

- Probability Estimation Through the Indicator Function
- System Reliability Example With Crystal Ball
- Same Seed to Compare Alternatives
- Modeling Correlation Between Random Numbers
- Option Pricing using Crystal Ball

# Probability Estimation Through $E[I_A]$

## Problem

Estimate the probability of event **A** happening.

## Method

Define an Indicator function  $I_A$  as the following

$I_A = 1$  if event **A** occurs and 0 otherwise

$$P(\mathbf{A} \text{ occurs}) = E[I_A]$$

# Probability Estimation Through $E[I_A]$

Proof:

$$P(\mathbf{A} \text{ occurs}) = E[I_A]$$

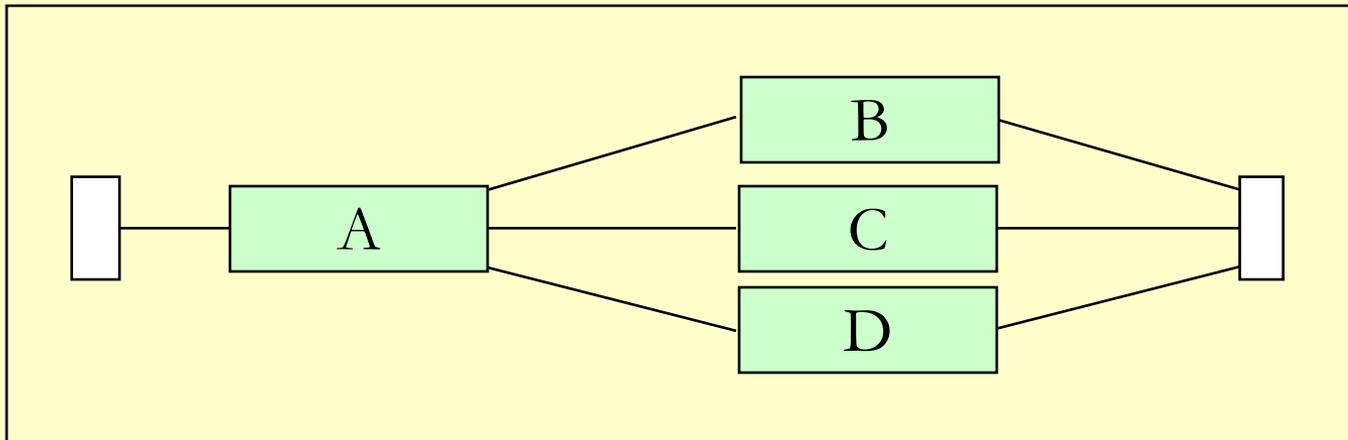
since,

$$\begin{aligned} E[I_A] &= 1 * P(\mathbf{A} \text{ occurs}) + 0 * P(\mathbf{A} \text{ doesn't occur}) \\ &= P(\mathbf{A} \text{ occurs}) \end{aligned}$$

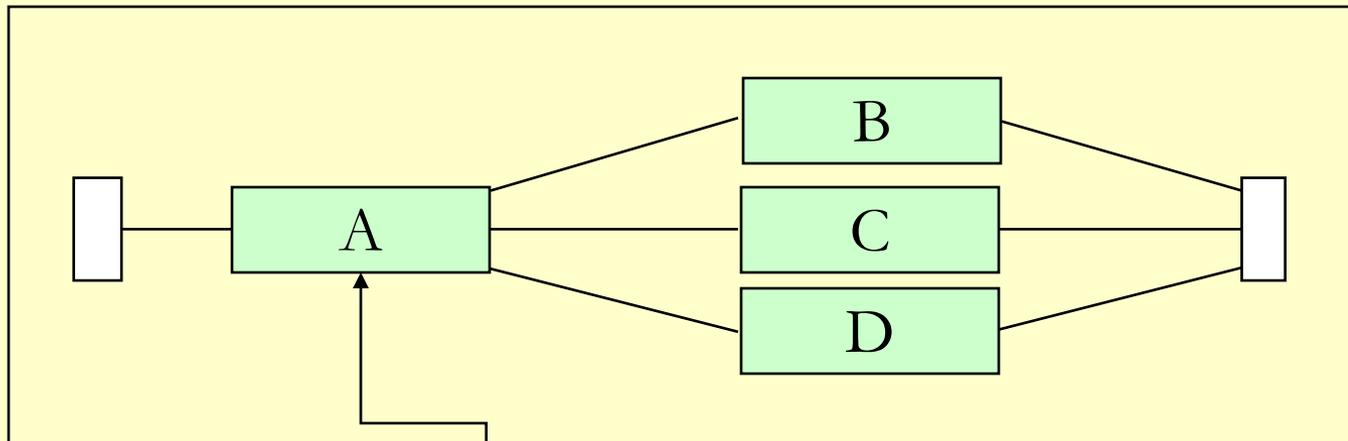
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# System Reliability Example

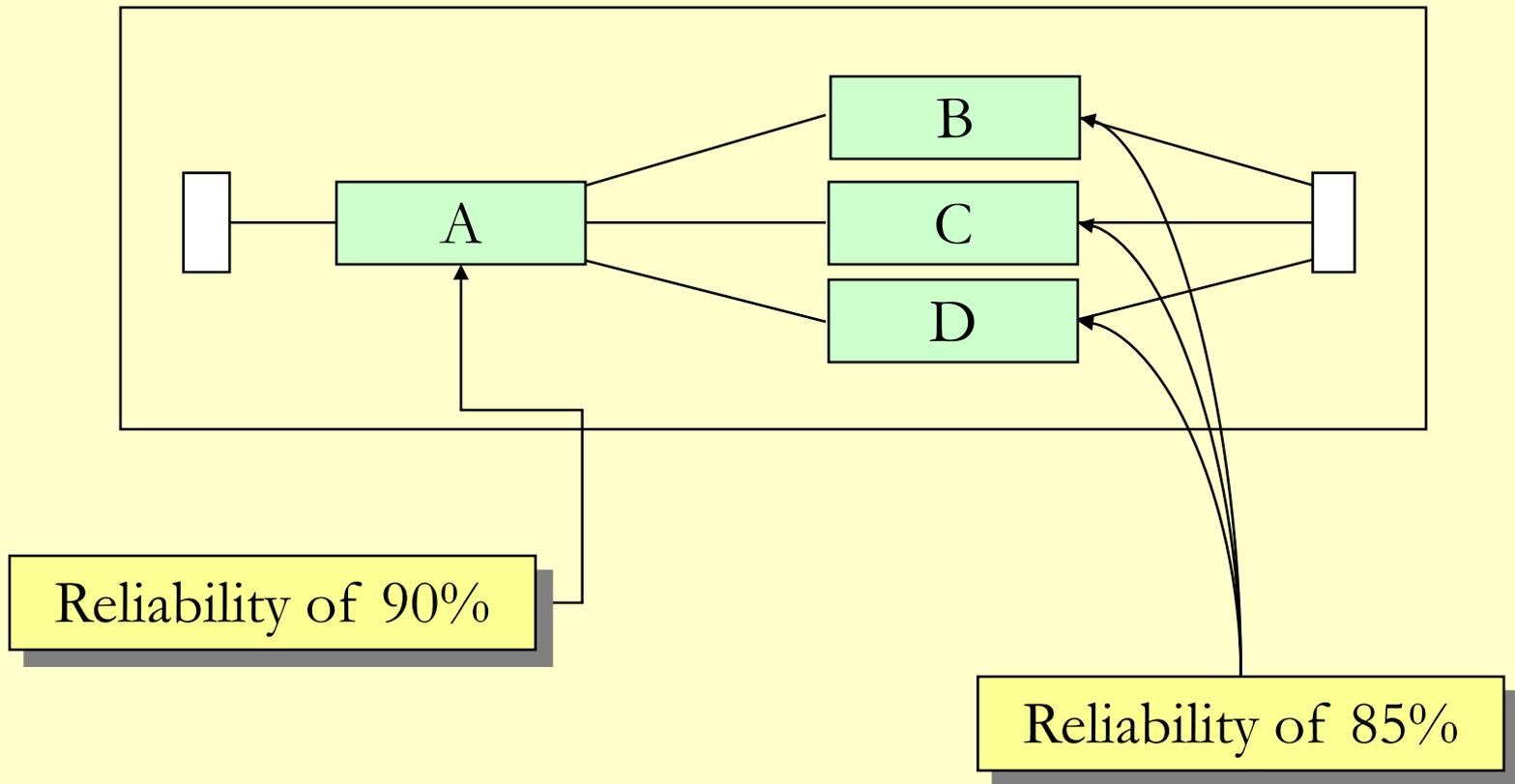


# System Reliability Example

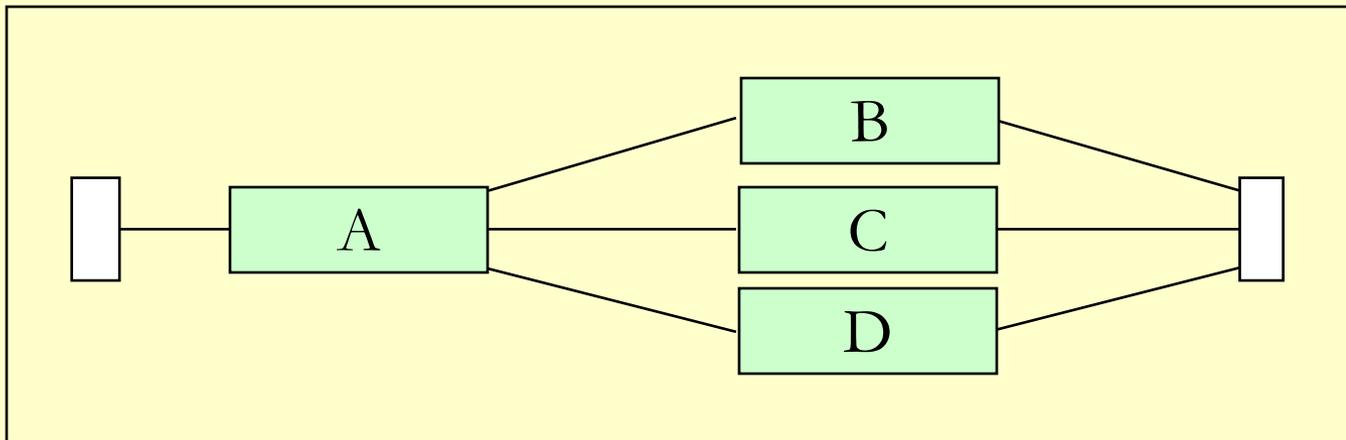


Reliability of 90%

# System Reliability Example



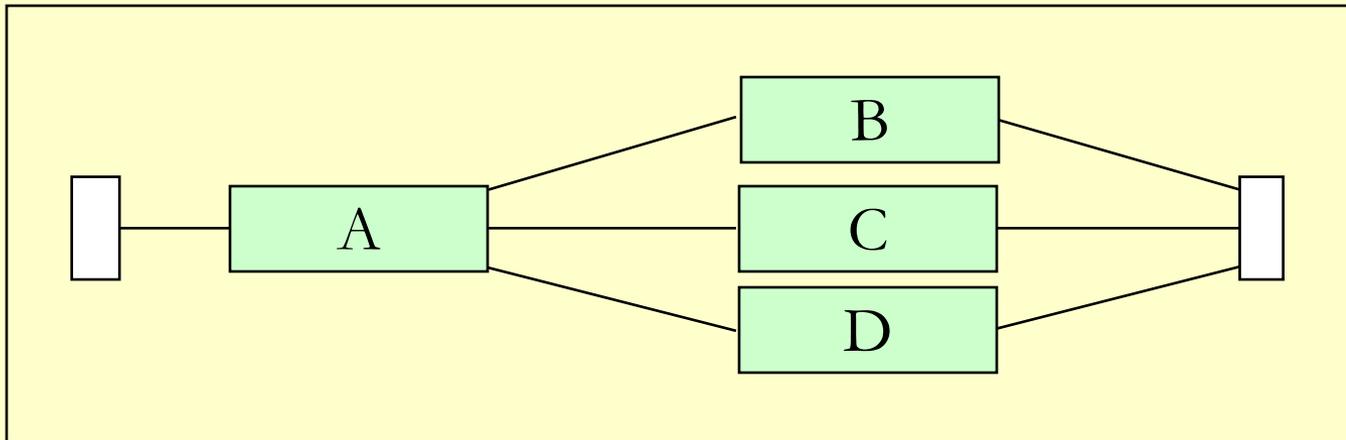
# System Reliability Example



System fails when either

1. A fails, or
2. B, C, and D fail together
3. or both

# System Reliability Example



See Excel file: Component Failure.xls

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# Using Same Seed for Simulation Runs

- Go to Run > Run Preferences > Sampling
- Enable “Use same sequence of random numbers”
- Set Initial Seed Value to a value of your choice

# Using Same Seed for Simulation Runs

## Uses

Compare two systems with different options for

- same arrival pattern of customers,
- same job schedules, etc.

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# Modeling Correlated Random Numbers

To model correlated random numbers

- Select cell
- Click on “Define Assumption” icon
- Click on “Correlate...” button.
- Set correlation with other random variables.

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# Option Pricing Using Crystal Ball

A stock for IBM incorporated was selling on the NASDAQ for \$71.49 on Tuesday, July 30, 2002.

It recorded a 52 week high of \$126.39 and a 52 week low of \$65.70 – Assume for our problem that this is the 2- $\sigma$  spread.

Thus,

$$\sigma = (\$126.39 - \$65.70)/4 = \$15.17$$

# Option Pricing Using Crystal Ball

For this problem we assume that the price of the stock a month from now is going to be Gaussian distributed with a mean of \$96 with a standard deviation of \$15.17

# Option Pricing Using Crystal Ball

## **Buttery Spreads**

A butterfly spread is a combination of a option positions that involves three strike prices.

To create a butterfly spread, a trader purchases an option with a low strike 1 price and an option with a high strike price and sells two options with an intermediate strike price.



# Option Pricing Using Crystal Ball

## Buttery Spreads

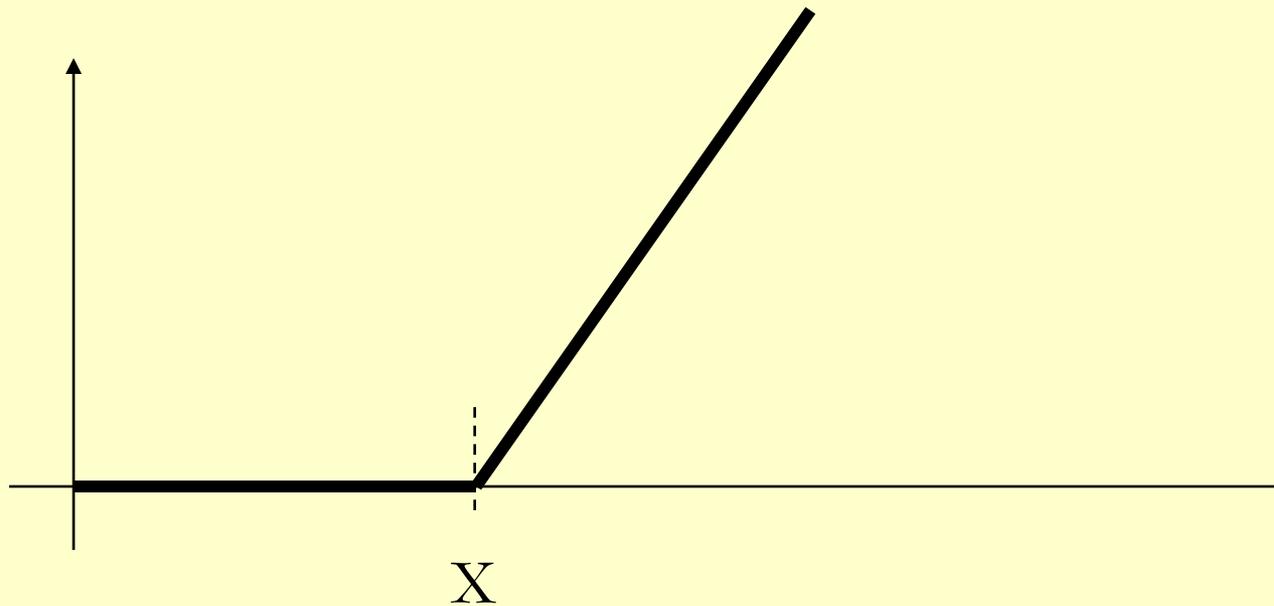
For this problem, assume that the intermediate strike price is halfway between the low and the high strike prices and that the options are European.

Denote the intermediate strike price by  $X$ , the low strike price by  $X - a$  and the high strike price by  $X + a$ .



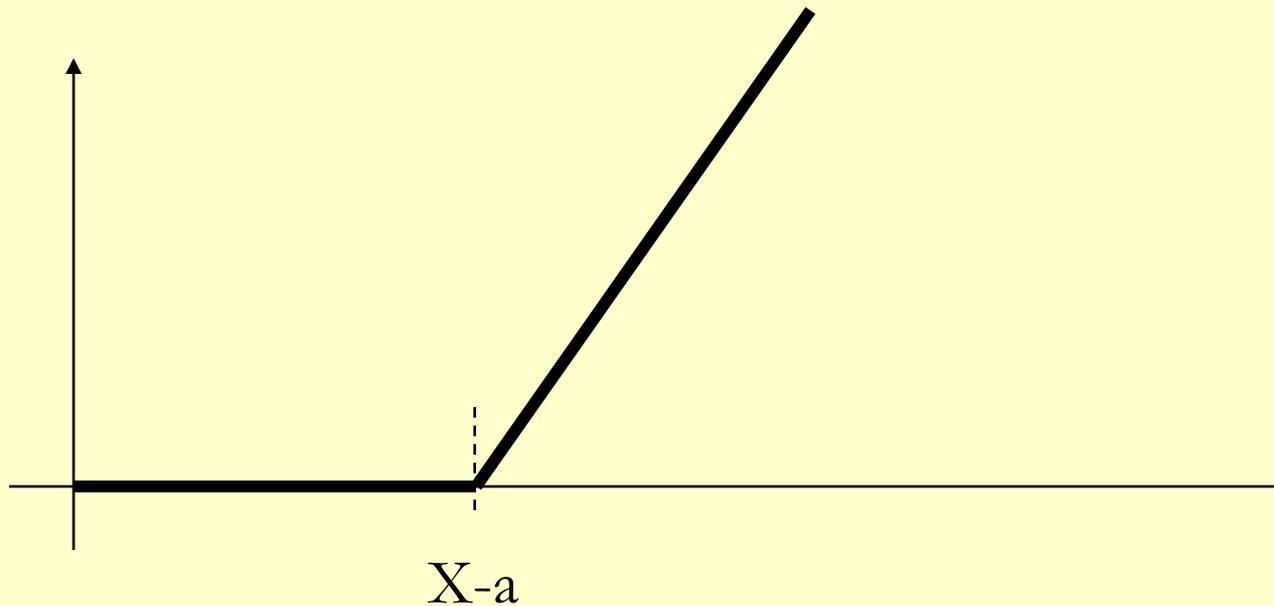
# Option Pricing Using Crystal Ball

## Simple Option with Strike Price $X$



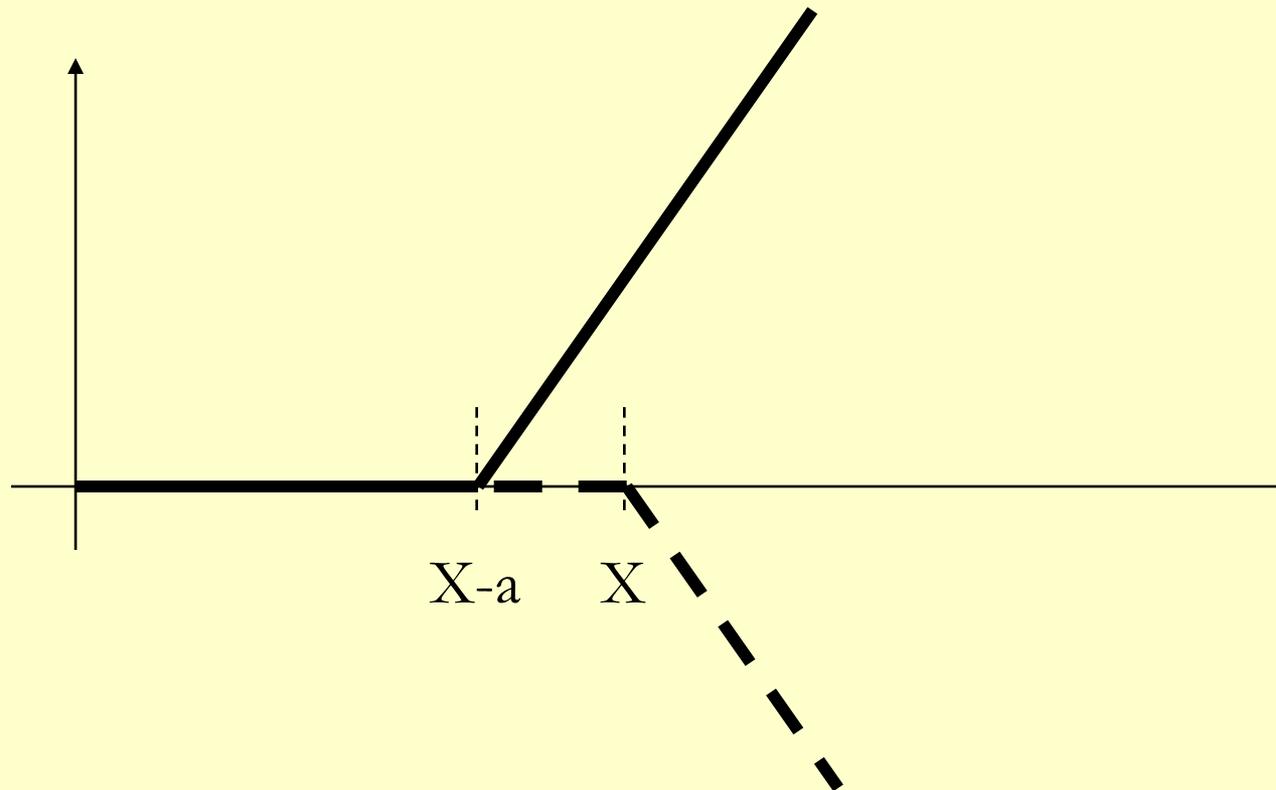
# Option Pricing Using Crystal Ball

Simple Option with Strike Price  $X-a$



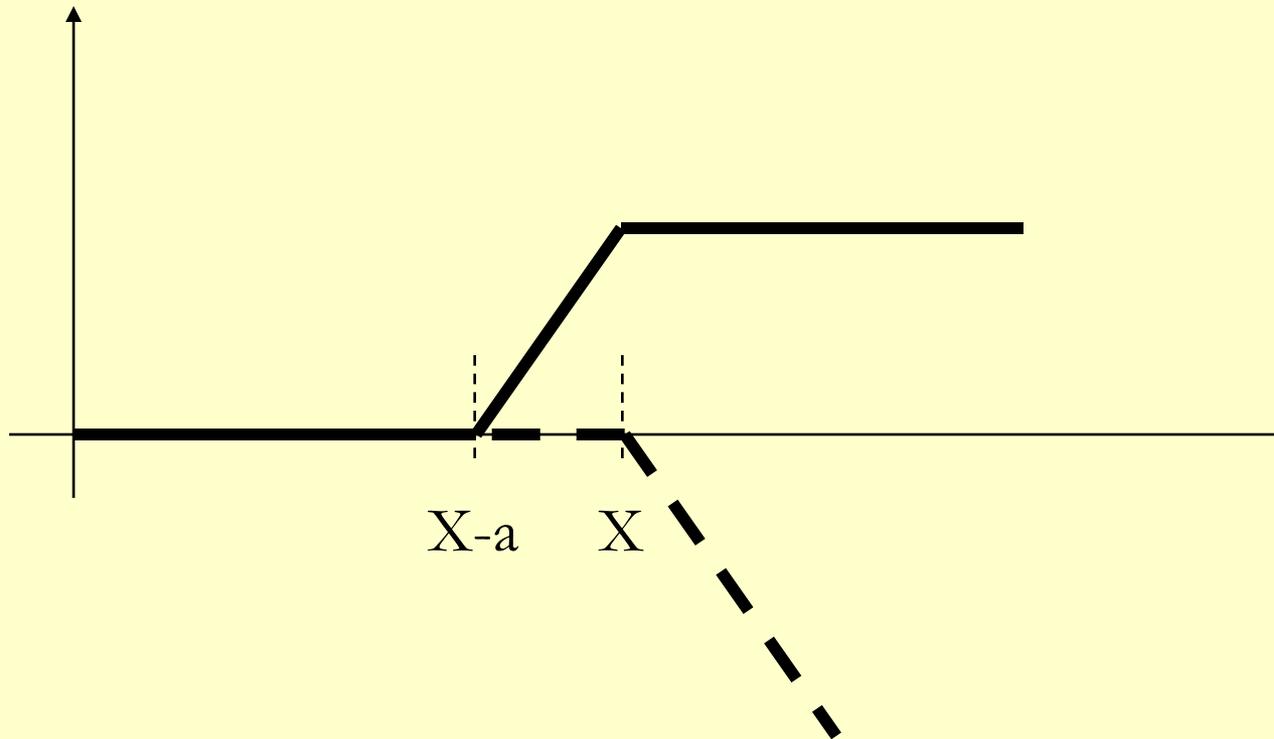
# Option Pricing Using Crystal Ball

**Sell an Option with Strike Price  $X$**



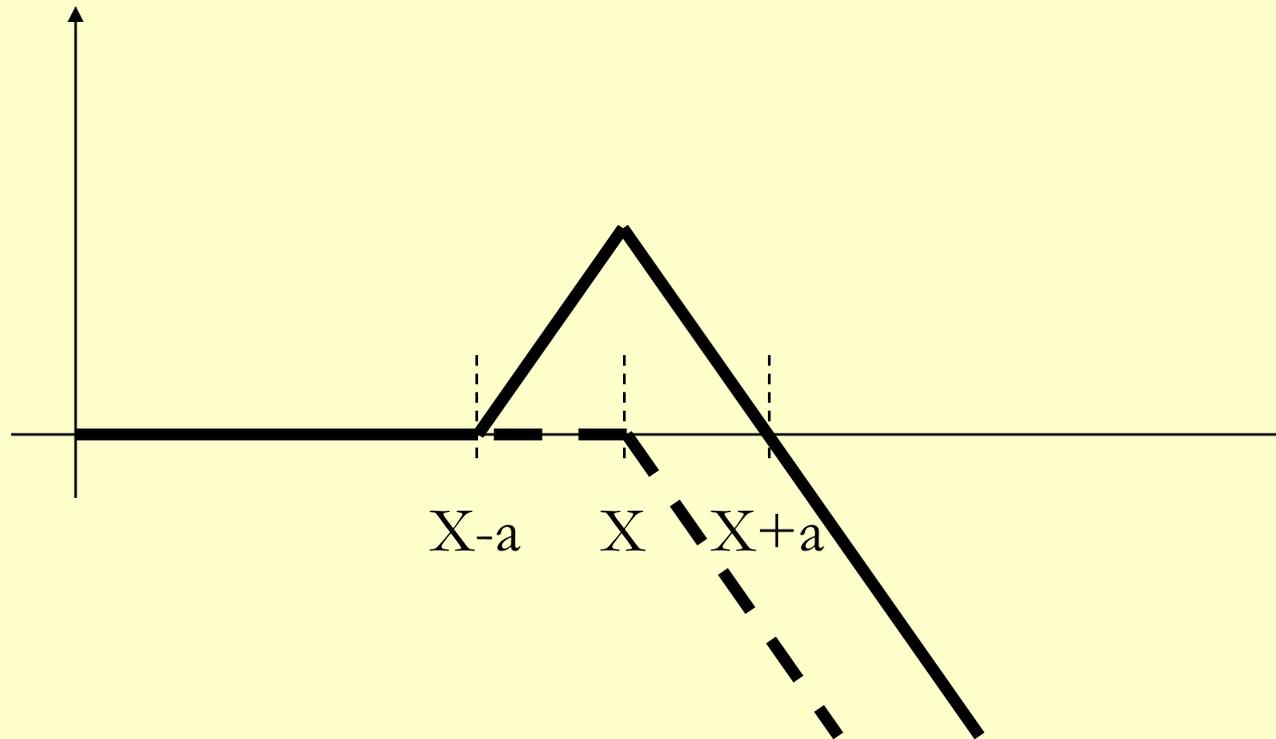
# Option Pricing Using Crystal Ball

**Sell an Option with Strike Price  $X$**



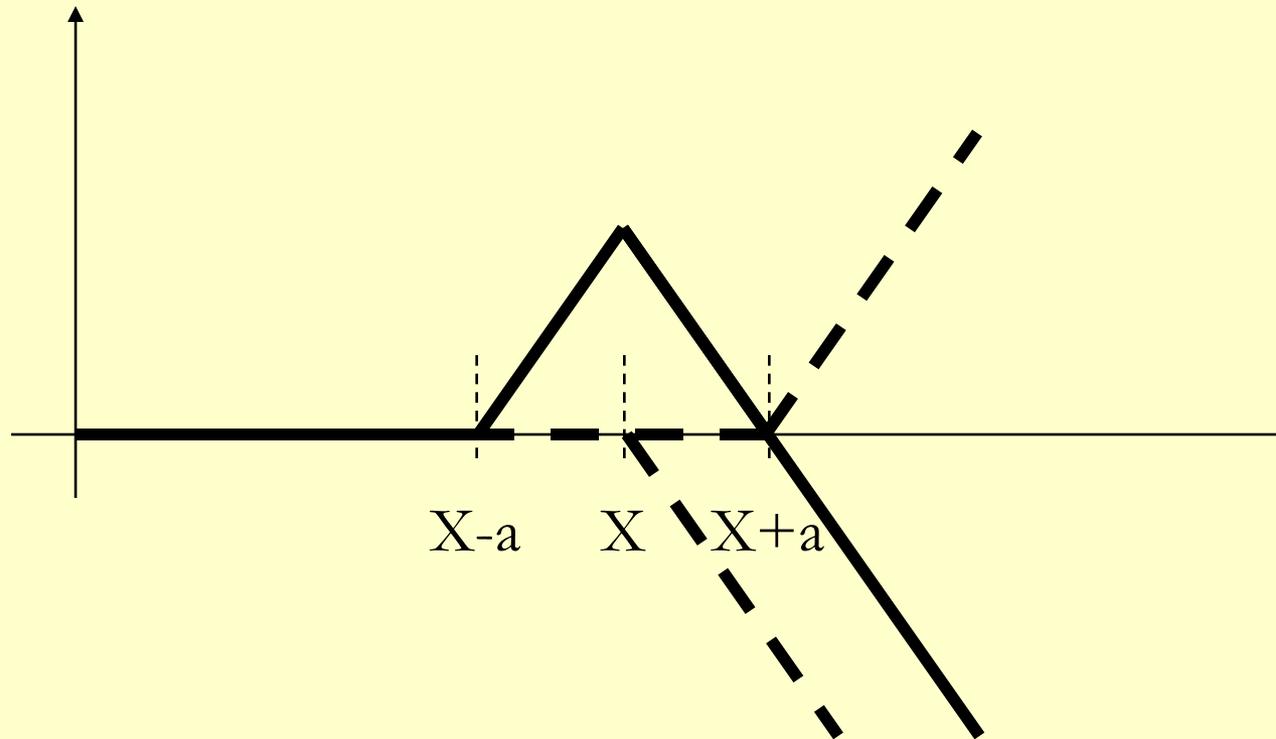
# Option Pricing Using Crystal Ball

**Sell another Option with Strike Price  $X$**



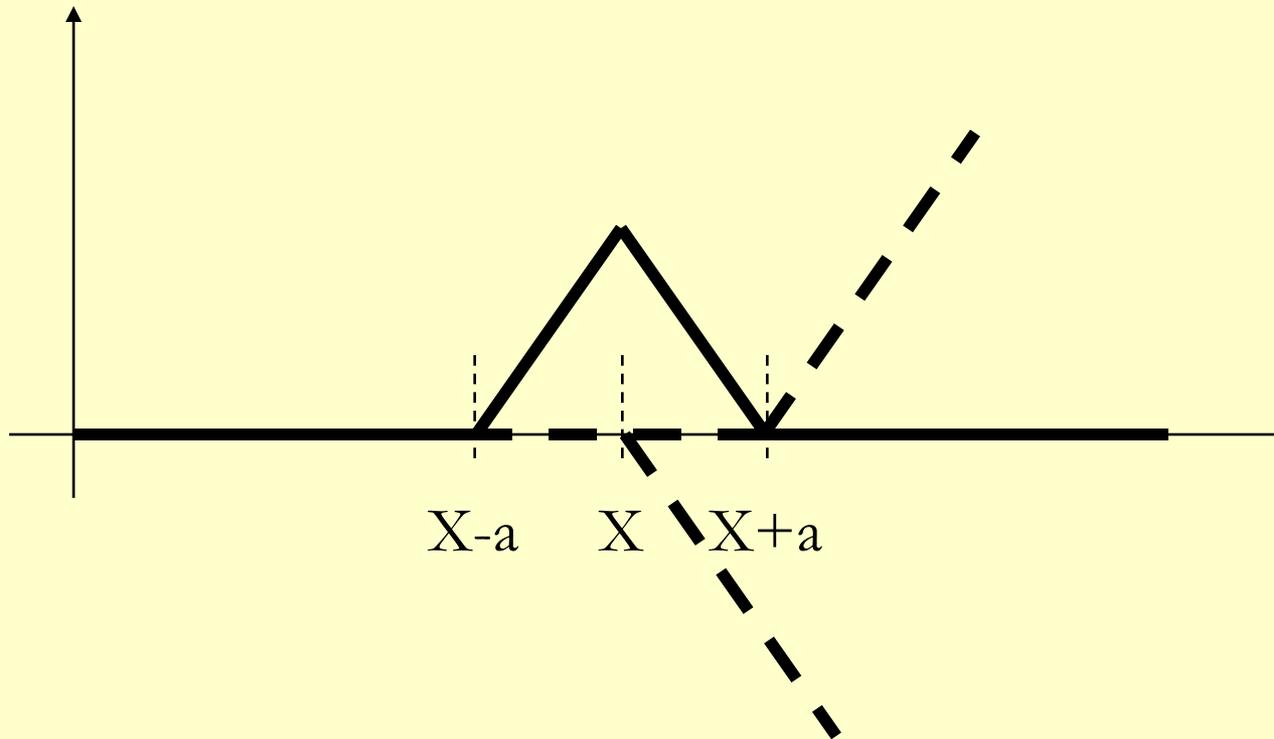
# Option Pricing Using Crystal Ball

**Sell another Option with Strike Price  $X$**



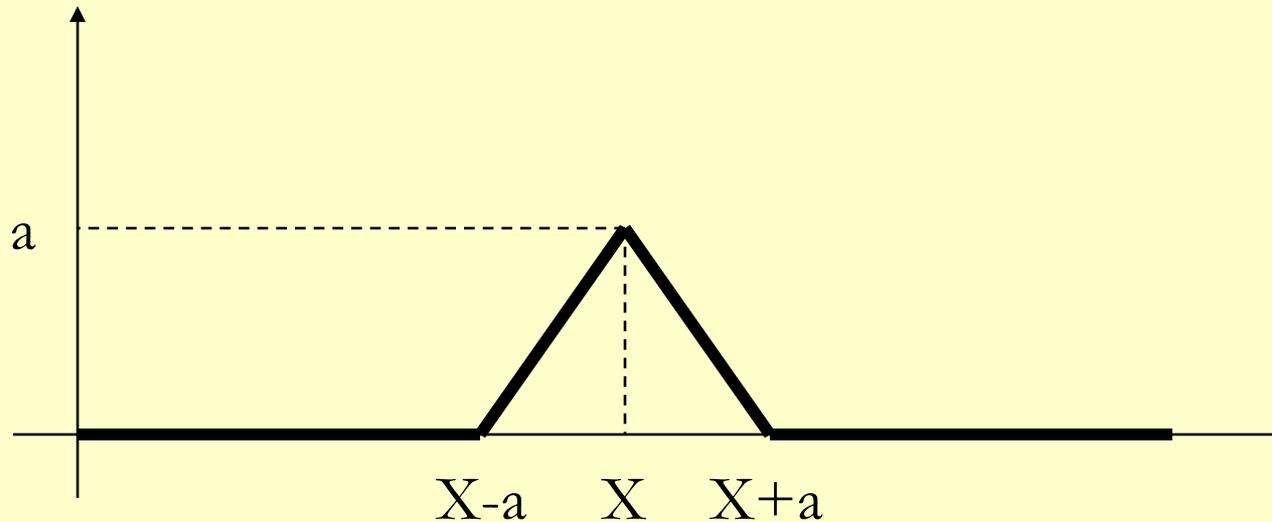
# Option Pricing Using Crystal Ball

**Sell another Option with Strike Price  $X$**



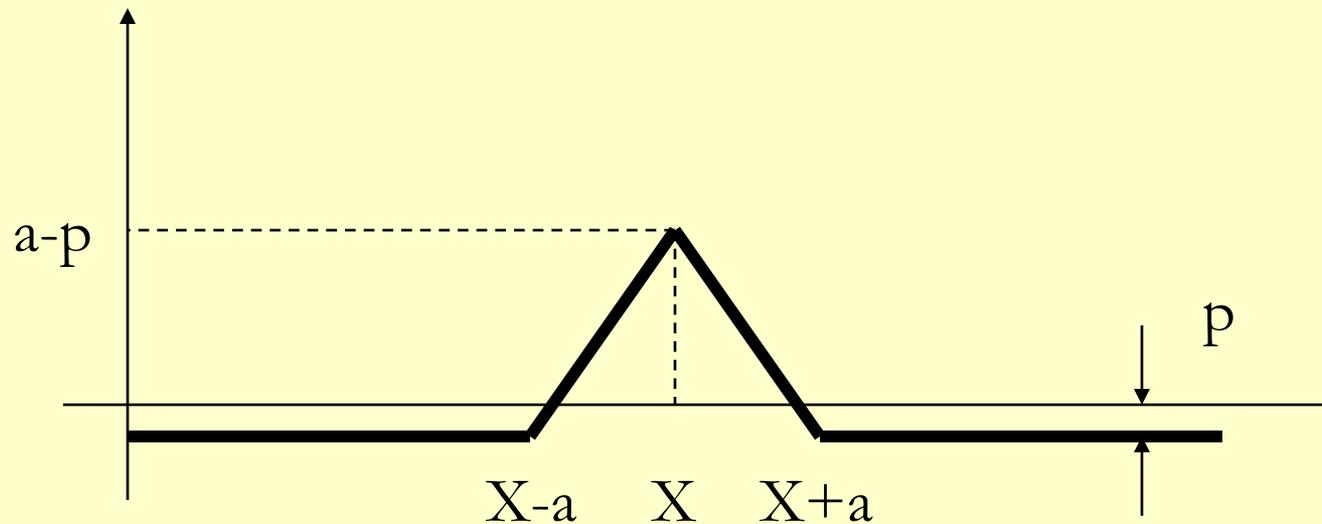
# Option Pricing Using Crystal Ball

## Buttery Spreads



# Option Pricing Using Crystal Ball

**Buttery Spreads – pay a price!**



# Option Pricing Using Crystal Ball

Question: What is the fair price for the butterfly option with a strike price ( $X$ ) of \$90 and a spread ( $a$ ) of \$10 assuming that the price of the stock a month from now is going to be Gaussian distributed with a mean of \$96 with a standard deviation of \$15.17 ?

See Excel file: Option Pricing.xls

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2.854 / 2.853 Introduction to Manufacturing Systems  
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