is supported by relationship

What determines quality of model?

Modeling and decision making

Determinism

- ❑ Does the system obey cause-effect (as observed)?
- ❑ Systematic error, random error

Repeatability

- ❑ How identical are repeated results?

Accuracy

- ❑ How close is the result to reality?

Non-dimensional analysis

- ❑ Qualitative & quantitative
- ❑ Rational process

Everything

Experience
or
Relative importance

Necessary & Sufficient

Input-output mapping

$$O_{MuSS} = G(SR) \cdot I_{MuSS}$$

$$\begin{vmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{vmatrix} \leftarrow \begin{vmatrix} f_{11}\left(SR_{\frac{Macro}{Macro}}\right) & f_{12}\left(SR_{\frac{Meso}{Macro}}\right) & f_{13}\left(SR_{\frac{Micro}{Macro}}\right) & f_{14}\left(SR_{\frac{Nano}{Macro}}\right) \\ f_{21}\left(SR_{\frac{Macro}{Meso}}\right) & f_{22}\left(SR_{\frac{Meso}{Meso}}\right) & f_{23}\left(SR_{\frac{Micro}{Meso}}\right) & f_{24}\left(SR_{\frac{Nano}{Meso}}\right) \\ f_{31}\left(SR_{\frac{Macro}{Micro}}\right) & f_{32}\left(SR_{\frac{Meso}{Micro}}\right) & f_{33}\left(SR_{\frac{Micro}{Micro}}\right) & f_{34}\left(SR_{\frac{Nano}{Micro}}\right) \\ f_{41}\left(SR_{\frac{Macro}{Nano}}\right) & f_{42}\left(SR_{\frac{Meso}{Nano}}\right) & f_{43}\left(SR_{\frac{Micro}{Nano}}\right) & f_{44}\left(SR_{\frac{Nano}{Nano}}\right) \end{vmatrix} \cdot \begin{vmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{vmatrix}$$

Conceptual

Input-output mapping

$$O_{MuSS} = G(SR) \cdot I_{MuSS}$$

$$\left| \begin{matrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{matrix} \right| = \left| \begin{matrix} C_{11} \cdot \left(\frac{SR_{Macro}}{Macro} \right)^{A_{11}} & C_{12} \cdot \left(\frac{SR_{Meso}}{Macro} \right)^{A_{12}} & C_{13} \cdot \left(\frac{SR_{Micro}}{Macro} \right)^{A_{13}} & C_{14} \cdot \left(\frac{SR_{Nano}}{Macro} \right)^{A_{14}} \\ C_{21} \cdot \left(\frac{SR_{Macro}}{Meso} \right)^{A_{21}} & C_{22} \cdot \left(\frac{SR_{Meso}}{Meso} \right)^{A_{22}} & C_{23} \cdot \left(\frac{SR_{Micro}}{Meso} \right)^{A_{23}} & C_{24} \cdot \left(\frac{SR_{Nano}}{Meso} \right)^{A_{24}} \\ C_{31} \cdot \left(\frac{SR_{Macro}}{Micro} \right)^{A_{31}} & C_{32} \cdot \left(\frac{SR_{Meso}}{Micro} \right)^{A_{32}} & C_{33} \cdot \left(\frac{SR_{Micro}}{Micro} \right)^{A_{33}} & C_{34} \cdot \left(\frac{SR_{Nano}}{Micro} \right)^{A_{34}} \\ C_{41} \cdot \left(\frac{SR_{Macro}}{Nano} \right)^{A_{41}} & C_{42} \cdot \left(\frac{SR_{Meso}}{Nano} \right)^{A_{42}} & C_{43} \cdot \left(\frac{SR_{Micro}}{Nano} \right)^{A_{43}} & C_{44} \cdot \left(\frac{SR_{Nano}}{Nano} \right)^{A_{44}} \end{matrix} \right| \cdot \left| \begin{matrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{matrix} \right|$$

Equivalent

What might G look like?

"Ideal" or perfect
scale interaction

$$\begin{Bmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{Bmatrix} \sim \begin{bmatrix} 10^{0} & 10^{3} & 10^{6} & 10^{9} \\ 10^{-3} & 10^{0} & 10^{3} & 10^{6} \\ 10^{-6} & 10^{-3} & 10^{0} & 10^{3} \\ 10^{-9} & 10^{-6} & 10^{-3} & 10^{0} \end{bmatrix} \cdot \begin{Bmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{Bmatrix}$$

$$\longrightarrow \quad G_p = \begin{bmatrix} \boxed{10^{0}} & 10^{3} & 10^{6} & 10^{9} \\ 10^{-3} & 10^{0} & 10^{3} & 10^{6} \\ 10^{-6} & 10^{-3} & 10^{0} & 10^{3} \\ 10^{-9} & 10^{-6} & 10^{-3} & 10^{0} \end{bmatrix}$$

$$G = \begin{bmatrix} \boxed{10^{100}} & 10^{3} & 10^{6} & \boxed{10^{20}} \\ 10^{-3} & 10^{0} & 10^{3} & 10^{6} \\ 10^{-6} & 10^{-3} & 10^{0} & 10^{3} \\ 10^{-9} & 10^{-6} & 10^{-3} & 10^{0} \end{bmatrix}$$

What does $| {}^{Gp\ ij} / {}_{G\ ij} |$ look like?

Why is this useful?

How will we use it?

Example: STM

$$i = C \cdot e^{\left(-2 \cdot K \cdot gap\right)}$$

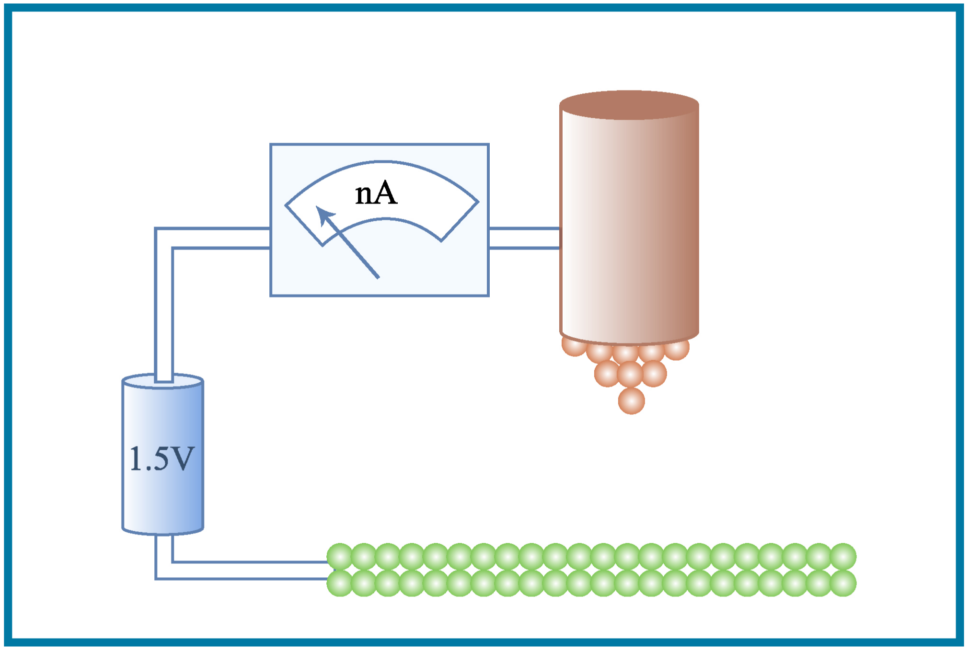


Figure by MIT OCW.

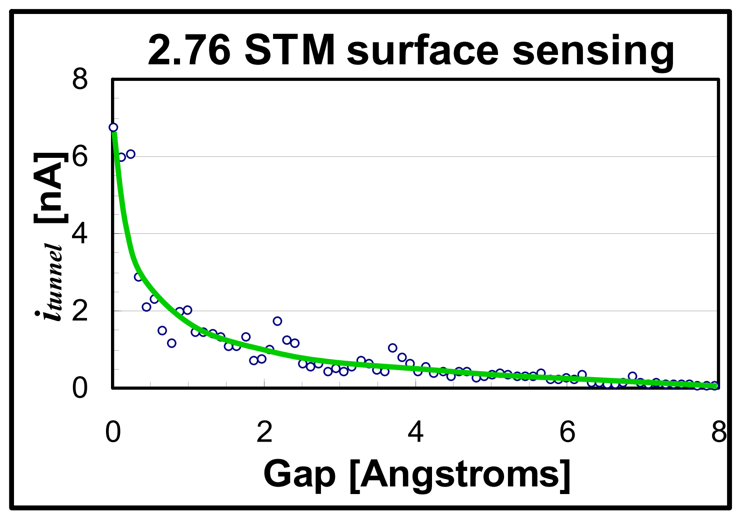
Is this the whole story?

Example: STM Signal

Vibration ↓   Noise ↓   Noise ↓   Gain ↓

$$\begin{vmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{vmatrix} \begin{vmatrix} f_{11}\; SR_{\frac{Macro}{Macro}} & f_{12}\; SR_{\frac{Meso}{Macro}} & f_{13}\; SR_{\frac{Micro}{Macro}} & f_{14}\; SR_{\frac{Nano}{Macro}} \\ f_{21}\; SR_{\frac{Macro}{Meso}} & f_{22}\; SR_{\frac{Meso}{Meso}} & f_{23}\; SR_{\frac{Micro}{Meso}} & f_{24}\; SR_{\frac{Nano}{Meso}} \\ f_{31}\; SR_{\frac{Macro}{Micro}} & f_{32}\; SR_{\frac{Meso}{Micro}} & f_{33}\; SR_{\frac{Micro}{Micro}} & f_{34}\; SR_{\frac{Nano}{Micro}} \\ f_{41}\; SR_{\frac{Macro}{Nano}} & f_{42}\; SR_{\frac{Meso}{Nano}} & f_{43}\; SR_{\frac{Micro}{Nano}} & f_{44}\; SR_{\frac{Nano}{Nano}} \end{vmatrix} \begin{vmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{vmatrix} \leftarrow i \quad C \;\; e^{2\,K\,gap}$$

Image removed for copyright reasons.
Source: http://www.almaden.ibm.com



Example: STM Displacement

$$\begin{vmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{vmatrix} \leftarrow \begin{vmatrix} f_{11}\left(SR_{\frac{Macro}{Macro}}\right) & f_{12}\left(SR_{\frac{Meso}{Macro}}\right) & f_{13}\left(SR_{\frac{Micro}{Macro}}\right) & f_{14}\left(SR_{\frac{Nano}{Macro}}\right) \\ f_{21}\left(SR_{\frac{Macro}{Meso}}\right) & f_{22}\left(SR_{\frac{Meso}{Meso}}\right) & f_{23}\left(SR_{\frac{Micro}{Meso}}\right) & f_{24}\left(SR_{\frac{Nano}{Meso}}\right) \\ f_{31}\left(SR_{\frac{Macro}{Micro}}\right) & f_{32}\left(SR_{\frac{Meso}{Micro}}\right) & f_{33}\left(SR_{\frac{Micro}{Micro}}\right) & f_{34}\left(SR_{\frac{Nano}{Micro}}\right) \\ f_{41}\left(SR_{\frac{Macro}{Nano}}\right) & f_{42}\left(SR_{\frac{Meso}{Nano}}\right) & f_{43}\left(SR_{\frac{Micro}{Nano}}\right) & f_{44}\left(SR_{\frac{Nano}{Nano}}\right) \end{vmatrix} \cdot \begin{vmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{vmatrix}$$

↑ Vibration ↑ Strain ↑ Slip ↑ 0

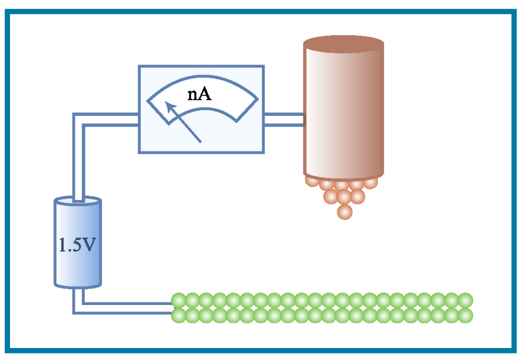


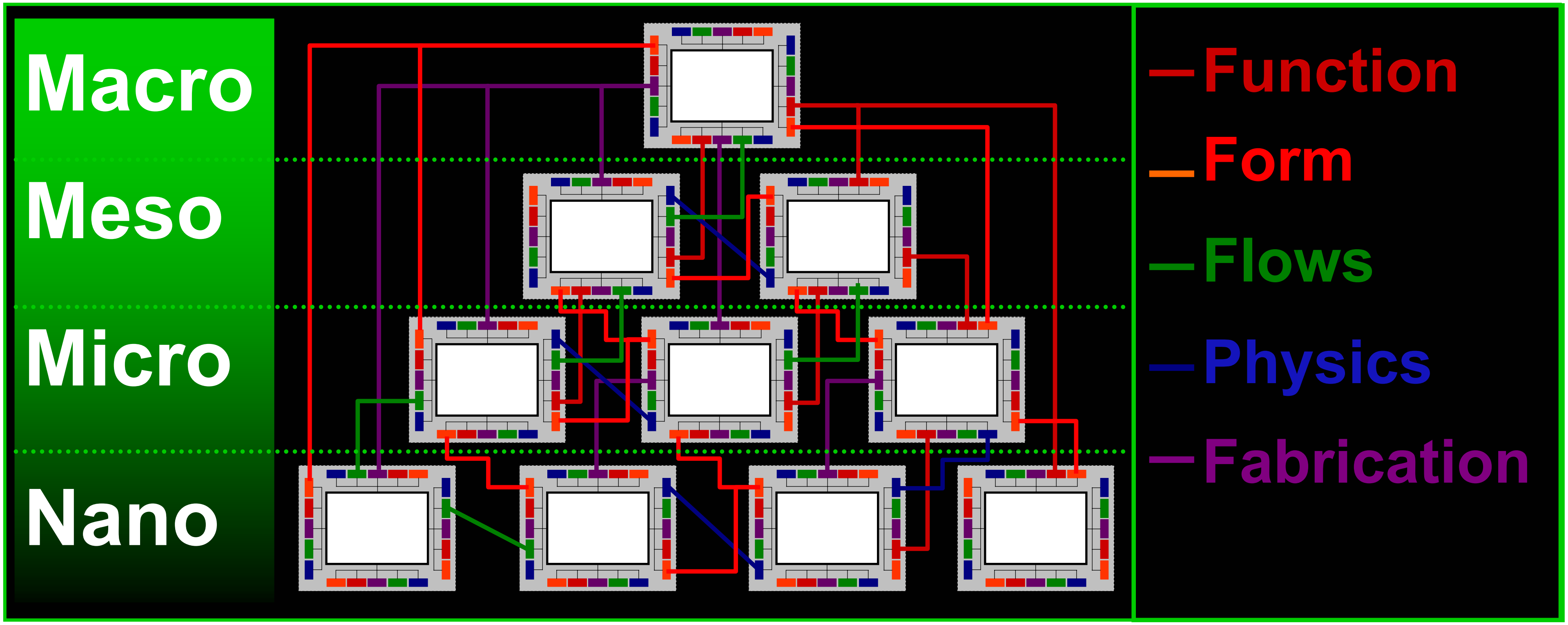
Figure by MIT OCW.

Purpose of today

$$
\begin{vmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{vmatrix} = \begin{vmatrix}
f_{11}\left(SR_{\frac{Macro}{Macro}}\right) & f_{12}\left(SR_{\frac{Meso}{Macro}}\right) & f_{13}\left(SR_{\frac{Micro}{Macro}}\right) & f_{14}\left(SR_{\frac{Nano}{Macro}}\right) \\
f_{21}\left(SR_{\frac{Macro}{Meso}}\right) & f_{22}\left(SR_{\frac{Meso}{Meso}}\right) & f_{23}\left(SR_{\frac{Micro}{Meso}}\right) & f_{24}\left(SR_{\frac{Nano}{Meso}}\right) \\
f_{31}\left(SR_{\frac{Macro}{Micro}}\right) & f_{32}\left(SR_{\frac{Meso}{Micro}}\right) & f_{33}\left(SR_{\frac{Micro}{Micro}}\right) & f_{34}\left(SR_{\frac{Nano}{Micro}}\right) \\
f_{41}\left(SR_{\frac{Macro}{Nano}}\right) & f_{42}\left(SR_{\frac{Meso}{Nano}}\right) & f_{43}\left(SR_{\frac{Micro}{Nano}}\right) & f_{44}\left(SR_{\frac{Nano}{Nano}}\right)
\end{vmatrix} \cdot \begin{vmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{vmatrix}
$$

Mechanical gain factors to make big machines work with little machines

What will this be applied to?



Function	Form	Flow	Physics	Fabrication
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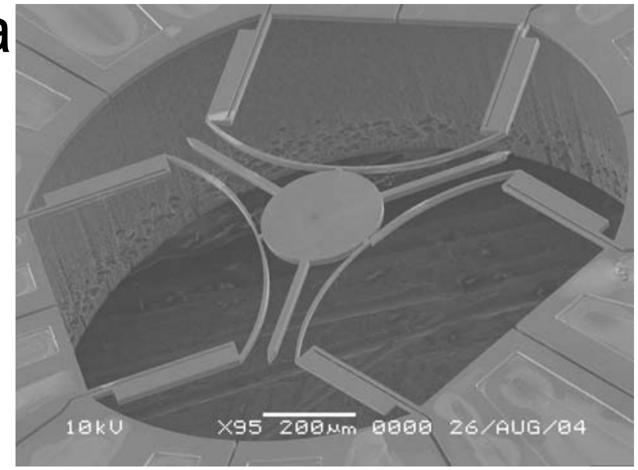
Where	Constraints	Information	Dominant	Cost
Etc…	Etc…	Etc…	Etc…	Etc…

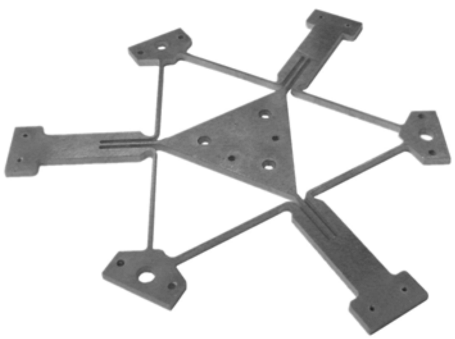
Early big machines made to work with the small

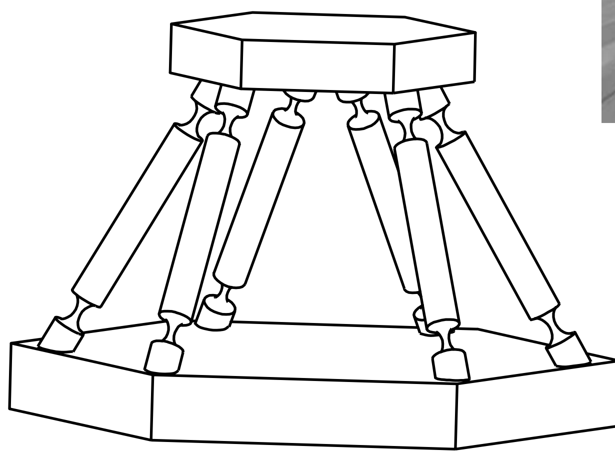
Big machines working with the small

What is the most critical requirements for a large-scale machine to live in a MuSS?

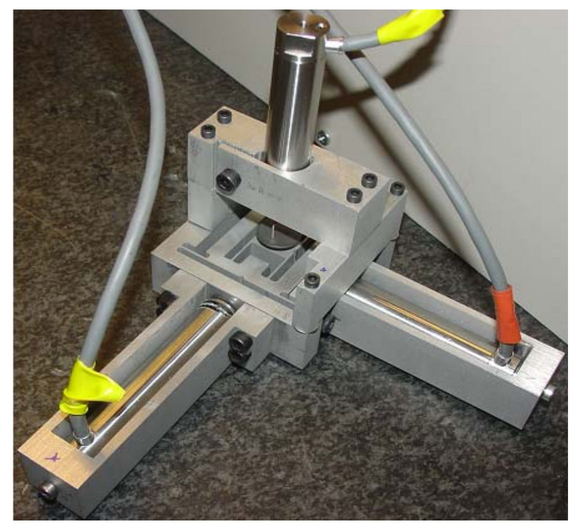
Motion stability, resolution, repeatability







Two diagrams removed for copyright reasons.



Strain management

Everything is compliant

- Strain error scales with size
- Large scale parts are kinematic bullies

Generally require “freedom to strain” to prevent over constraint & energy storage

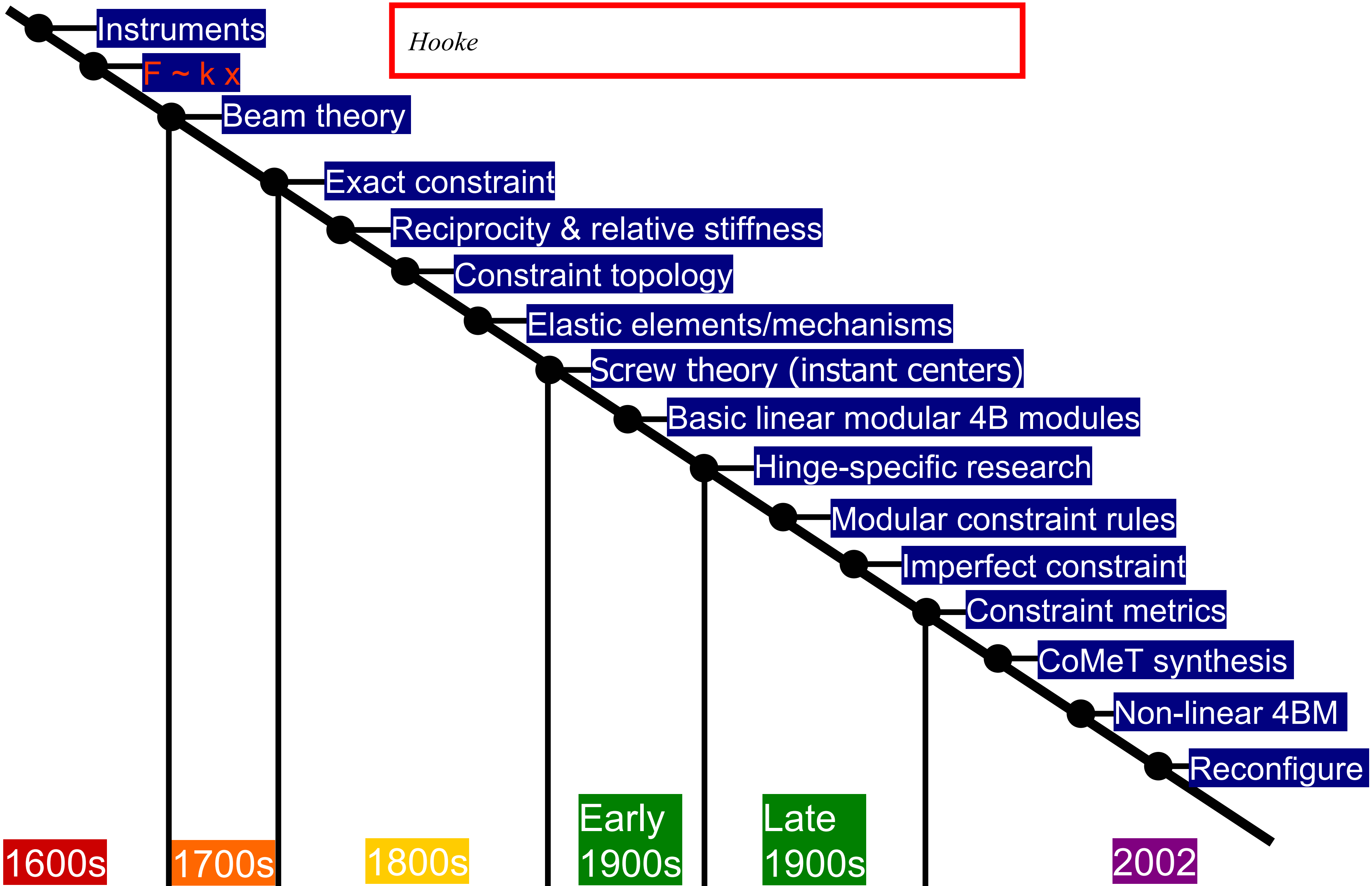
Generally seek to minimize strain

Mechanism/fixture/structure design

- Necessary & sufficient constraint topology → concepts
- Exact constraint

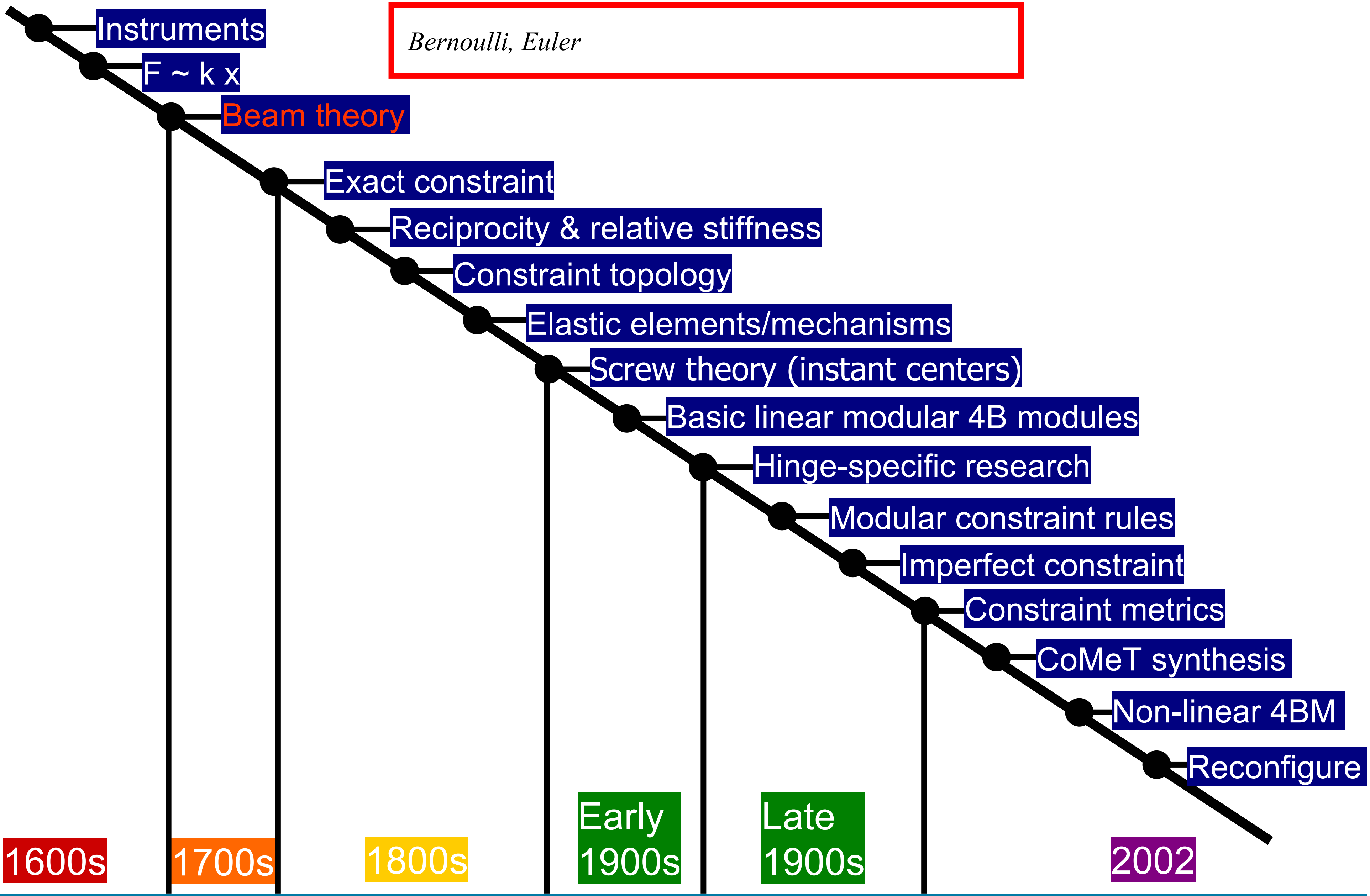
History of compliant machines

Hooke

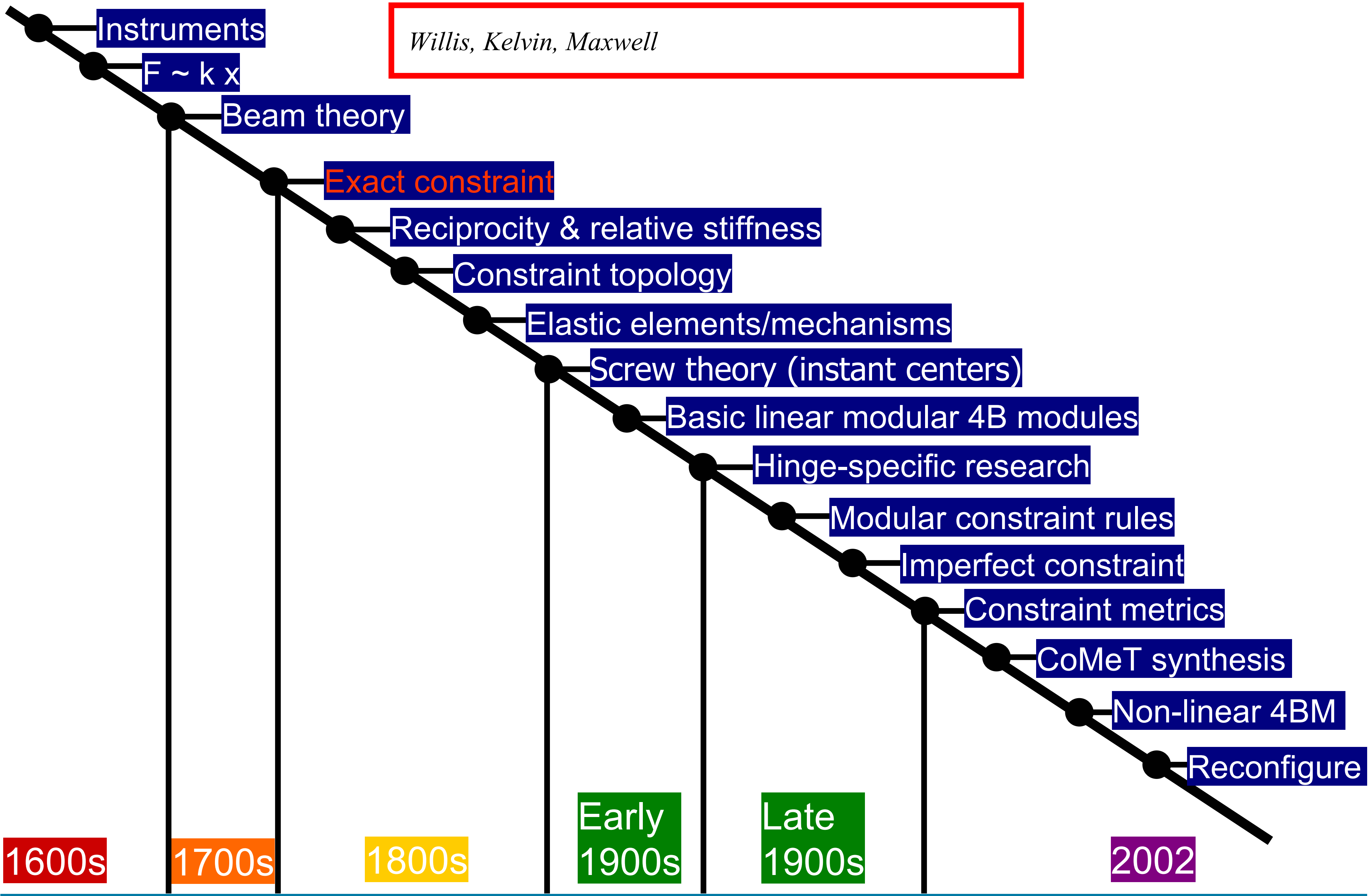


History of compliant machines

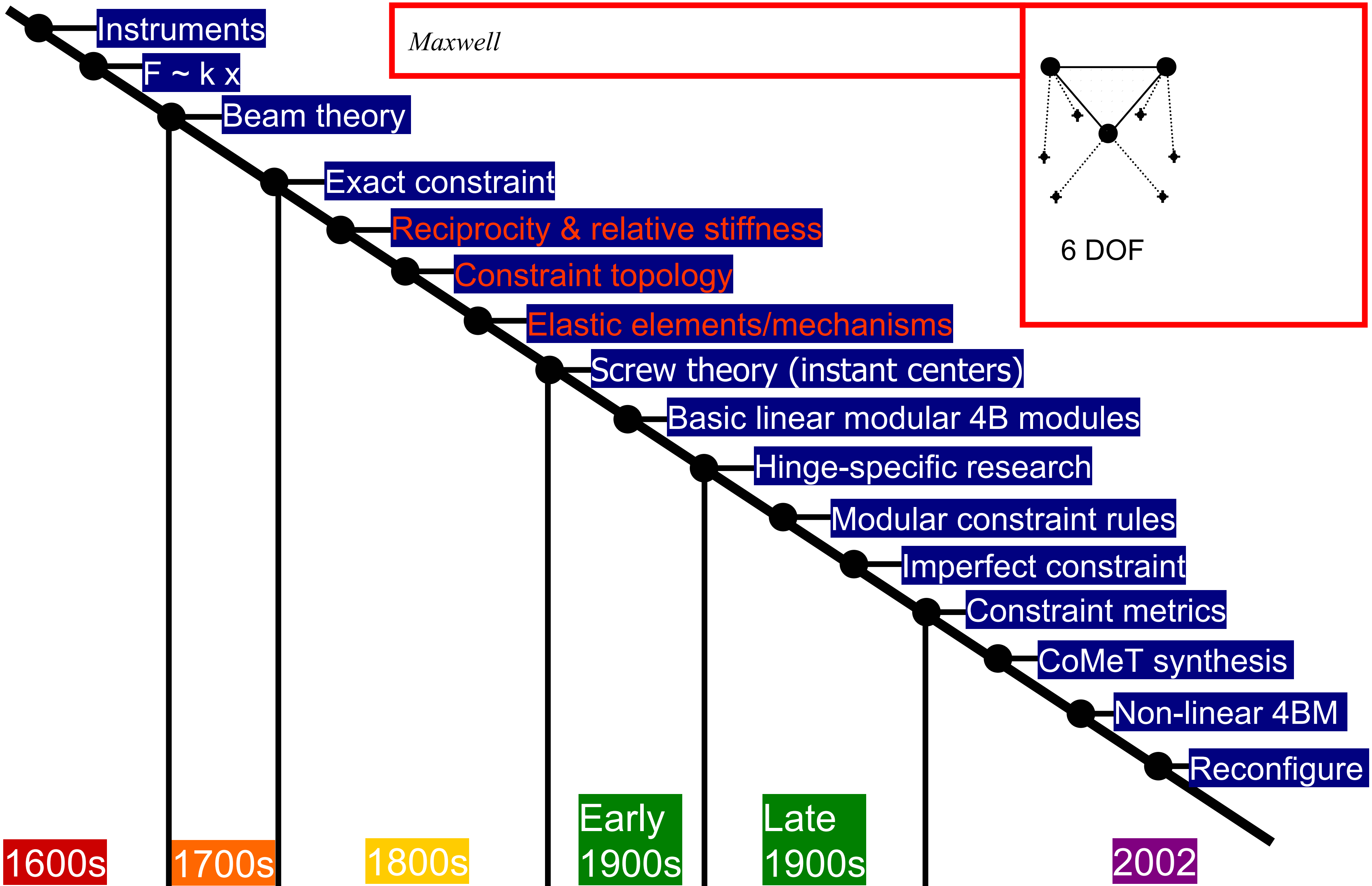
Bernoulli, Euler



History of compliant machines

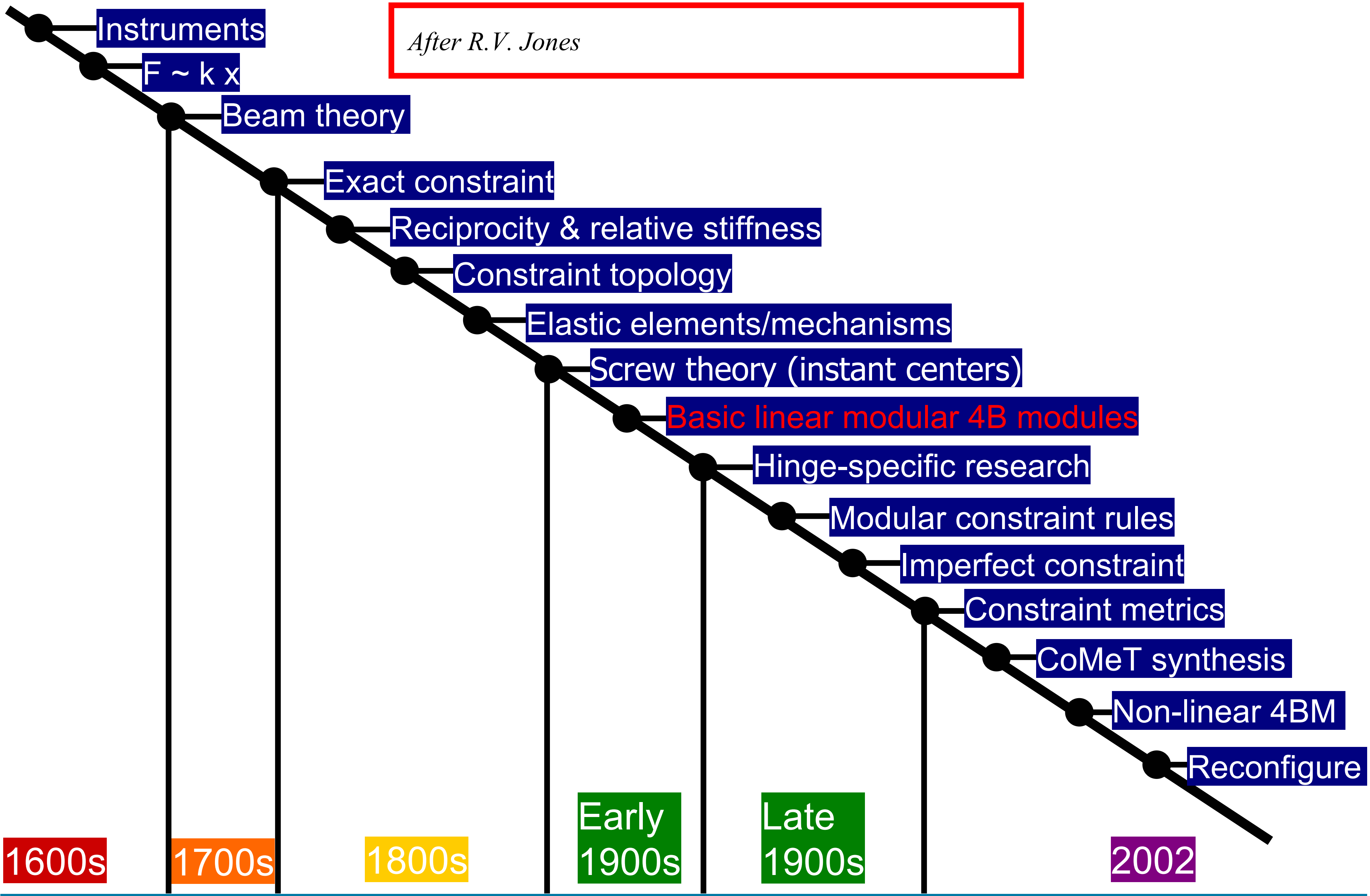


History of compliant machines



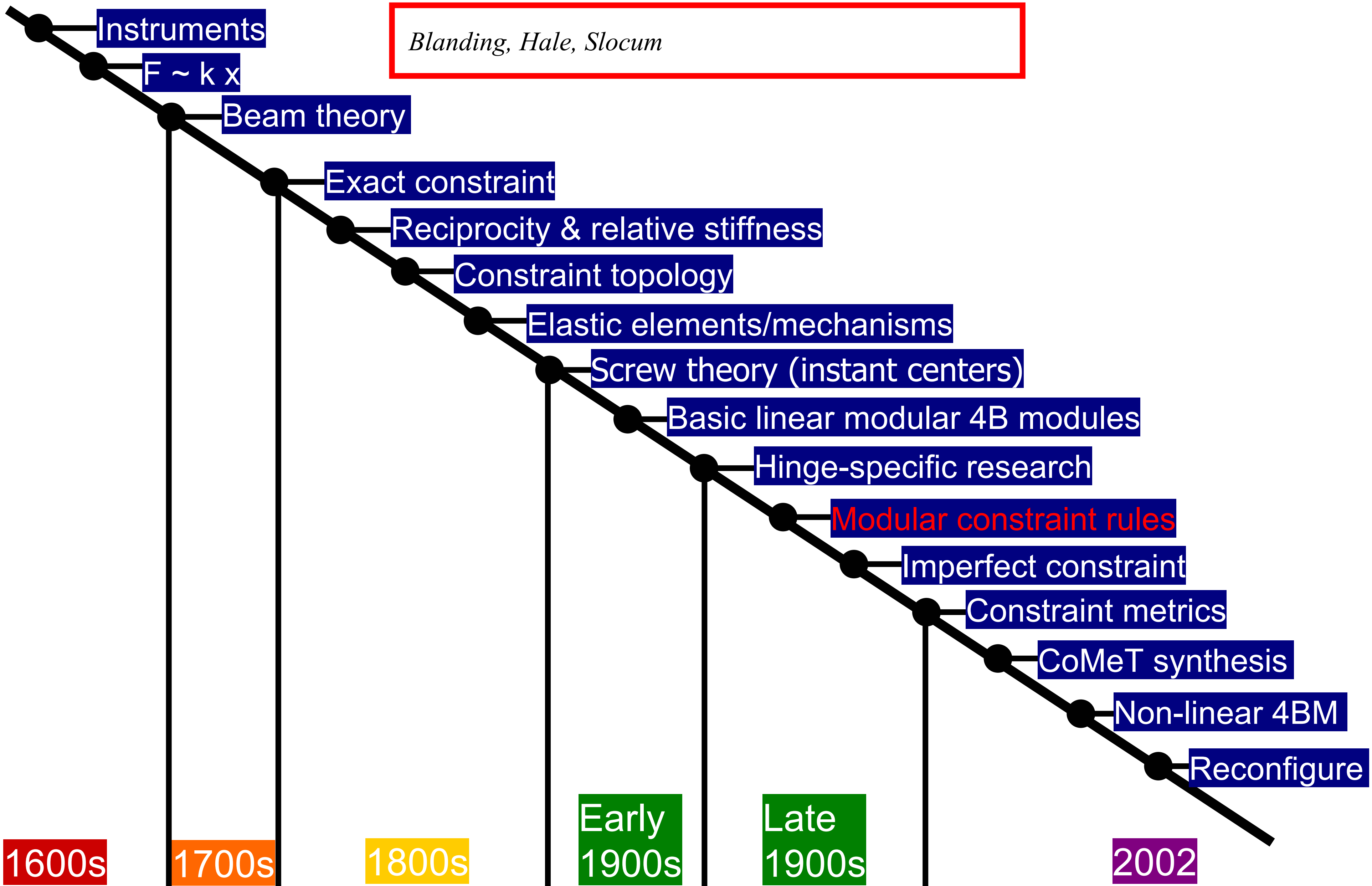
History of compliant machines

After R.V. Jones



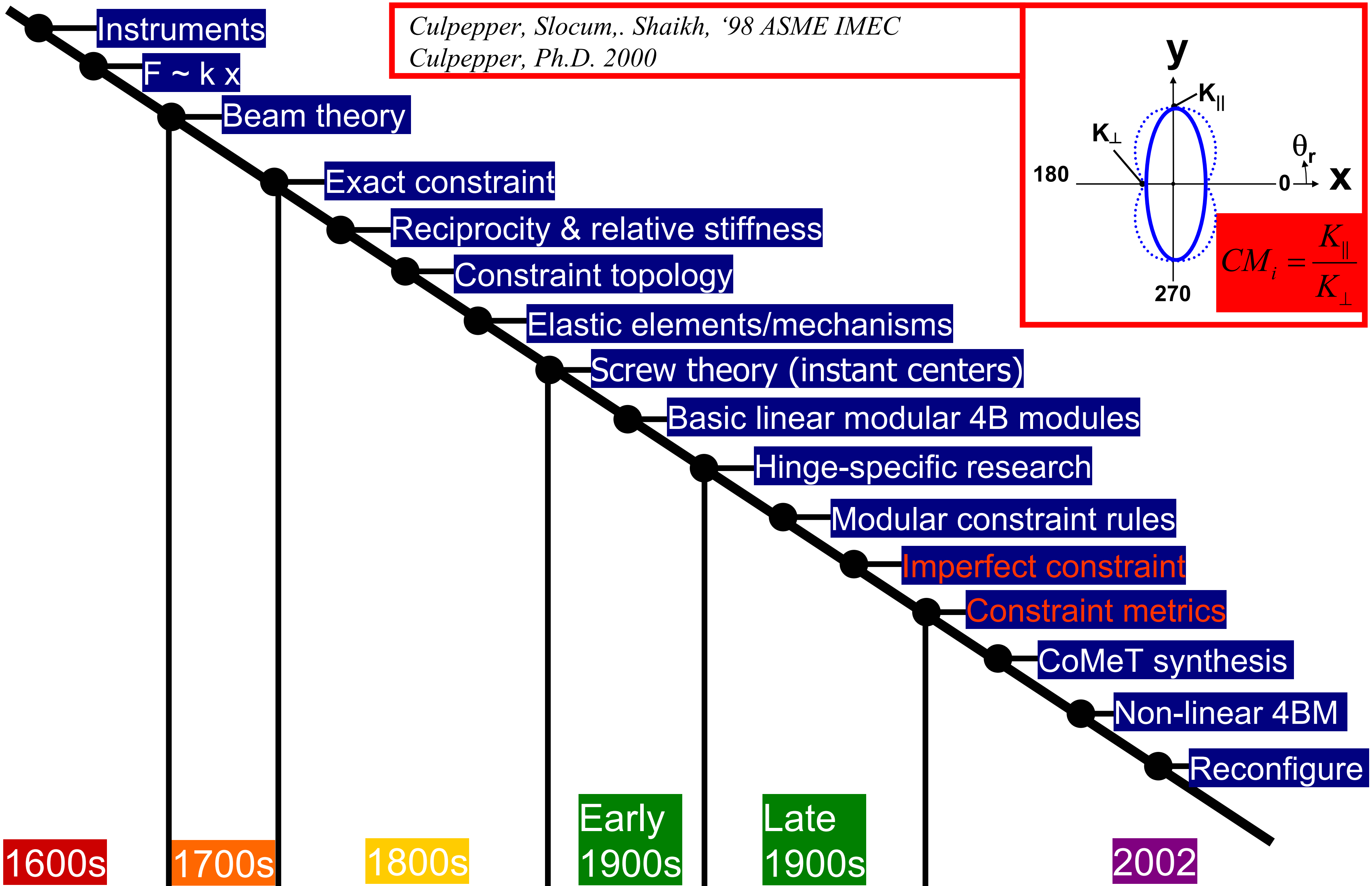
History of compliant machines

Blanding, Hale, Slocum



History of compliant machines

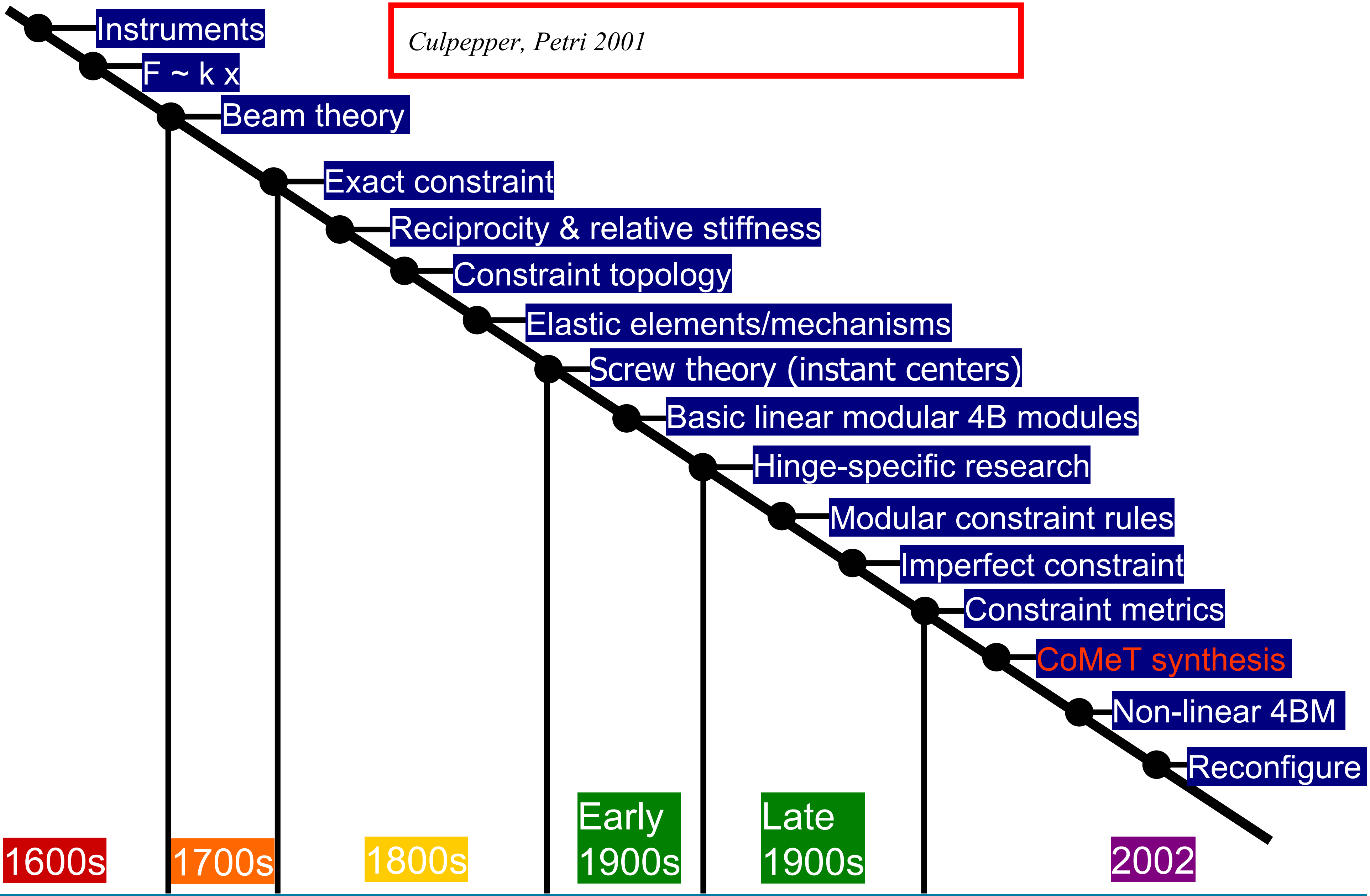
Culpepper, Slocum,. Shaikh, '98 ASME IMEC
Culpepper, Ph.D. 2000



$$CM_i = \frac{K_{\parallel}}{K_{\perp}}$$

History of compliant machines

Culpepper, Petri 2001

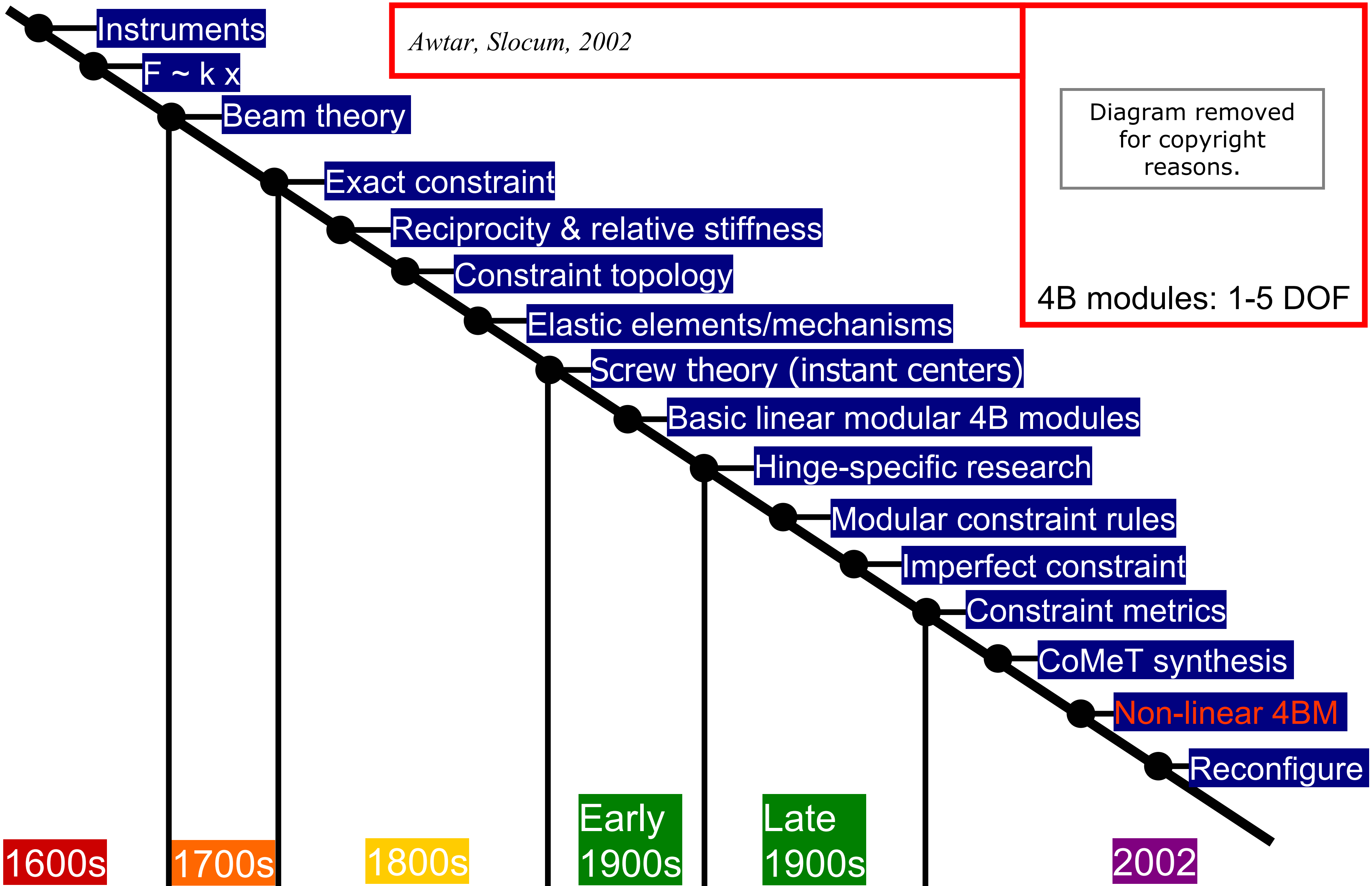


History of compliant machines

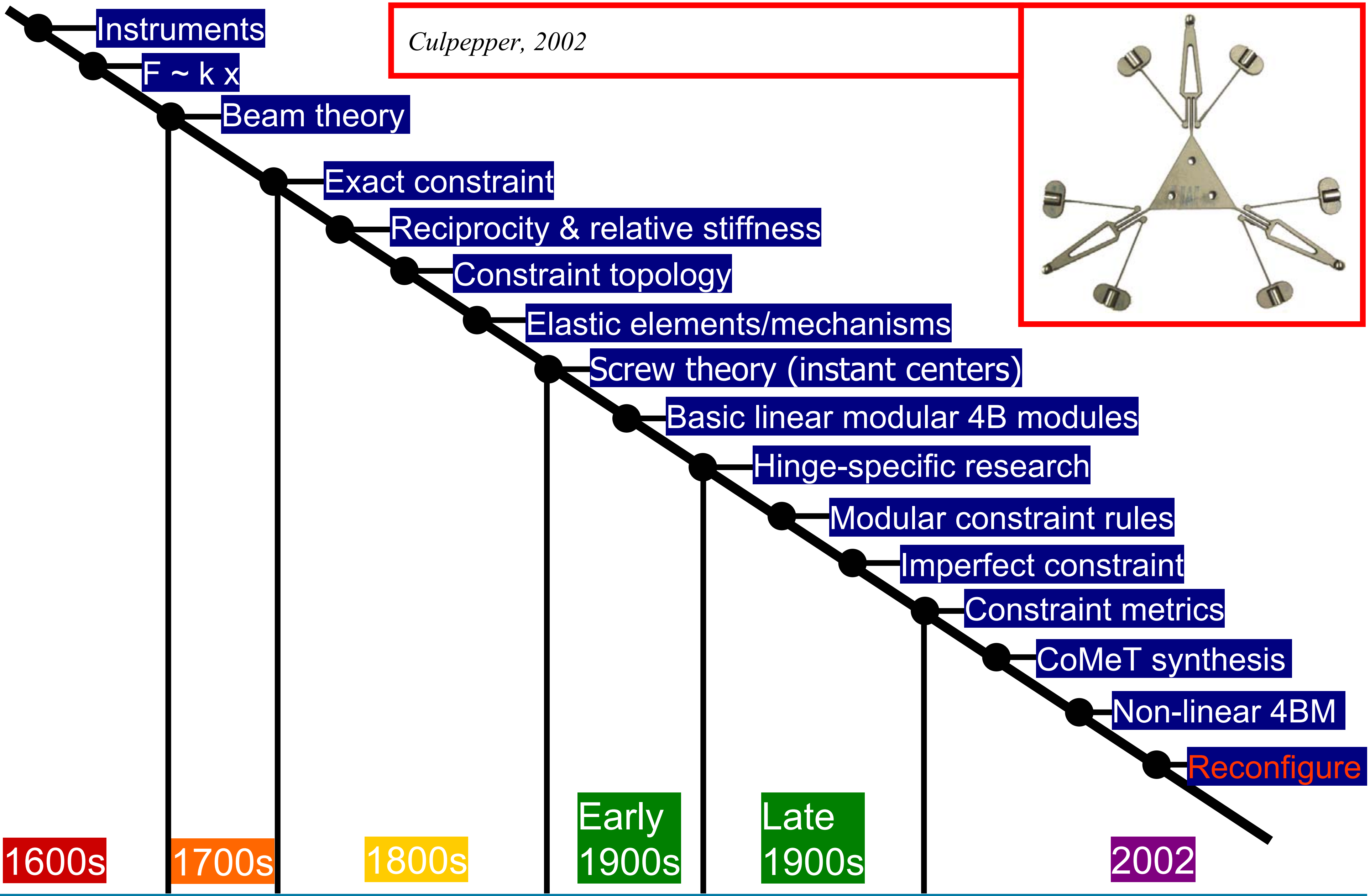
Awtar, Slocum, 2002

Diagram removed for copyright reasons.

4B modules: 1-5 DOF



History of compliant machines



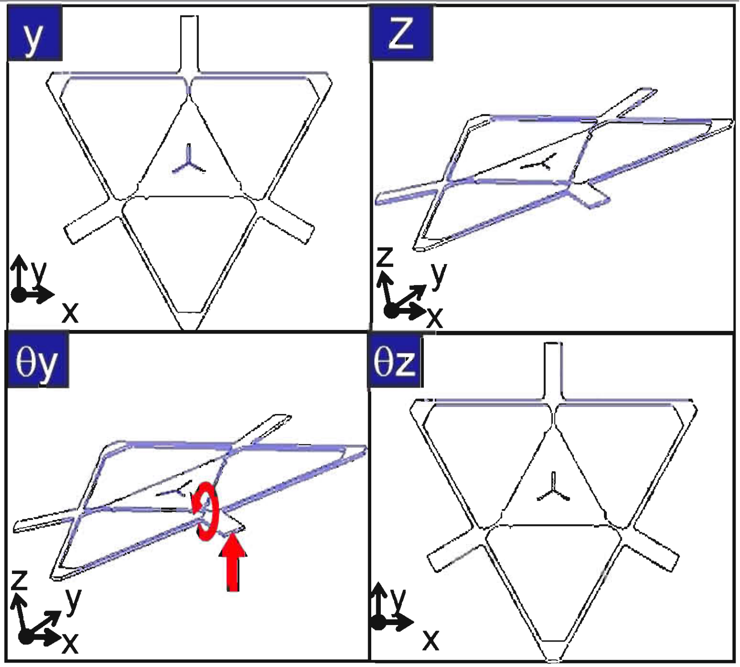
Culpepper, 2002

Principles of cross-scale motion and constraint

To flex or not to flex

Good

- Smooth, “hysteresis free” motion
- Symmetry easy to implement
- Linearity



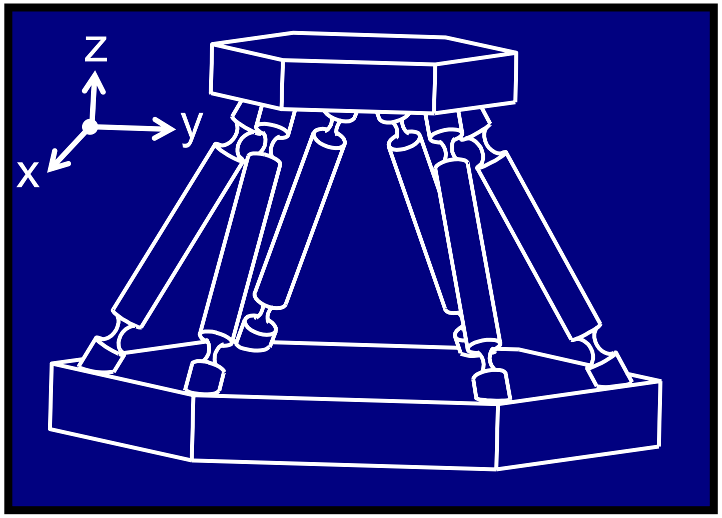
Unpleasant

- Accuracy and repeatability sensitive to several variables
- Limited motion/stroke (20% of yield unless single crystal Si)
- Stiffness ratio compared to other guide/bearing mechanisms is low

Design for compliant constraint

1. Stability

- ❑ Maximize passive stability
- ❑ "Self-help" (symmetry & cancellation)

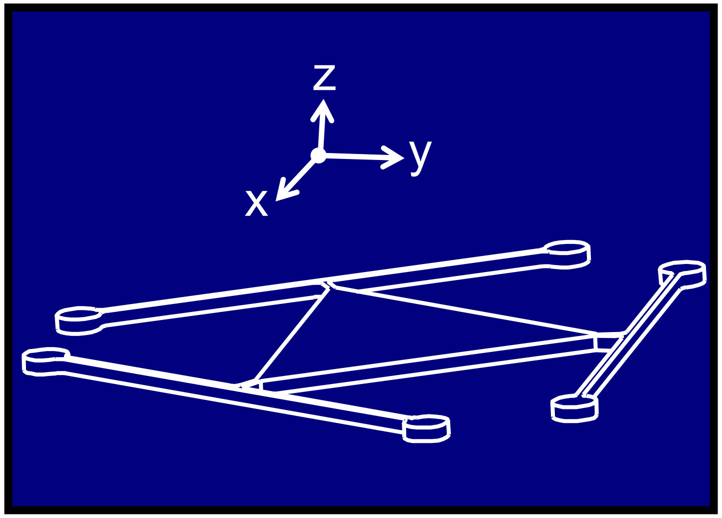


2. Envelope

- ❑ Strain errors (compliance, thermal)
- ❑ Packaging

3. Manufacturing

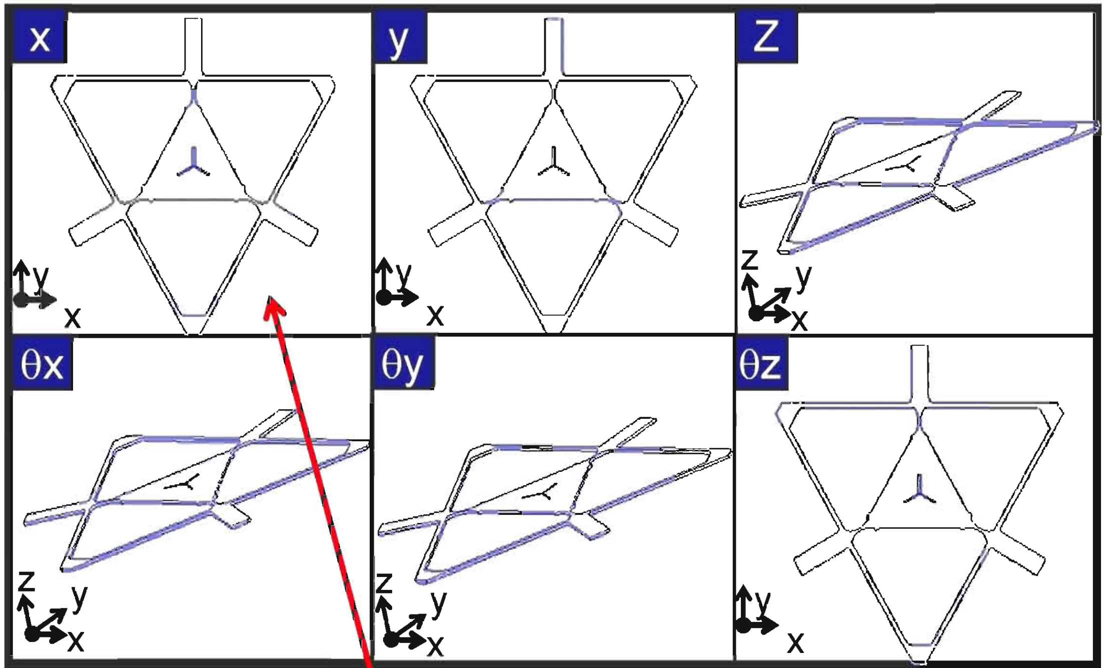
- ❑ Monolithic
- ❑ Minimum information

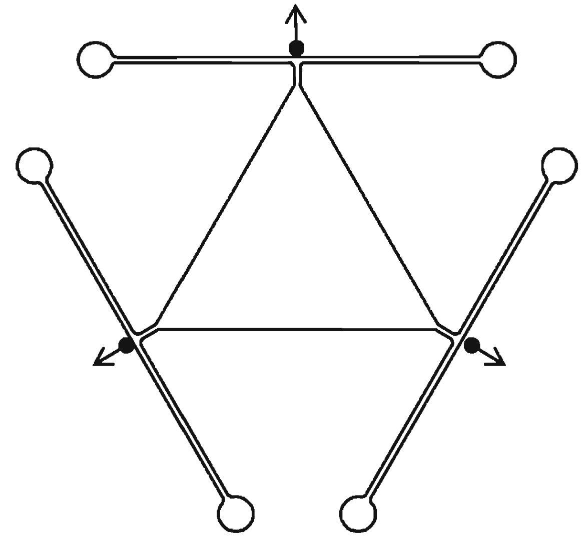


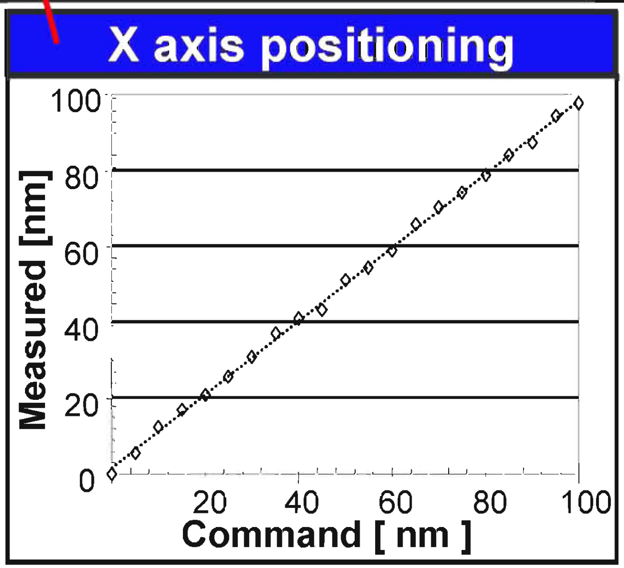
4. Constraint

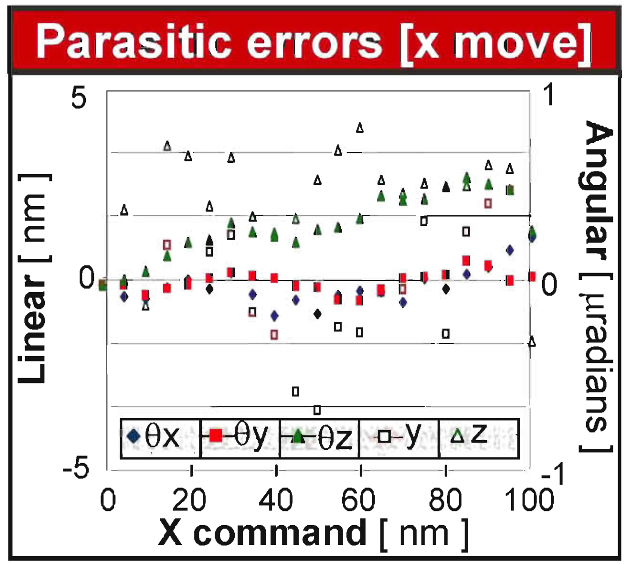
- ❑ Maximize linear independence
- ❑ Parasitic errors

Parasitic errors



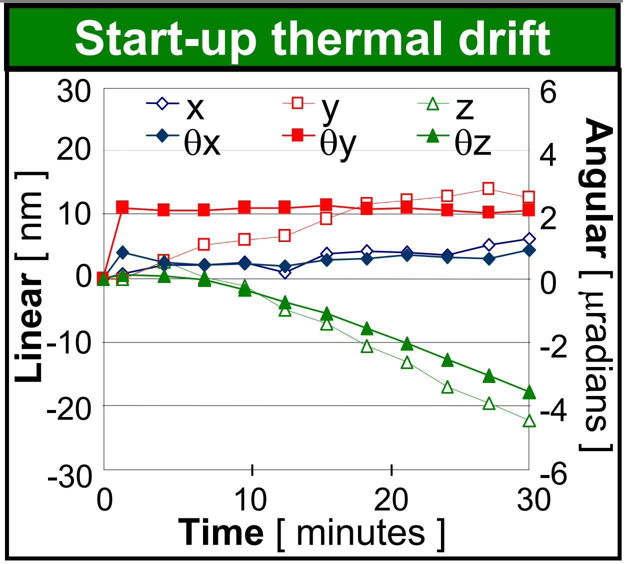




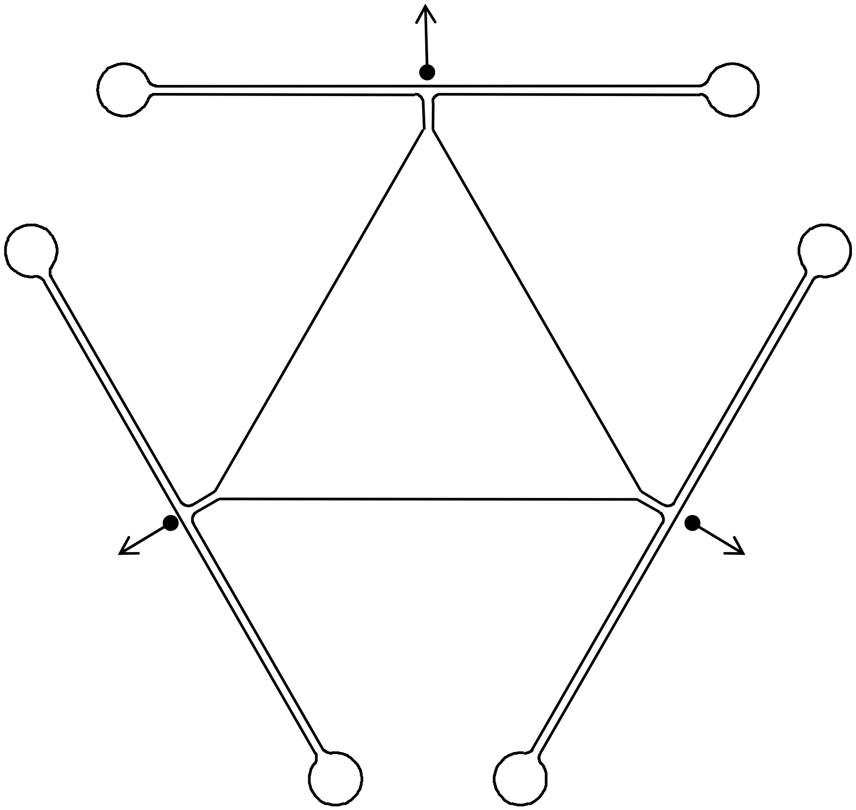


Principle of symmetry

Thermal strain error



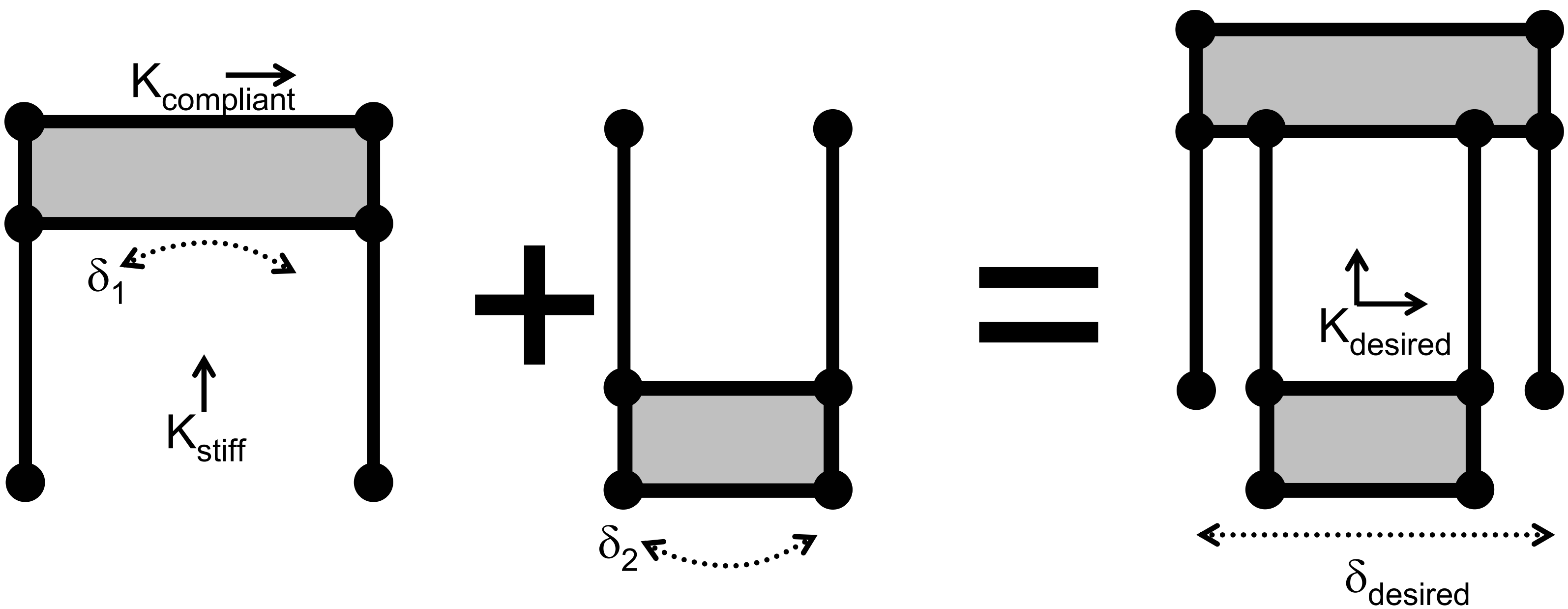
Over constraint → Force



Principle of cancellation

Kinematic path building blocks

- ❑ Straight lines
- ❑ Rotation (instant centers)



Principle of center of stiffness

Loading matters

Center of stiffness: Load = no rotation

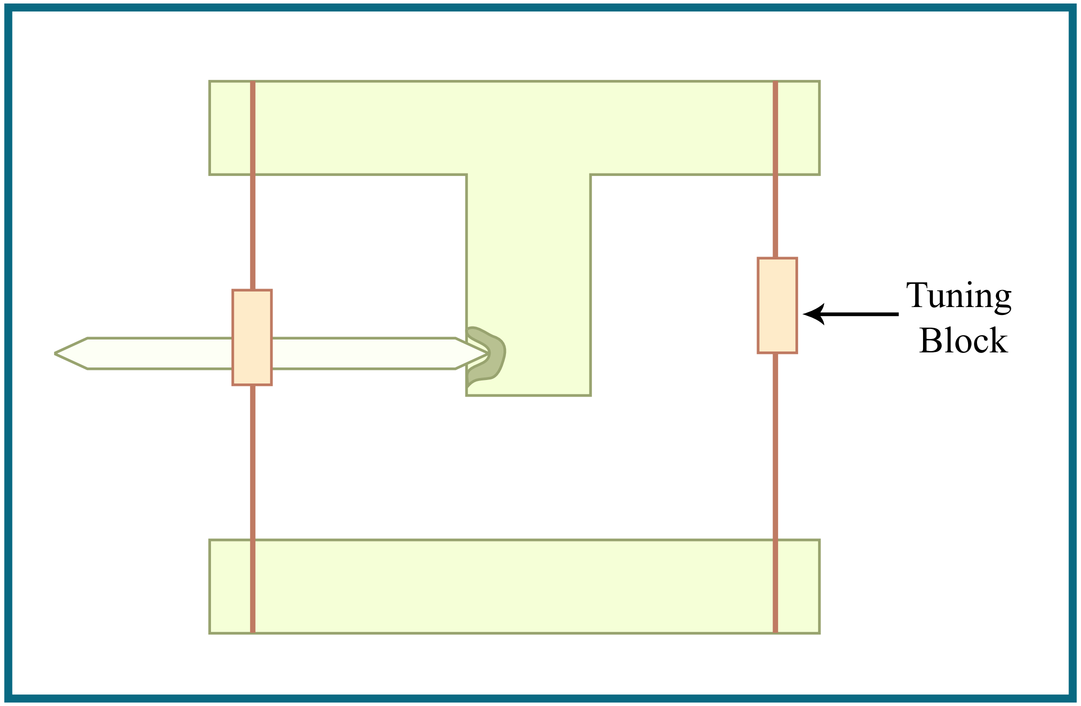


Figure by MIT OCW. After R. V. Jones.

Modeling

General Exact Constraint

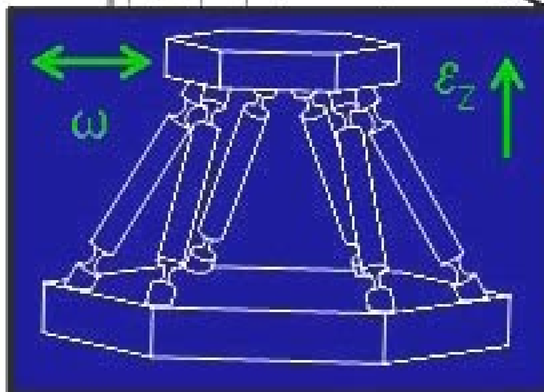
$$\delta_{Stage} = A \cdot \Delta_{Actuator}$$

$$\begin{vmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{vmatrix} = \begin{vmatrix} E_{x1} & E_{x2} & E_{x3} & E_{x4} & E_{x5} & E_{x6} \\ E_{y1} & E_{y2} & E_{y3} & E_{y4} & E_{y5} & E_{y6} \\ E_{z1} & E_{z2} & E_{z3} & E_{z4} & E_{z5} & E_{z6} \\ E_{\theta x1} & E_{\theta x2} & E_{\theta x3} & E_{\theta x4} & E_{\theta x5} & E_{\theta x6} \\ E_{\theta y1} & E_{\theta y2} & E_{\theta y3} & E_{\theta y4} & E_{\theta y5} & E_{\theta y6} \\ E_{\theta z1} & E_{\theta z2} & E_{\theta z3} & E_{\theta z4} & E_{\theta z5} & E_{\theta z6} \end{vmatrix} \begin{vmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{vmatrix}$$

Nominal

$$\delta_{Stage} = N \cdot \Delta_{Actuator}$$

$$\begin{vmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{vmatrix} = \begin{vmatrix} N_{x1} & N_{x2} & N_{x3} & 0 & 0 & 0 \\ N_{y1} & N_{y2} & N_{y3} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{z4} & N_{z5} & N_{z6} \\ & & & N_{\theta x4} & N_{\theta x5} & N_{\theta x6} \\ & & & N_{\theta y4} & N_{\theta y5} & N_{\theta y6} \\ & & & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{vmatrix}$$



Non-Calibrated

$$\delta_{Stage} = U \cdot \Delta_{Actuator}$$

$$\begin{vmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{vmatrix} = \begin{vmatrix} U_{x1} & U_{x2} & U_{x3} & P_{x4} & P_{x5} & P_{x6} \\ U_{y1} & U_{y2} & U_{y3} & P_{y4} & P_{y5} & P_{y6} \\ P_{z1} & P_{z2} & P_{z3} & U_{z4} & U_{z5} & U_{z6} \\ P_{\theta x1} & P_{\theta x2} & P_{\theta x3} & U_{\theta x4} & U_{\theta x5} & U_{\theta x6} \\ P_{\theta y1} & P_{\theta y2} & P_{\theta y3} & U_{\theta y4} & U_{\theta y5} & U_{\theta y6} \\ U_{\theta z1} & U_{\theta z2} & U_{\theta z3} & P_{\theta z4} & P_{\theta z5} & P_{\theta z6} \end{vmatrix} \begin{vmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{vmatrix}$$

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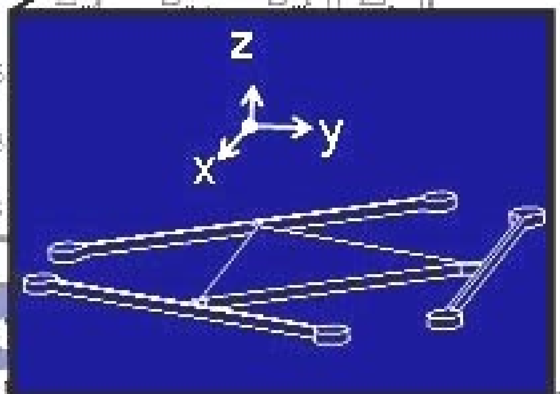
Modeling

General Exact Constraint

$$\delta_{Stage} = A \cdot \Delta_{Actuator}$$

$$\begin{vmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{vmatrix} = \begin{vmatrix} E_{x1} & E_{x2} & E_{x3} & E_{x4} & E_{x5} & E_{x6} \\ E_{y1} & E_{y2} & E_{y3} & E_{y4} & E_{y5} & E_{y6} \\ E_{z1} & E_{z2} & E_{z3} & E_{z4} & E_{z5} & E_{z6} \\ E_{\theta x1} & E_{\theta x2} & E_{\theta x3} & & & \\ E_{\theta y1} & E_{\theta y2} & E_{\theta y3} & & & \\ E_{\theta z1} & E_{\theta z2} & E_{\theta z3} & & & \end{vmatrix} \begin{vmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{vmatrix}$$



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"Perfectly" Calibrated

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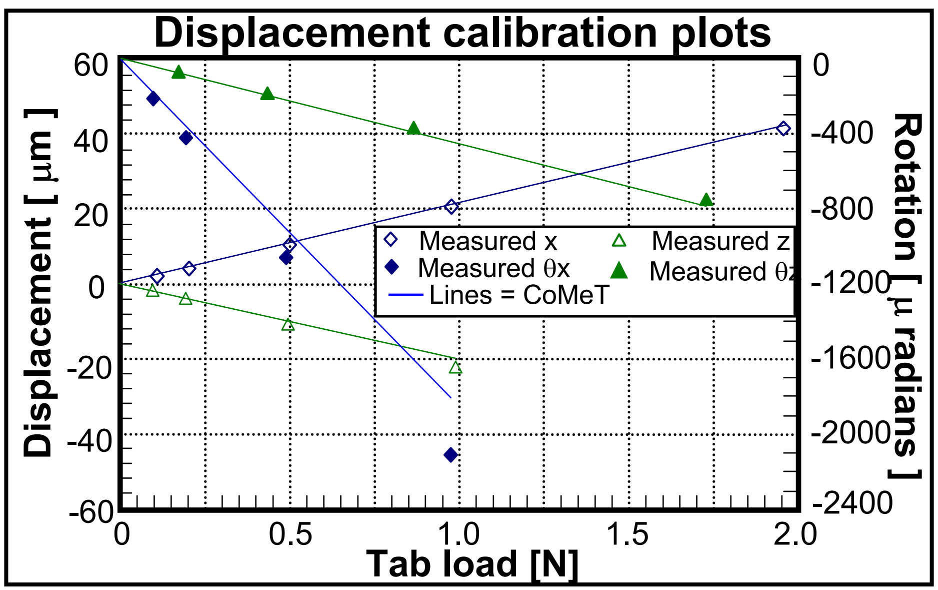
Actuator sensitivity & calibration

Source of errors

- ❑ Tolerance (5000 nm vs 1nm?)
- ❑ Mounting
- ❑ Material props
- ❑ Stress stiffening
- ❑ Linear assumptions

- ❑ Calibration and mapping…



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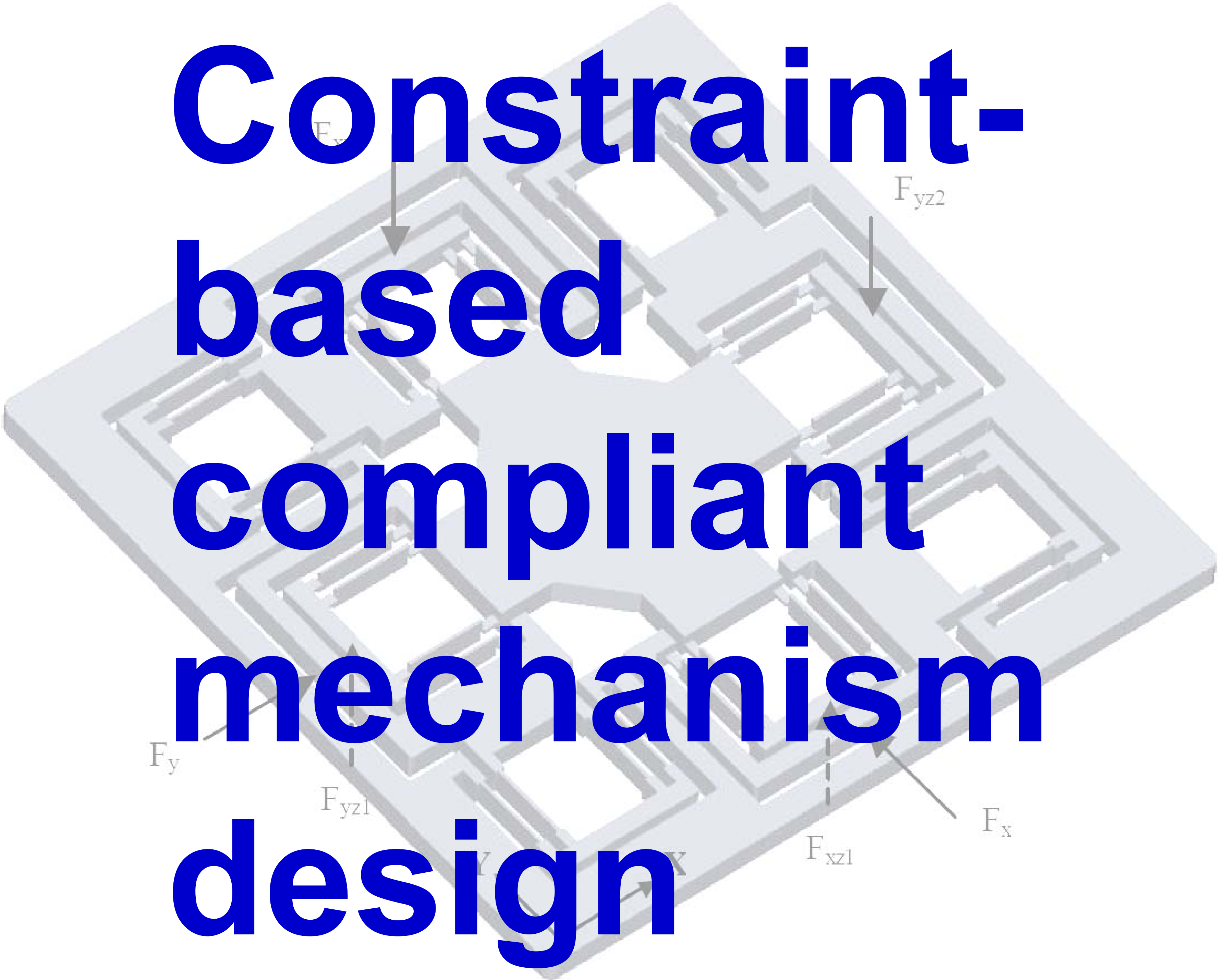
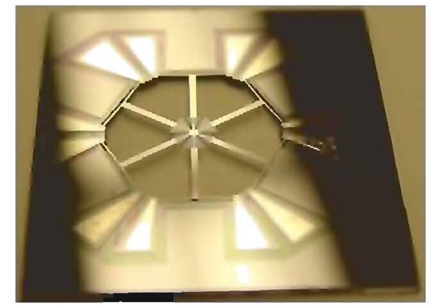
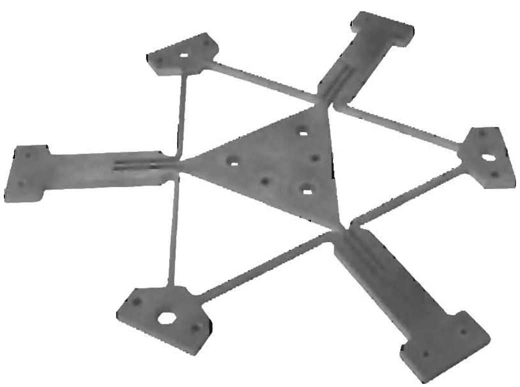
Constraint-based compliant mechanism design

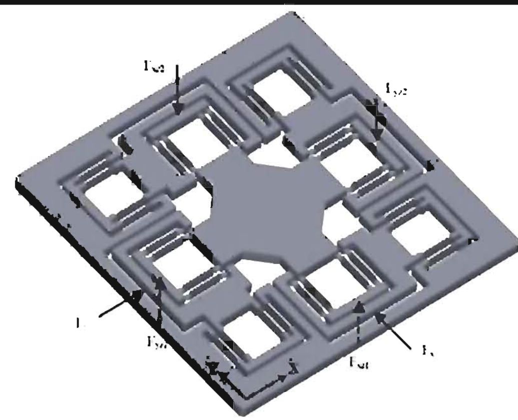


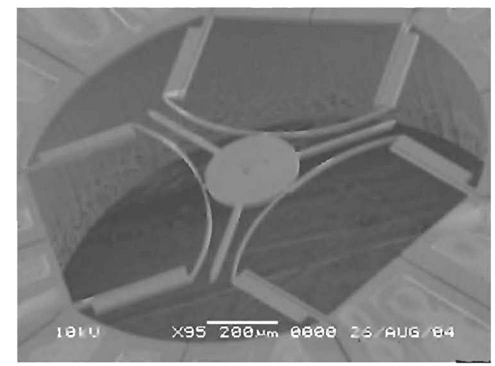
Constraint-based CM design

Compliant mechanism design methods		
Pseudo-rigid body (lumped compliance)	**Continuum /topology (distributed compliance)**	**Constraint-based (modular kinematic concepts)**
↓	↓	↓
• Combinations of rigid and compliant elements • Compliant elements function and are modeled as hinges	• Continuum-topology generation •Topology optimization	• Modular concepts combined according to rational constraint & motion rules/constraint metrics • Constraint and displacement rules generate parametric concepts, optimization finalizes design









Constraint-based method ≠ Intuition

This is a rational process

Rules of constraint

DOC = # of linearly independent constraints

DOF = 6 - DOC

Constraints have lines of action

Lines of action intersect at instant centers

Instant centers (via constraint) define motion

Basic elements

Diagrams removed for copyright reasons.
Source: Blanding, D. L. *Exact Constraint: Machine Design using Kinematic Principles*. New York: ASME Press, 1999. ISBN: 0791800857

Bars

Beams

Plates

Cross Beam Hinge

Notch Hinge

Common precision constraint types

Constraints

- 5 DOF

Diagrams removed for copyright reasons.
Source: Blanding, D. L. *Exact Constraint: Machine Design using Kinematic Principles*. New York: ASME Press, 1999. ISBN: 0791800857

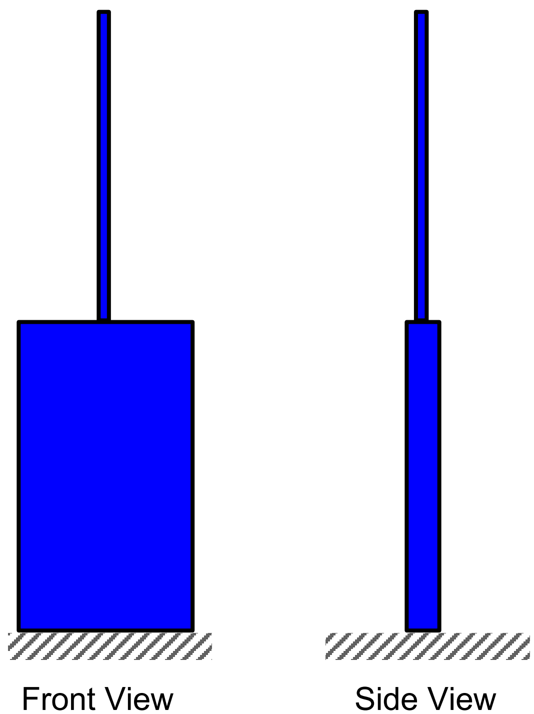
- 3 planar DOF

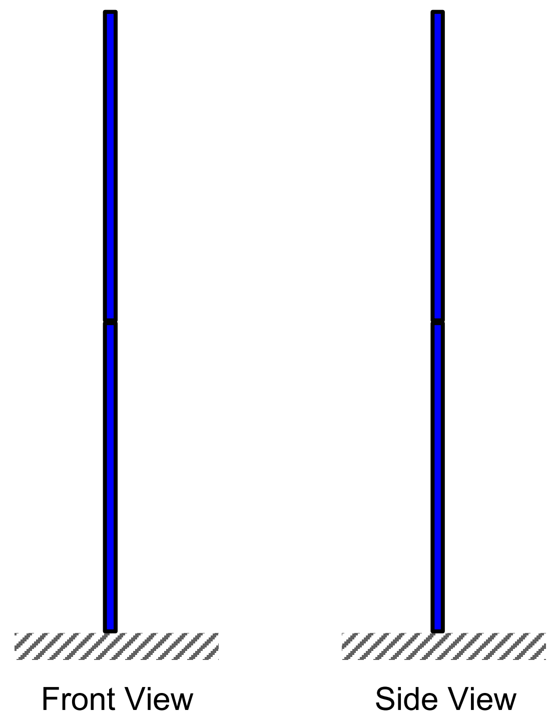
Constraint and Freedom

When connecting in series

- ❑ Add degrees of freedom with exception of redundant degrees
- ❑ Examples:

Rod at end of plate & Rod on Rod



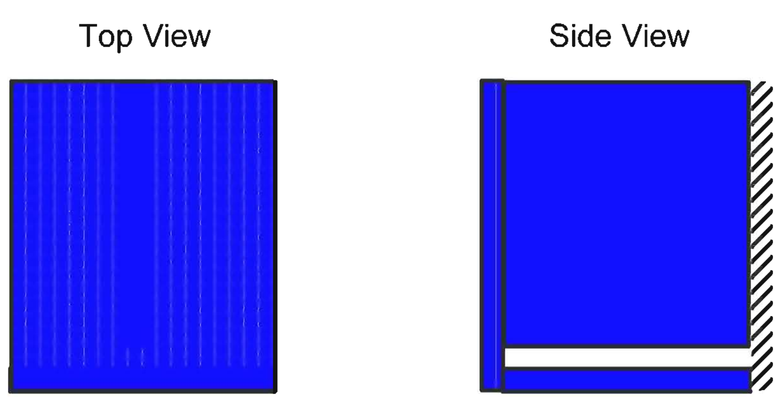


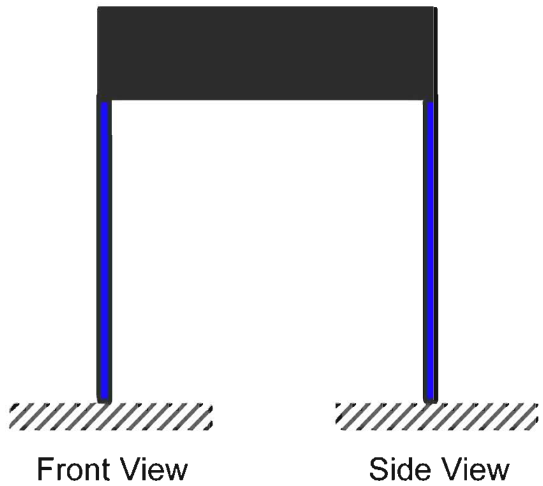
Constraint and freedom

When connecting in parallel

- Add constraints
- Example: Two sheets @ 90 degrees

Two Bars





Example: Constraint-based design

Constraint-based compliant mechanism design

STEP 1: Design requirements
Motion path, stiffness, load capacity, etc…

STEP 2: Motion path decomposition
Arcs, lines, rotation pts. sub-paths

STEP 3:Kinematic parametric concepts
Motions, constraint metric, symmetry, etc.

STEP 4:Constraint-motion addition rules
Serial, parallel, hybrid

STEP 5: Topology concept generation
Path & constraint driven

STEP 6: Concept selection phase I
Path errors & over constraint

STEP 7: Size and shape optimization
Stiffness, load capacity, efficiency, etc…

STEP 8: Concept selection phase II
Direct comparison with design requirements

Diagram of automobile steering column, rack and rotor - removed for copyright reasons.

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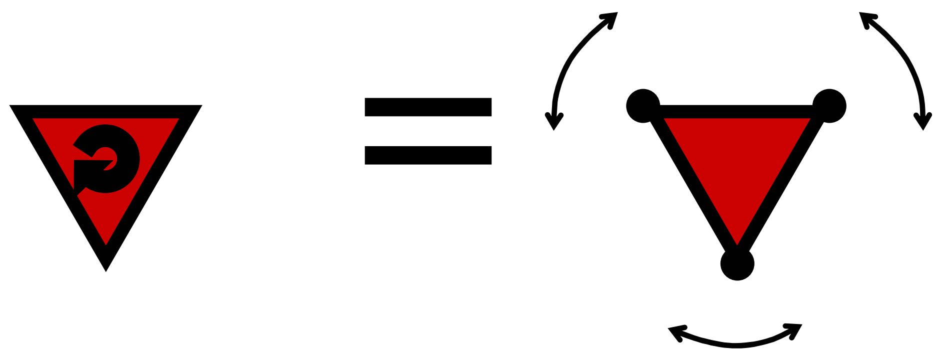

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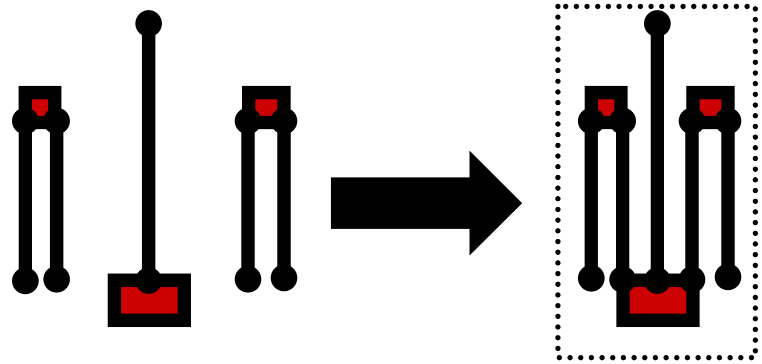

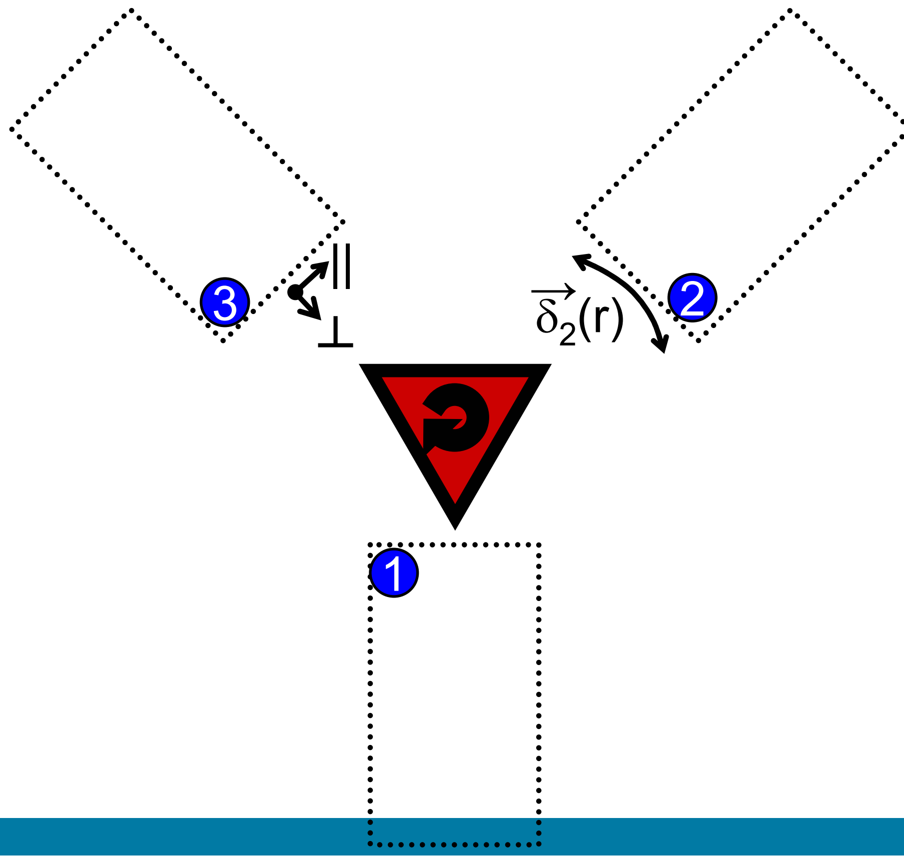

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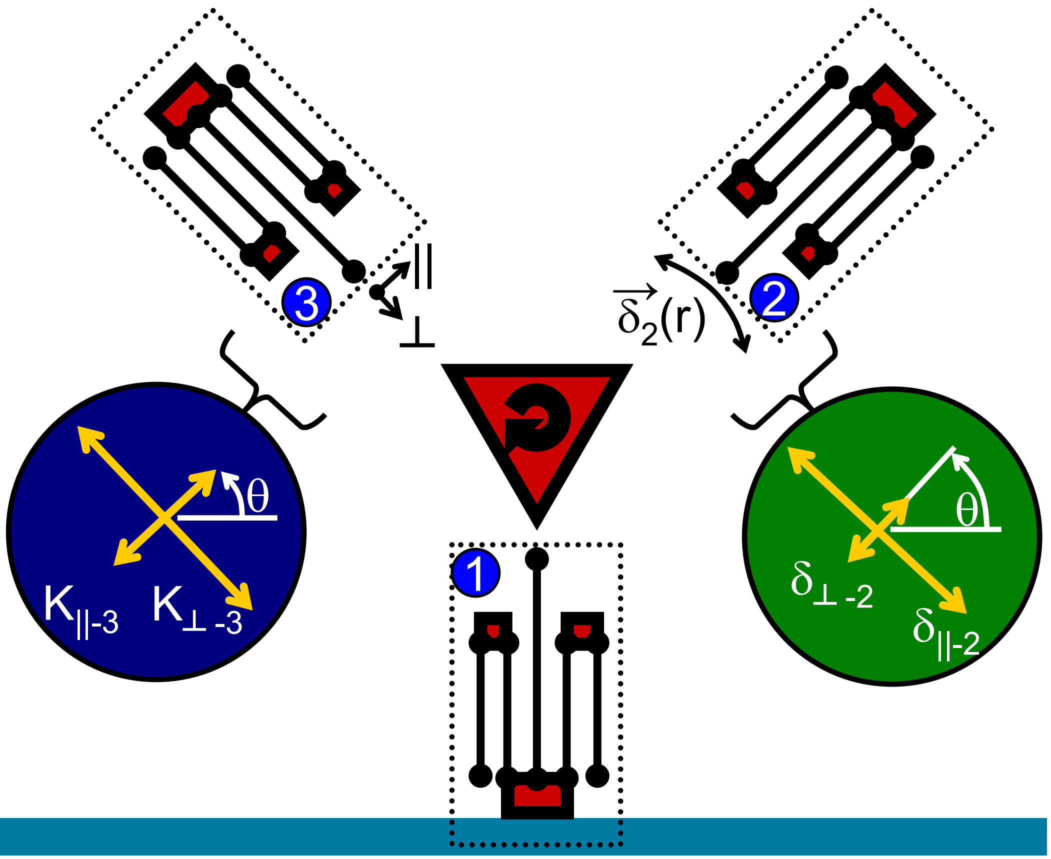
Direct comparison with design requirements

1. Series topology: Add DOF
2. Parallel topology: Add Constraints
3. Over constraint: $\frac{K_{\parallel}}{K_{\perp}} \cdot \frac{\delta_{\perp}}{\delta_{\parallel}} \rightarrow CM_k \cdot CM_\delta << 1$



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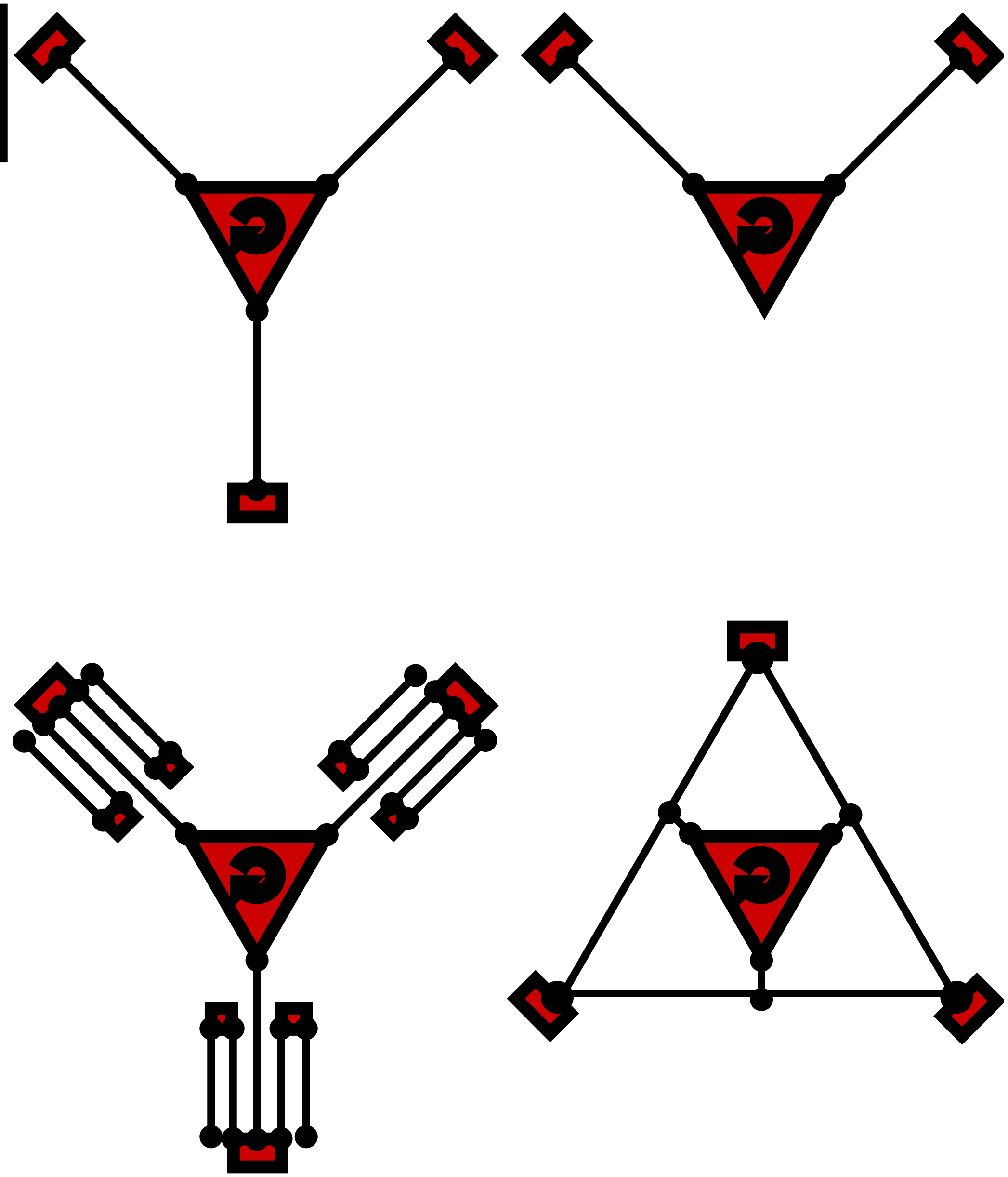

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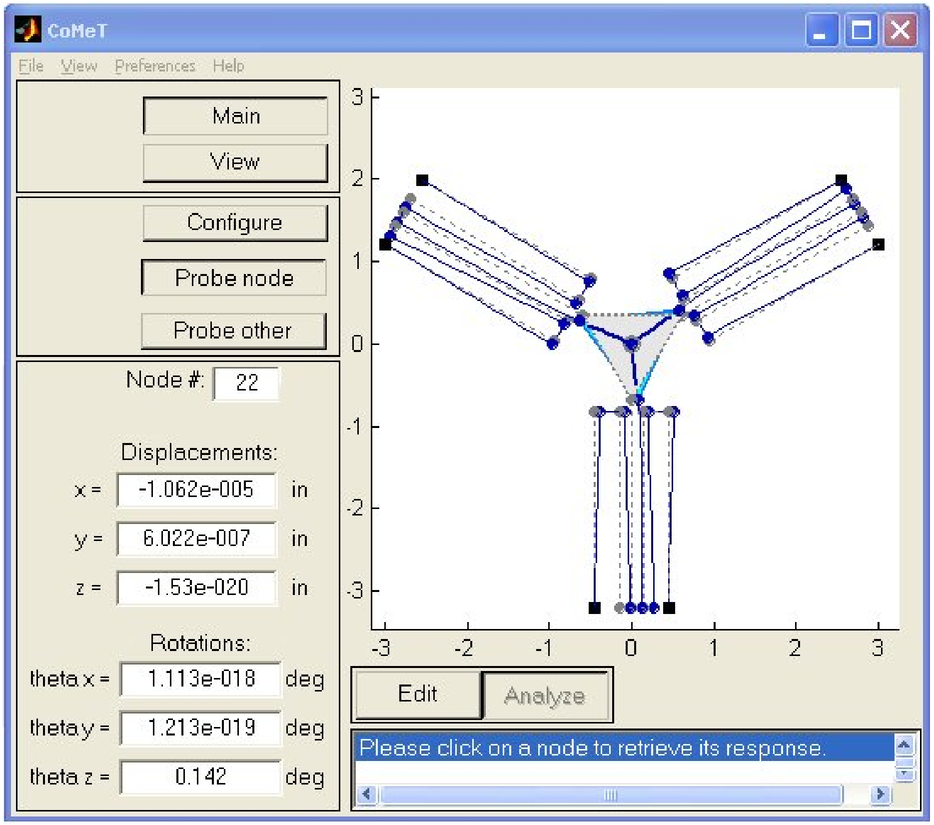
Direct comparison with design requirements

CoMeT: Compliant Mechanism Tool



Example: Constraint-based design

Constraint-based compliant mechanism design

STEP 1: Design requirements
Motion path, stiffness, load capacity, etc…

STEP 2: Motion path decomposition
Arcs, lines, rotation pts. sub-paths

STEP 3:Kinematic parametric concepts
Motions, constraint metric, symmetry, etc.

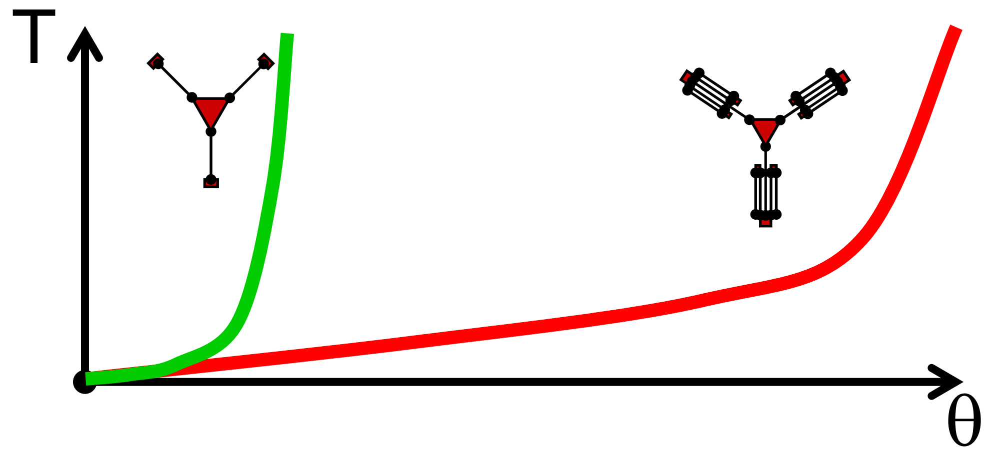
STEP 4:Constraint-motion addition rules
Serial, parallel, hybrid

STEP 5: Topology concept generation
Path & constraint driven

STEP 6: Concept selection phase I
Path errors & over constraint

STEP 7: Size and shape optimization
Stiffness, load capacity, efficiency, etc…

STEP 8: Concept selection phase II
Direct comparison with design requirements



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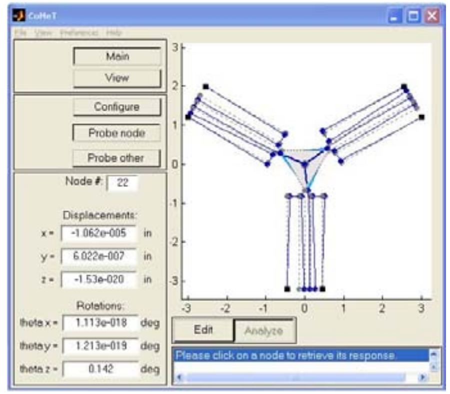

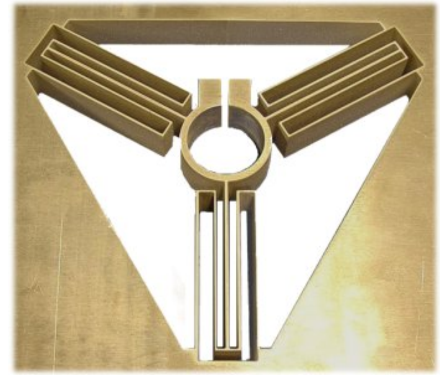

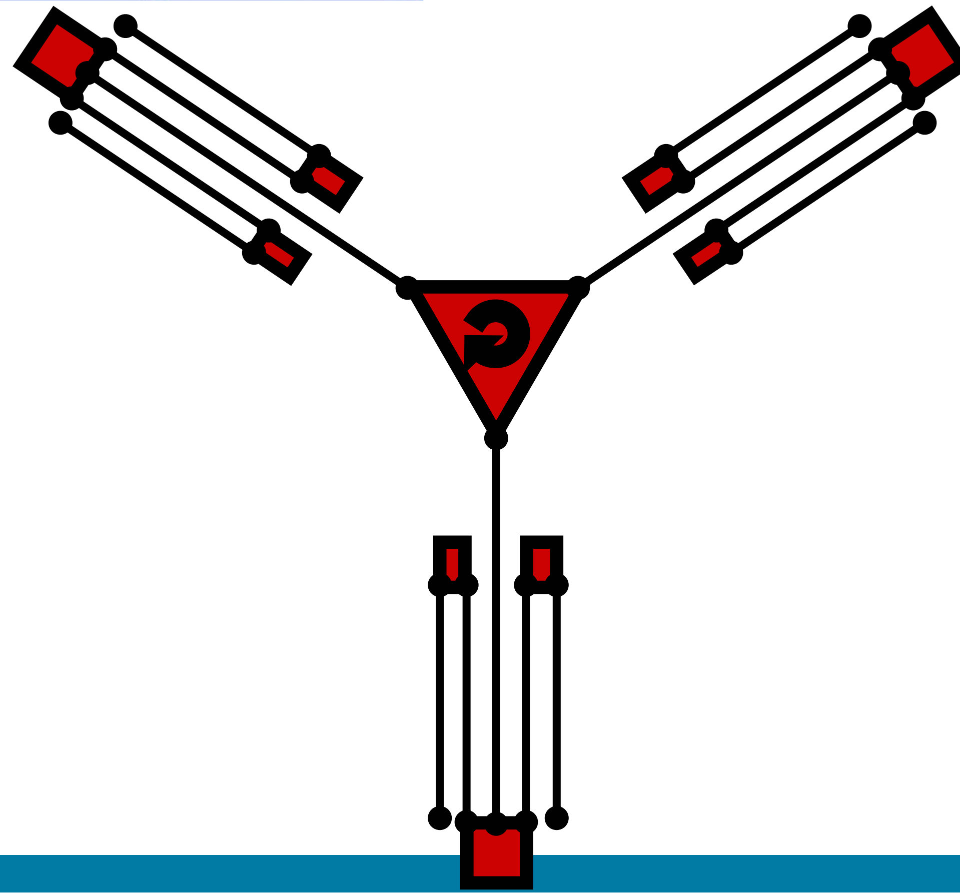

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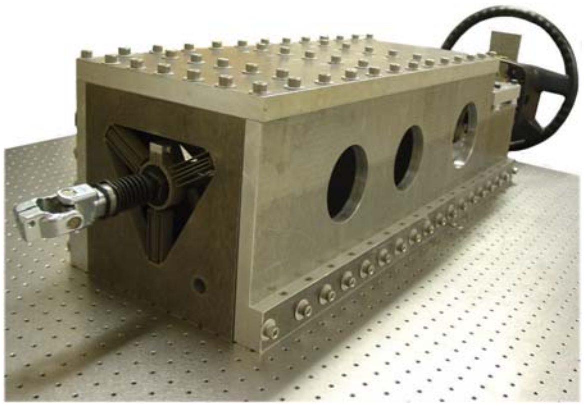
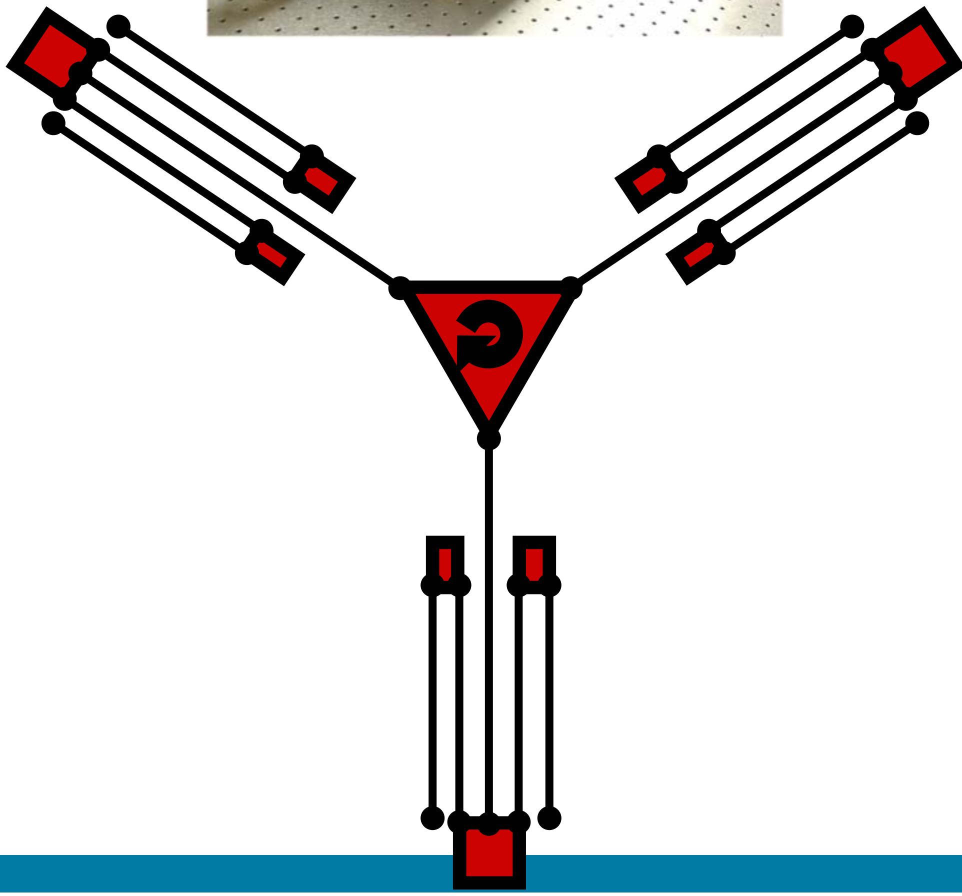
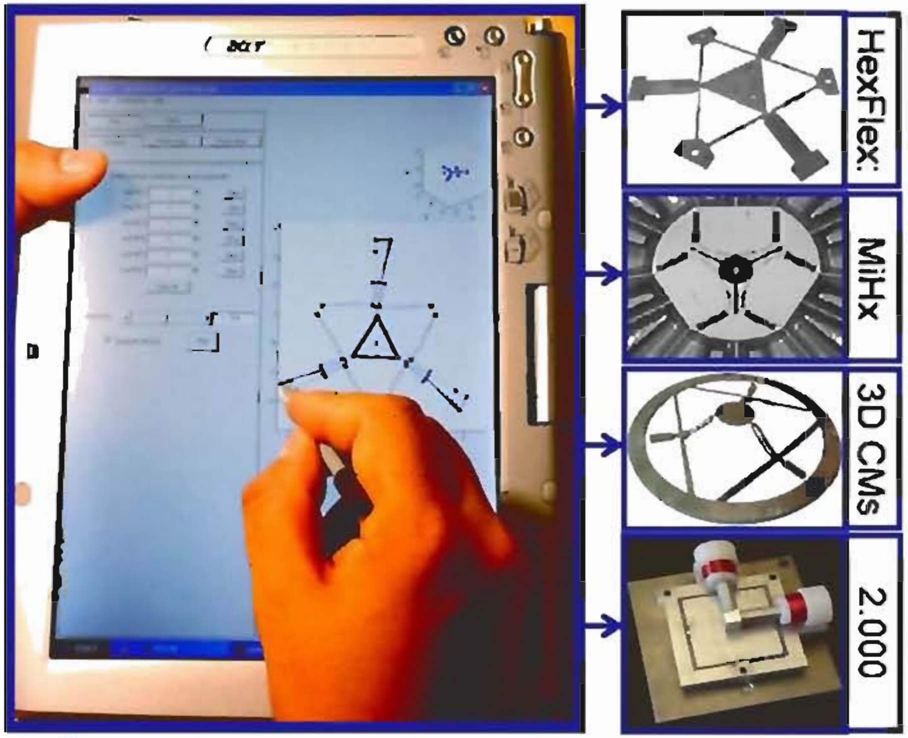




Compliant mechanism tool: CoMeT



$$\delta_{Stage} = S_D \cdot \Delta_{Actuator}$$

Or

$$\delta_{Stage} = S_F \cdot F_{Actuator}$$

$$\begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} N_{x1} & N_{x2} & N_{x3} & 0 & 0 & 0 \\ N_{y1} & N_{y2} & N_{y3} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{z4} & N_{z5} & N_{z6} \\ 0 & 0 & 0 & N_{\theta x4} & N_{\theta x5} & N_{\theta x6} \\ 0 & 0 & 0 & N_{\theta y4} & N_{\theta y5} & N_{\theta y6} \\ N_{\theta z1} & N_{\theta z2} & N_{\theta z3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{bmatrix}$$

Design activity

Problem

Design a mechanical filter which:

- Gij = 0.05 (factor of 20 filtering)
- Range of 0.5 mm with less than 5 micron PE
- Envelope: 5 x 5 x 0.125 inches

Give us enough information to:

- Understand your constraint topology
- Fabricate it
- Assume it is aluminum

Email journal file at end of class

You may ask any question at any time…

Assignment

Form teams of 4 now, email members to TA by 5pm Friday

Flexure reading (pp. 67-82 & 174-205)

CoMeT tutorials 1 - 3

Learn a CAD package (3 wks!!!)

Create CoMeT model of your flexure, send to TA by Monday 9am