Lecture on Temperature Distribution (11/29-04)
(Ref. Appendix 8C)

- Temperature rise at the interface is a function of the following:
  - Contact geometry (asperity, plowing particles, A_a/A_r, etc)
  - Sliding speed
  - Plowing vs sliding at the asperities
  - Applied load
  - Presence of lubricant
  - Plastic work done in the deforming material
- Temperature rise at the interface can be 1D, 2D & 3D.
- Metal cutting at high loads and speeds -- 1-D
- Sliding at low loads -- 3-D
- Accuracy of theoretical models depends on the assumptions involved. The existing models can be improved.

Temperature Distribution at the Sliding Interface

Governing Equation

$$\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial^2 \Theta}{\partial z^2} + \frac{\dot{W}}{k} = \frac{1}{\alpha}\frac{\partial \Theta}{\partial t}$$

where Θ = temperature

W = internal work done per unit volume $= \dfrac{\mathrm{d}}{\mathrm{dt}} \int \bar{\sigma}\, d\bar{\varepsilon}$

k = thermal conductivity

$\alpha = k/\rho c$ = thermal diffusivity [length^2/time]

Partition Function

$$\Theta_1(x, y, z, t) = R_1 q f_1(k_1, \rho_1, c_1, v, x, y, z, t)$$

$$\Theta_2(x, y, z, t) = (1 - R_1) q f_2(k_2, \rho_2, c_2, v, x, y, z, t)$$

@ z=0, $\Theta_1 = \Theta_2$

Moving-Heat-Source Problem

- The temperature rise at the point (x, y, z) at time t in an infinite solid due to a quantity of heat Q instantaneously released at (x', y', z') with no internal heat generation is given by

$$\theta - \theta_i = \frac{Q\alpha}{8k(\pi\alpha t)^{3/2}} \exp\left[-\frac{(x - x')^2 + (y - y')^2 + (z - z')^2}{4\alpha t} \right] \qquad (8.C3)$$

where θ_i is the initial temperature, which will be assumed to be equal to zero.

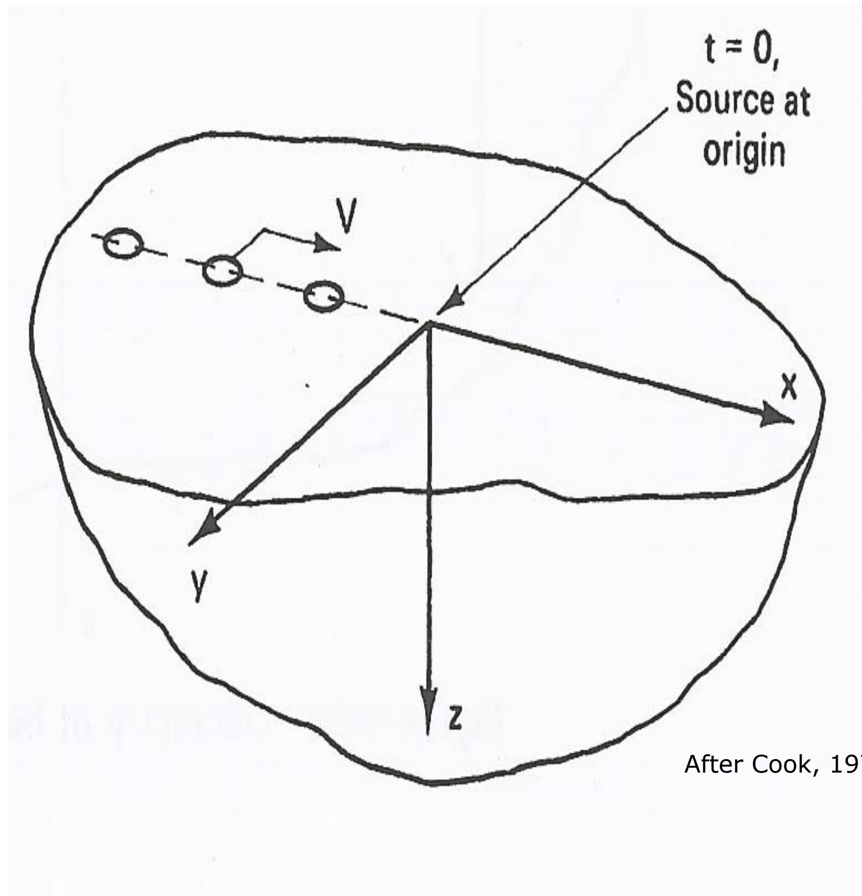
Moving Line-Heat-Source Problem

(Replacing Q with Q dy' and integrating with respect to y' from - infinity to + infinity)

$$\Theta - \Theta_i = \frac{Q}{4\pi k t} \exp\left[-\frac{(x - x')^2 + (z - z')^2}{4\alpha t} \right] \qquad (8.C4)$$

Moving-Heat-Source Problem

Point heat source moving at a constant velocity along the x-axis on the surface of a semi-infinite half space $z > 0$



After Cook, 1970.

Moving-Heat-Source Problem

If we let heat source be at the origin at t = 0, then at time t ago, the heat source was at x' = V t. Temperature due to heat (dQ =Q dt) liberated at (x = -Vt) is

$$d\Theta_{x,y,z} = \frac{2Q\,dt}{8\rho c(\pi \alpha t)^{3/2}} \exp\left[-\frac{(x+Vt)^2+y^2+z^2}{4\alpha t}\right] \qquad (8.C5)$$

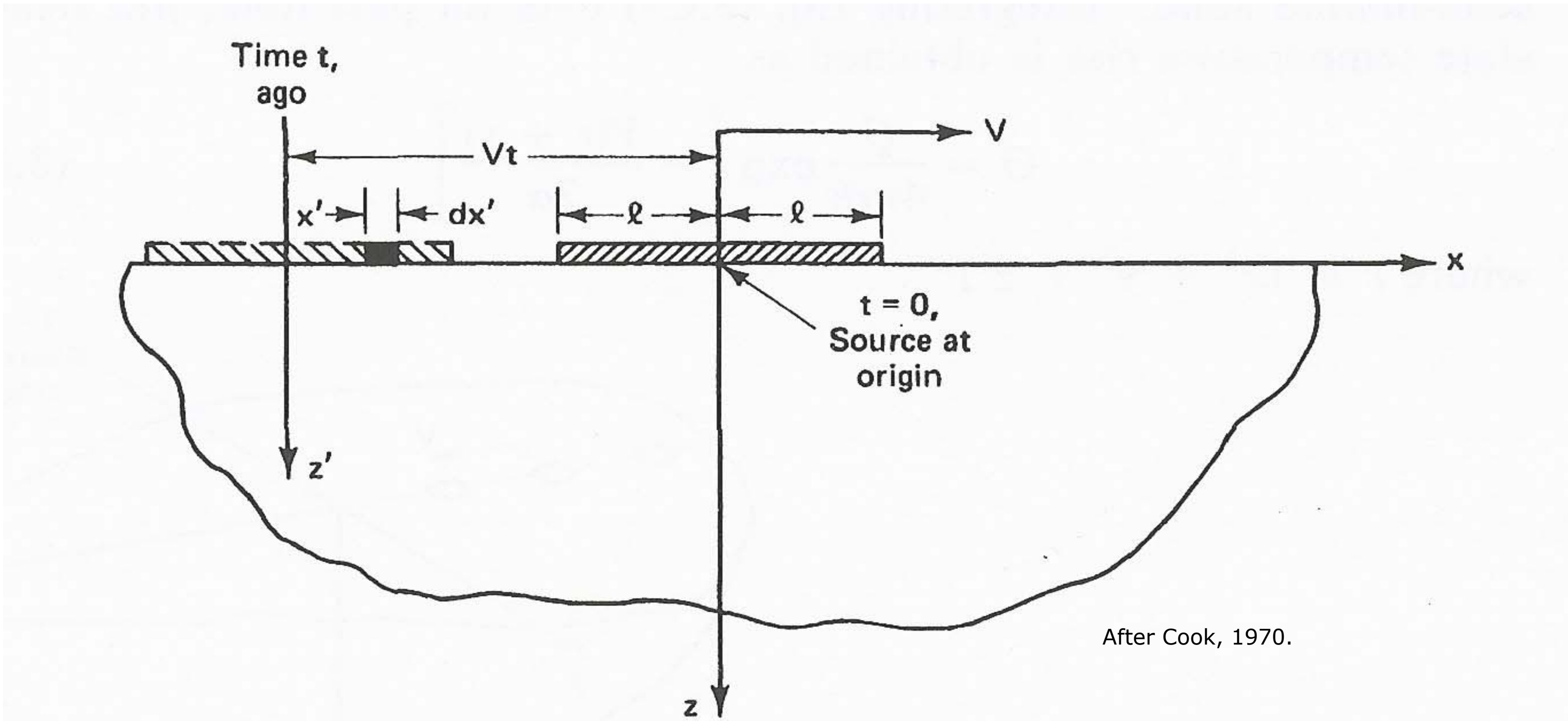
Point Source Problem

Integrating Eq. (8.C5) over all past time, the steady state temperature rise is

$$\theta = \frac{Q}{4\pi r k} \exp\left[-\frac{V(r + x)}{2\alpha}\right] \tag{8.C6}$$

where $r = (x^2 + y^2 + z^2)^{1/2}$

Geometry of Band Source Problem



Band Source Problem

The temperature at (x, y. z) at t = 0 due to a line heat source at dQ= 2q dx'dt per unit length, parallel to the y-axis and rough the point (x'-Vt, 0, 0) is

$$\Theta = \frac{q\,dx'\,dt}{2\pi kt} \exp\left[-\frac{(x - x' + Vt)^2 + z^2}{4\alpha t} \right] \qquad (8.C7)$$

Band Source Problem

To find the temperature at zero time for a band of length $2l$ which has been moving for an infinite time, Eq. (8.C7) may be integrated with respect to x' from $-l$ to l and with respect to t from $-\infty$ to 0. The solution may be written as

$$\Theta = \frac{2q\alpha}{\pi k V} \int_{X-L}^{X+L} K_0(Z^2 + u^2)^{1/2} \exp(-u)\, du \tag{8.C8}$$

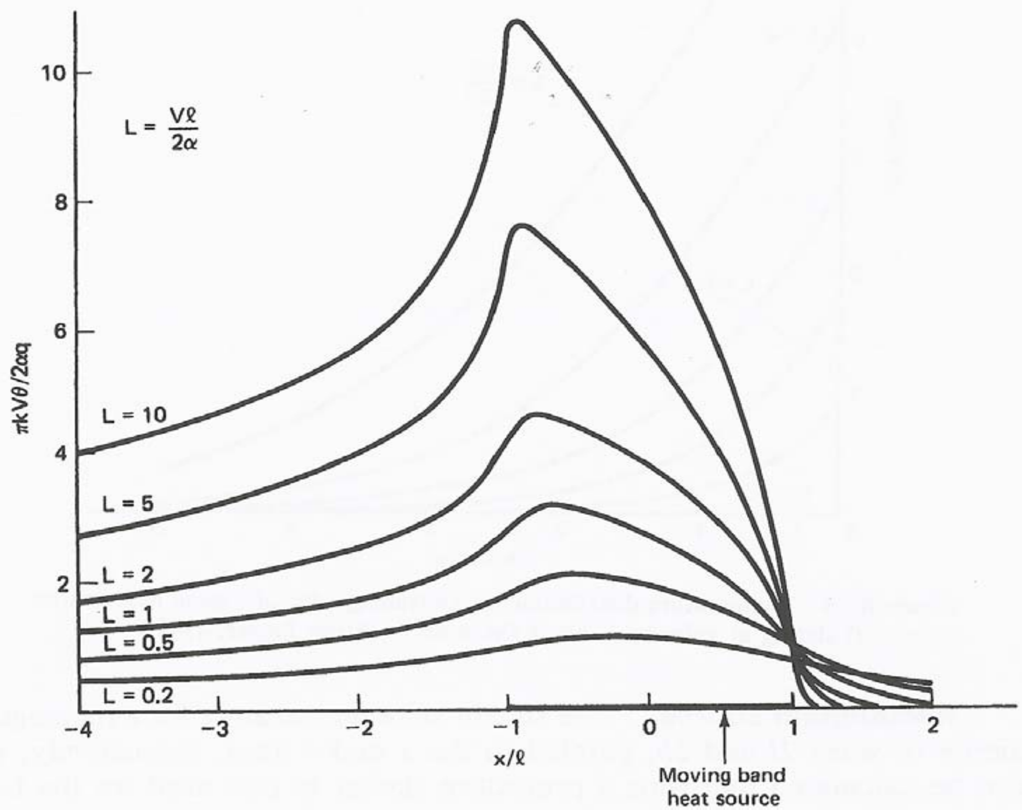
where $K_0(s)$ is the modified Bessel function of the second kind and the dimensionless quantities are defined as

$$X = \frac{Vx}{2\alpha} \qquad Y = \frac{Vy}{2\alpha} \qquad Z = \frac{Vz}{2\alpha} \qquad L = \frac{Vl}{2\alpha} \tag{8.C9}$$

Band Source Problem

$$\Theta_{max} \simeq 1.6 \frac{ql}{k} \left(\frac{Vl}{\alpha} \right)^{-1/2} \qquad (8.C10)$$

$$\overline{\Theta} \simeq \frac{2}{3} \Theta_{max}$$



Temperature rise at the sliding surface as a function of position and sliding speed (after Jaeger, 1942).

Band Source Problem

L > 10 (high sliding speed)

$$\Theta_{max} \simeq 1.6 \frac{ql}{k} \left(\frac{Vl}{\alpha} \right)^{-1/2}$$

$$\overline{\Theta} \simeq \frac{2}{3} \Theta_{max}$$

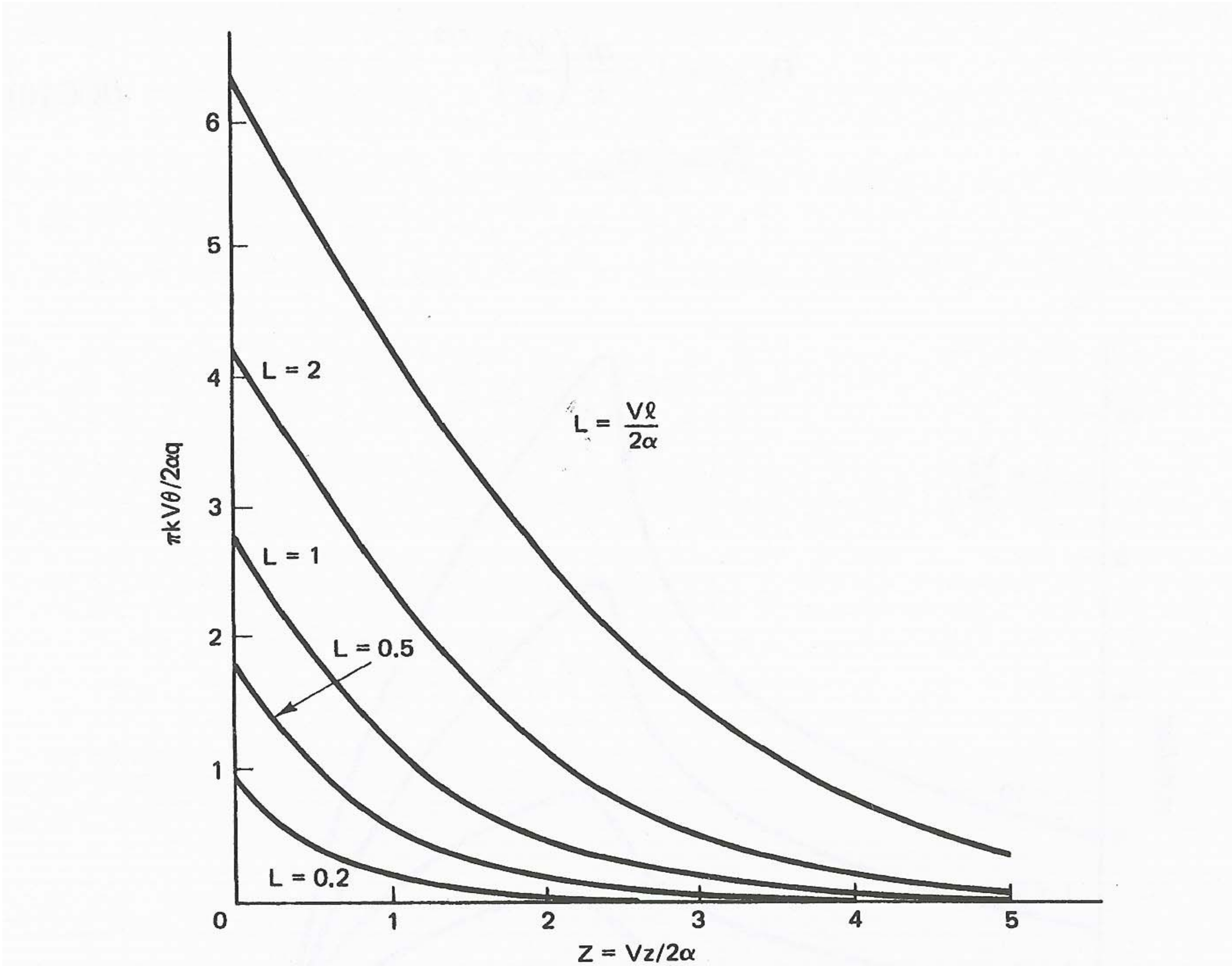
Band Source Problem

- L < 0.5 (low speeds)

$$\Theta_{max} \simeq 0.64 \frac{ql}{k} \ln \frac{6.1\alpha}{Vl}$$

$$\overline{\Theta} \simeq 0.64 \frac{ql}{k} \ln \frac{5\alpha}{Vl}$$

Band Source Problem



Temperature distribution at the trailing edge of a band heat source (z = -l) sliding velocity V along the x axis. (after Jaeger, 1942).

(2*l*x2*l*) Square Source Problem

- L > 10 (high sliding speed)

$$\theta_{max} = 1.6 \frac{ql}{k}\left(\frac{Vl}{\alpha}\right)^{-\frac{1}{2}}$$

$$\overline{\theta} = \frac{2}{3}\theta_{max} \frac{ql}{k}\left(\frac{Vl}{\alpha}\right)^{-\frac{1}{2}}$$

$(2l\text{x}2l)$ Square Source Problem

- L < 0.5 (low sliding speed)

$$\theta_{\max} = 1.1 \frac{ql}{k}$$

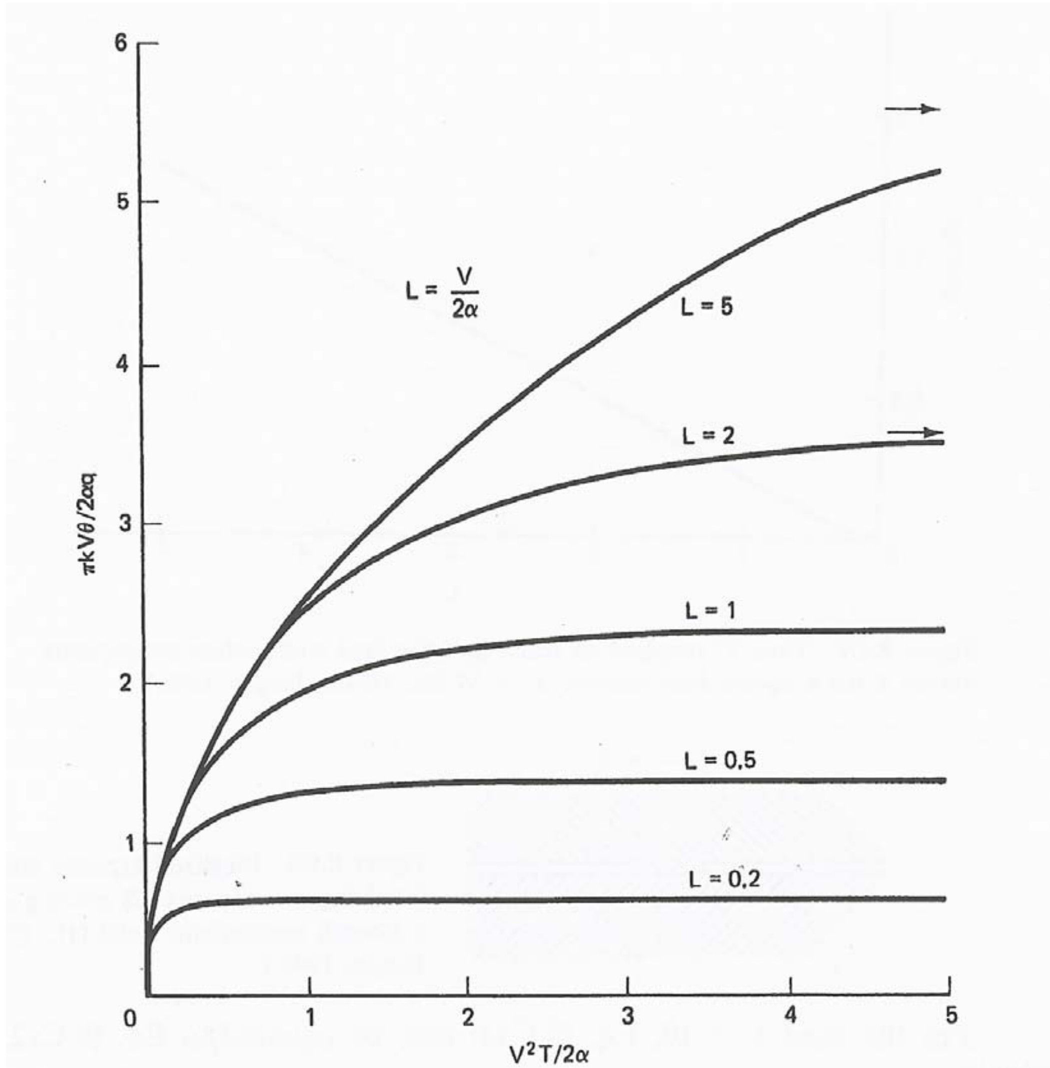
$$\overline{\theta} = 0.95 \frac{ql}{k}$$

Stationary Heat Source Problem

$$\theta_{\max} = 1.1 \frac{ql}{k}$$

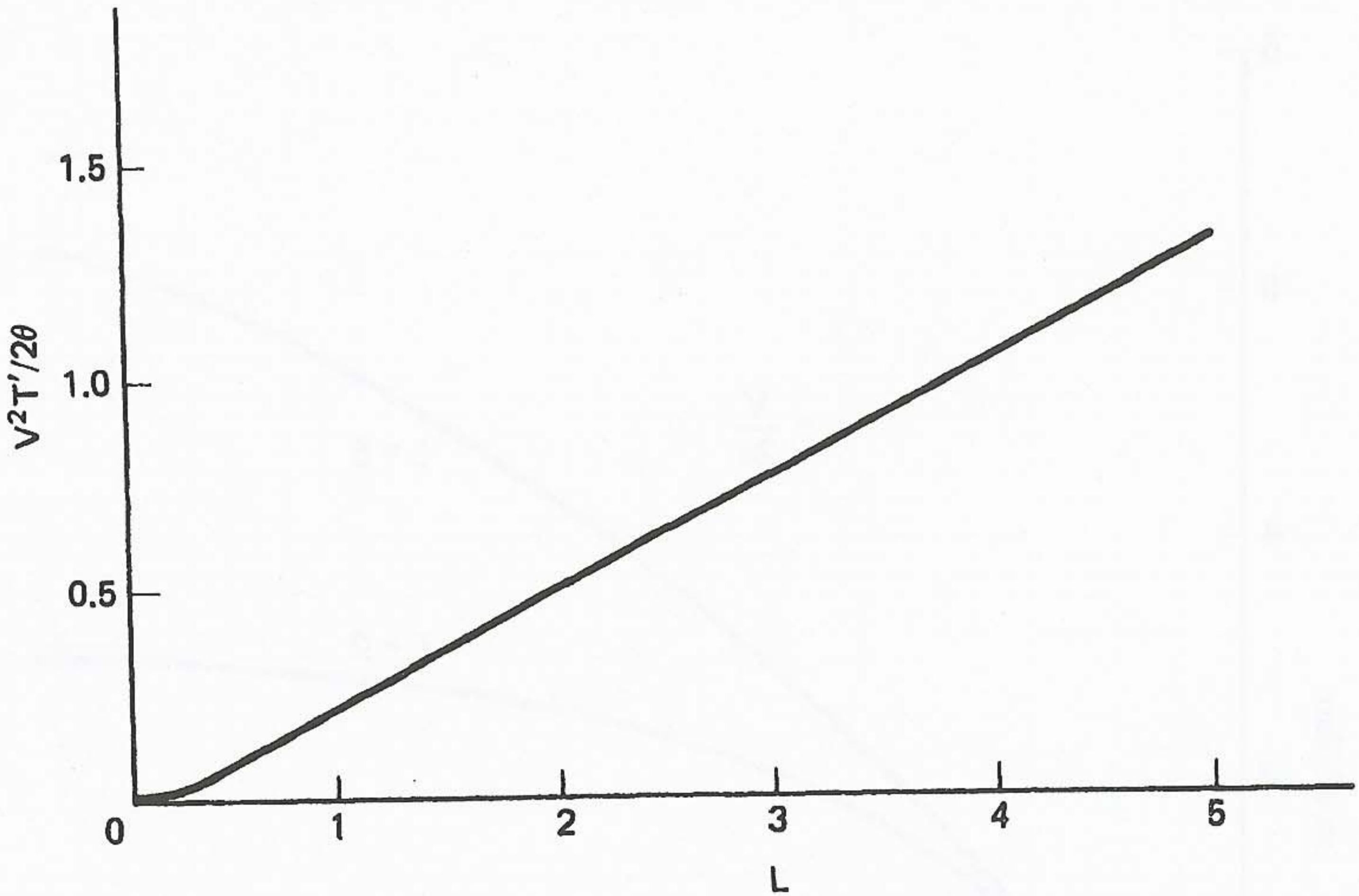
$$\overline{\theta} = 0.95 \frac{ql}{k}$$

Transient Heat Source Problem



Temperature rise at the center of a square heat source that has been moving for a finite period of time ***T***. (after Jaeger, 1942).

Transient Heat Source Problem



Time T' to reach half the final steady-state temperature versus L
for a square heat source. $L=Vl/2\alpha$ (after Jaeger, 1942).

Transient Heat Source Problem

Relationship between the time taken to reach the final temperature and the sliding velocity

$$VT = 2.5\ell$$

Square asperity contact sliding on a smooth semi-infinite solid

When $L = \dfrac{V\ell}{2\alpha_1}$ is small,

$$\bar{\theta} = 0.95\frac{\mathrm{rq}\ell}{\mathrm{k}_1} = 0.95\frac{(1-\mathrm{r})\mathrm{q}\ell}{\mathrm{k}_2}$$

$$r = \frac{k_1}{k_1 + k_2}$$

Square asperity contact sliding on a smooth semi-infinite solid

When $L > 10$,

$$\overline{\theta} = 1.6\frac{\mathrm{r}\mathrm{q}\ell}{\mathrm{k}_1}\left(\frac{\alpha_1}{V\ell}\right)^{1/2} = 1.1\frac{(1-\mathrm{r})\mathrm{q}\ell}{\mathrm{k}_2}$$

$$r = \frac{1}{1 + 1.45\left(k_2 / k_1\right)\left(\alpha_1 / V\ell\right)^{1/2}}$$

$$q = \tau\, V$$

Many square asperity contacts sliding on a smooth semi-infinite solid

When $L = \dfrac{Vd}{2\alpha} < \dfrac{1}{2}$, & assuming $q = \mu VH$

$$\overline{\theta}_i = 0.48 \frac{\mu H}{\rho c} \left(\frac{Vd}{\alpha} \right)$$

Many square asperity contacts sliding on a smooth semi-infinite solid

When $L = \dfrac{Vd}{2\alpha} > 10$, & assuming $q = \mu VH$

$$\bar{\theta}_i = 0.71 \frac{\mu H}{\rho c} \left(\frac{Vd}{\alpha}\right)^{1/2}$$

where

$$d = 2\ell, \qquad \frac{\mu H}{\rho c} \approx 300^o F$$

Many square asperity contacts sliding on a smooth semi-infinite solid

$$\overline{\theta} = \theta_i + \theta_a - \theta_s$$

For high veleocity

$$\overline{\theta} = (\frac{Vd}{\alpha})^{1/2} + \frac{\overline{\sigma}}{H}(\frac{V\ell}{\alpha})^{1/2} - \frac{\overline{\sigma}}{H}(\frac{Vs}{2\alpha})^{1/2}$$

For low velocity

$$\overline{\theta} = 0.95(\frac{Vd}{2\alpha} + \frac{\overline{\sigma}}{H}\frac{V\ell}{\alpha} - \frac{\overline{\sigma}}{H}\frac{Vs}{2\alpha})$$

$s =$ mean contact spacing

Rough surface sliding over another rough surface

For $L > 10$

$$\bar{\theta}_{\max} = 0.59 \frac{\mu H}{\rho c} \left(\frac{Vd}{\alpha}\right)^{1/2} + \frac{\mu \sigma}{\rho c} \left(\frac{V\ell}{\alpha}\right)^{1/2}$$

For $L < 1$

$$\bar{\theta}_{\max} = 0.26 \frac{\mu H}{\rho c} \frac{Vd}{\alpha} + \frac{\mu \sigma}{\rho c} \frac{V\ell}{\alpha}$$

$s =$ mean contact spacing

Experimental results

Diagram removed for copyright reasons.
See Figure 8.C9 and 8.C10 in [Suh 1986]: Suh, N. P. *Tribophysics*.
Englewood Cliffs NJ: Prentice-Hall, 1986. ISBN: 0139309837.