

# **Multi-Stage Control and Scheduling**

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# Definitions

- Events may be *controllable* or not, and *predictable* or not.

	<i>controllable</i>	<i>uncontrollable</i>
<i>predictable</i>	loading a part	lunch
<i>unpredictable</i>	???	machine failure

# Definitions

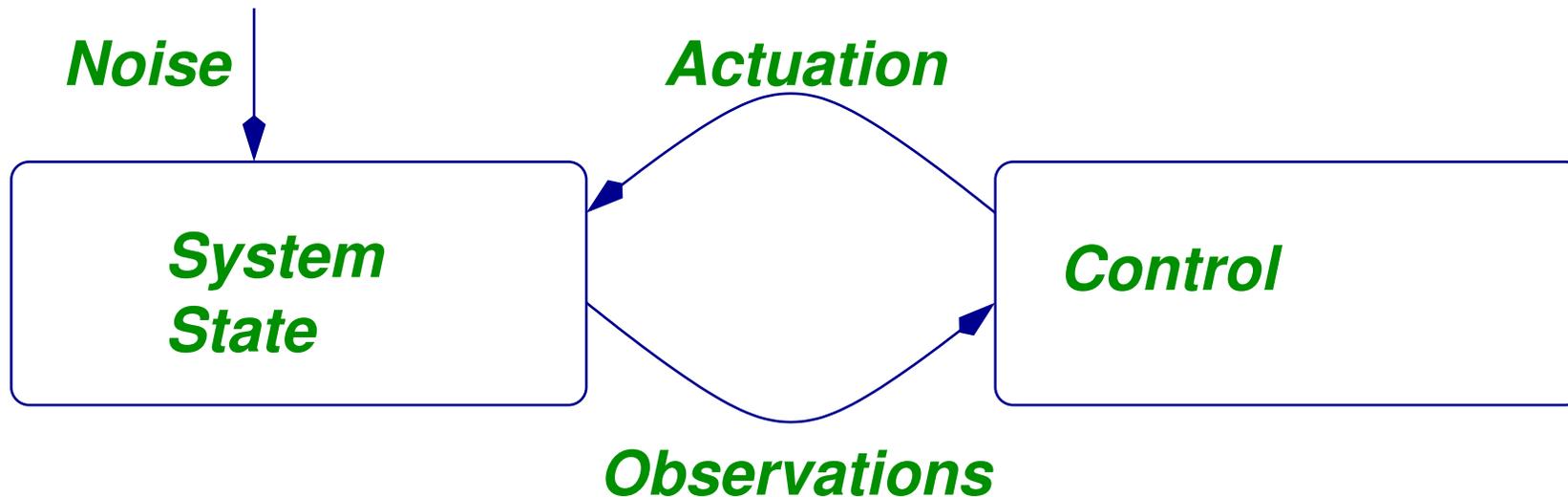
- *Scheduling is the selection of times for future controllable events.*
- Ideally, scheduling systems should deal with *all* controllable events, and not just production.
  - ★ That is, they should select times for operations, set-up changes, preventive maintenance, etc.
  - ★ They should at least be *aware* of set-up changes, preventive maintenance, etc. when they select times for operations.

# Definitions

- Because of recurring random events, scheduling is an on-going process, and not a one-time calculation.
- Scheduling, or shop floor control, is the bottom of the scheduling/planning hierarchy. It translates *plans* into *events*.

# Definitions

## Control Paradigm



This is the general paradigm for control theory and engineering.

# Definitions

In a factory,

- *State*: distribution of inventory, repair/failure states of machines, etc.
- *Control*: move a part to a machine and start operation; begin preventive maintenance, etc.
- *Noise*: machine failures, change in demand, etc.

### Definitions

- *Release*: Authorizing a job for production, or allowing a raw part onto the factory floor.
- *Dispatch*: Moving a part into a workstation or machine.
- *Release is more important than dispatch*. That is, improving release has more impact than improving dispatch, if both are reasonable.

## Requirements

## Definitions

Scheduling systems or methods should ...

- deliver good factory performance.
- compute decisions quickly, in response to changing conditions.

# Performance Goals

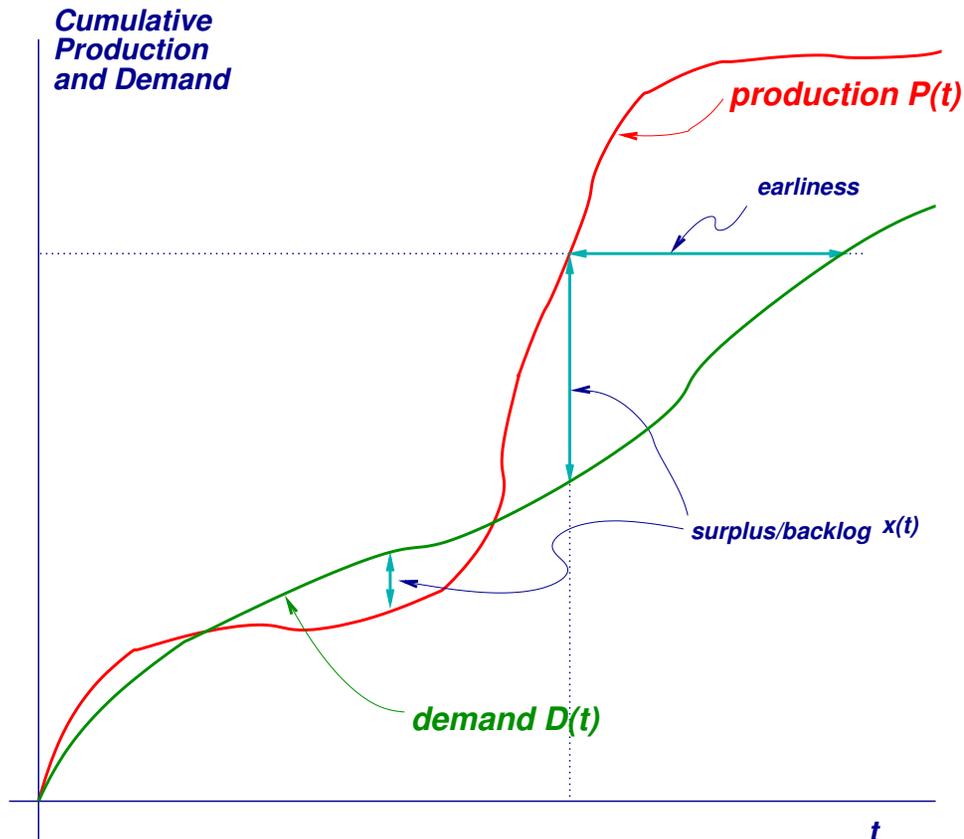
- To minimize inventory and backlog.
- To maximize probability that customers are satisfied.
- To maximize predictability (ie, minimize performance variability).

# Performance Goals

- For MTO (Make to Order)
  - ★ To meet delivery promises.
  - ★ To make delivery promises that are both *soon* and *reliable* .
- For MTS (Make to Stock)
  - ★ to have FG (finished goods) available when customers arrive; and
  - ★ to have minimal FG inventory.

# Performance Goals

## Objective of Scheduling



Objective is to keep cumulative production close to cumulative demand.

# Performance Goals

## Difficulties

- Complex factories
- Unpredictable demand (ie  $D$  uncertainty)
- Factory unreliability (ie  $P$  uncontrollability)

# Basic approaches

- Simple rules — *heuristics*
  - ★ Dangers:
    - \* Too simple — may ignore important features.
    - \* Rule proliferation.
- Detailed calculations
  - ★ Dangers:
    - \* Too complex — impossible to develop intuition.
    - \* Rigid — had to modify — may have to lie in data.

# Basic approaches

## Detailed calculations

- Deterministic optimization.
  - ★ Large linear or mixed integer program.
  - ★ Re-optimize periodically or after important event.
- Scheduling by simulation.

# Basic approaches

## Detailed calculations

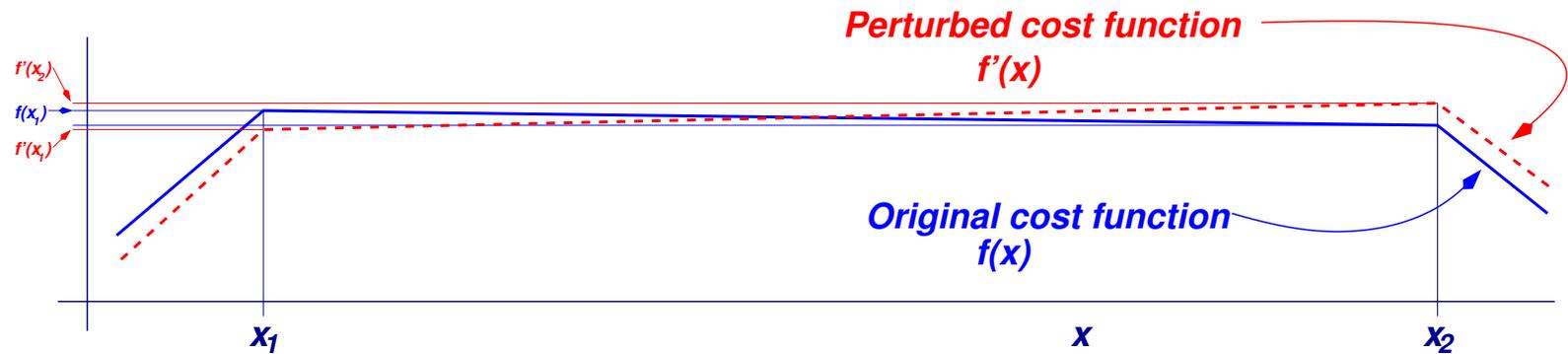
### Dangers

- Nervousness or scheduling volatility (fast but inaccurate response):
  - ★ The optimum may be very flat. That is, many very different schedule alternatives may produce similar performance.
  - ★ A small change of conditions may therefore cause the optimal schedule to change substantially.

# Basic approaches

## Detailed calculations

### Dangers



- Original optimum performance was  $f(x_1)$ .
- New optimum performance is  $f'(x_2)$ .
- If we did not change  $x$ , new performance would be  $f'(x_1)$ .
- Benefit from change:  $f'(x_2) - f'(x_1)$ , *small*.
- Cost of change:  $x_2 - x_1$ , *large*.

# Basic approaches

## Detailed calculations

### Dangers

- Slow response:
  - ★ Long computation time.
  - ★ Freezing.
- Bad data:
  - ★ Factory data is often very poor, especially when workers are required to collect it manually.
  - ★ GIGO

## Characteristics

## Heuristics

- A *heuristic* is a proposed solution to a problem that *seems* reasonable but cannot be rigorously justified.
- In reentrant systems, heuristics tend to favor *older* parts.
  - ★ This keeps inventory low.

## Characteristics

## Heuristics

## Desirable Characteristics

- Good heuristics deliver good performance.
- Heuristics tend to be simple and intuitive.
  - ★ People should be able to understand why choices are made, and anticipate what will happen.
  - ★ Relevant information should be simple and easy to get access to.
  - ★ Simplicity helps the development of simulations.
- Good heuristics are insensitive to small perturbations or errors in data.

## Characteristics

## Heuristics

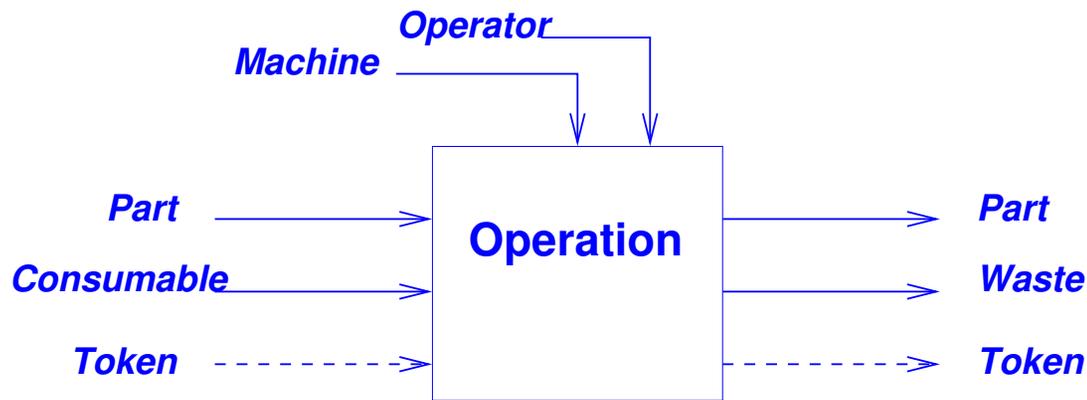
### Decentralization

- It is often desirable for people to make decisions on the basis of local, current information.
  - ★ Centralized decision-making is most often bureaucratic, slow, and inflexible.
- Most heuristics are naturally decentralized, or can be implemented in a decentralized fashion.

# Heuristics

## Material/token policies

### Performance evaluation



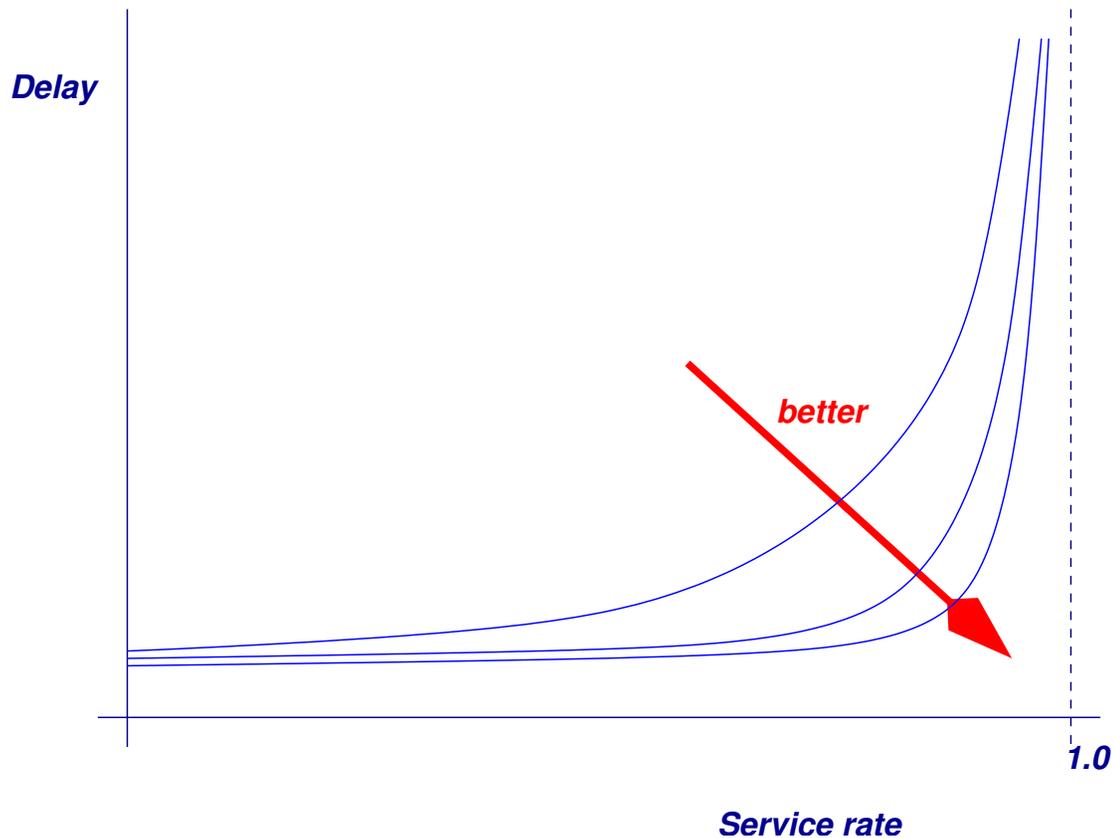
- An operation cannot take place unless there is a token available.
- Tokens *authorize* production.

- These policies can often be implemented *either* with finite buffer space, or a finite number of tokens. Mixtures are also possible.
- Buffer space could be shelf space, or floor space indicated with paint or tape.

# Heuristics

## Material/token policies

### Performance evaluation



- Tradeoff between service rate and average cycle time.

# Heuristics

## Material/token policies

### Finite buffer

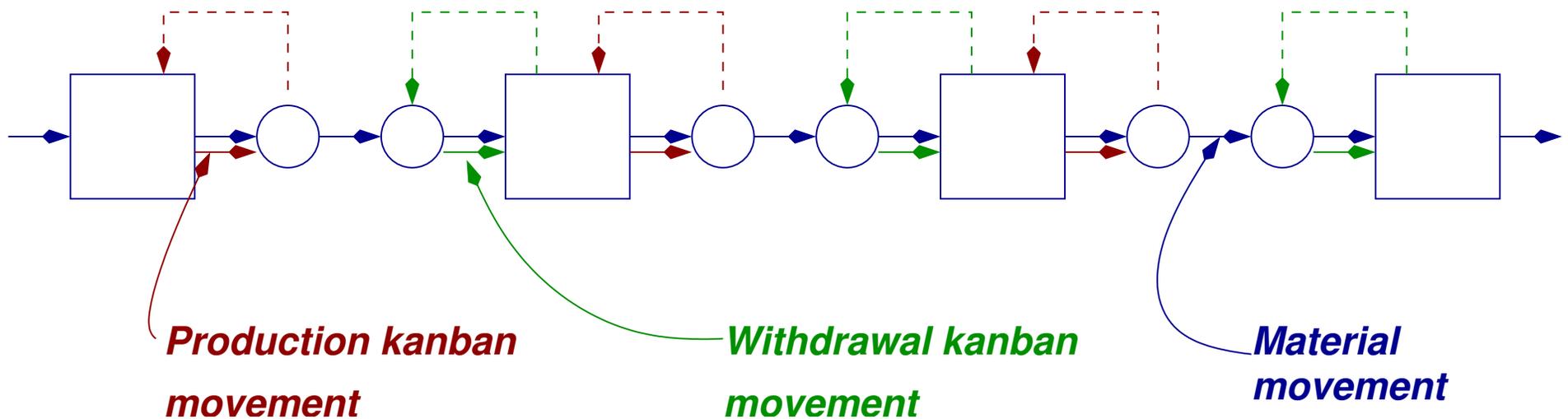


- Buffers tend to be close to full.
- Sizes of buffers should be related to magnitude of disruptions.
- Not practical for large systems, unless each box represents a *set* of machines.

# Heuristics

## Material/token policies

### Kanban



- Performance slightly better than finite buffer.
- Sizes of buffers should be related to magnitude of disruptions.

# Heuristics

## Material/token policies

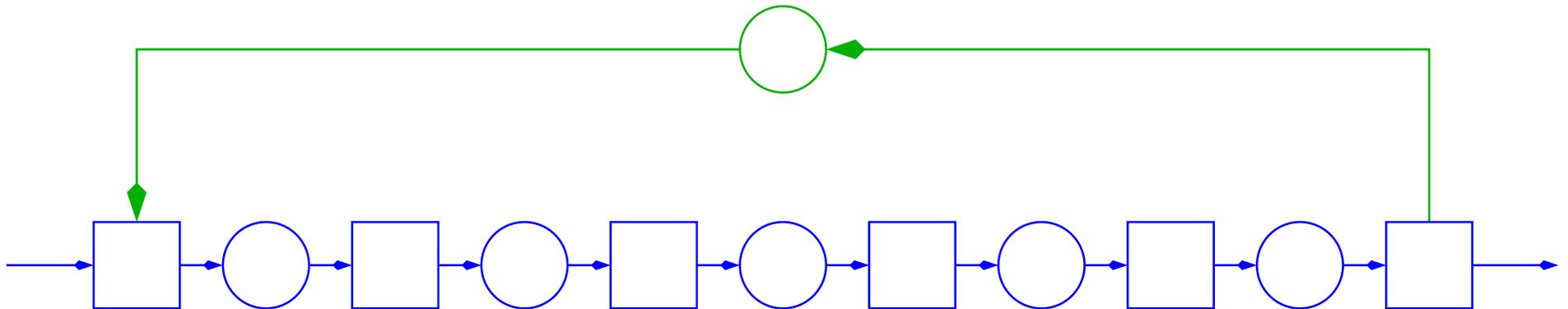
### CONWIP

- *Constant Work in Progress*
- Variation on kanban in which the number of parts in an area is limited.
- When the limit is reached, no new part enters until a part leaves.
- Variations:
  - ★ When there are multiple part types, limit work hours or dollars rather than number of parts.
  - ★ Or establish individual limits for each part type.

# Heuristics

## Material/token policies

### CONWIP



- If token buffer is not empty, attach a token to a part when  $M_1$  starts working on it.
- If token buffer is empty, do not allow part into  $M_1$ .
- Token and part travel together until they reach last machine.
- When last machine completes work on a part, the part leaves and the token moves to the token buffer.

# Heuristics

## Material/token policies

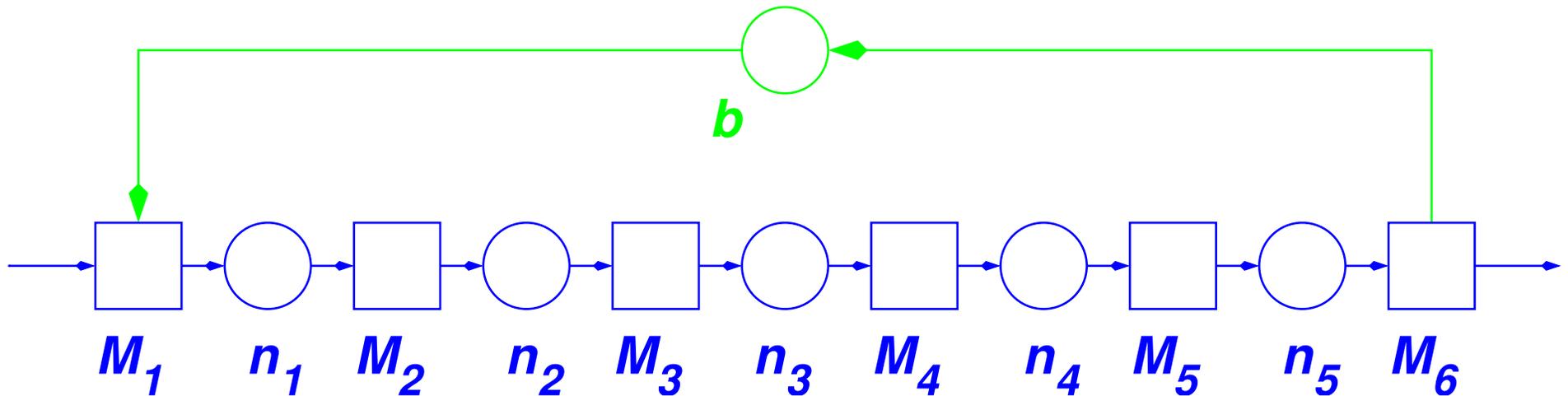
### CONWIP

- Infinite material buffers.
- Infinite token buffer.
- Limited material population at all times.
- Population limit should be related to magnitude of disruptions.

# Heuristics

## Material/token policies

### CONWIP

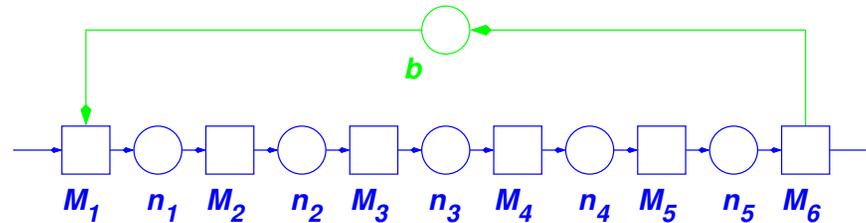


- *Claim:*  $n_1 + n_2 + \dots + n_6 + b$  is constant.

# Material/token policies

## Heuristics

### CONWIP Proof

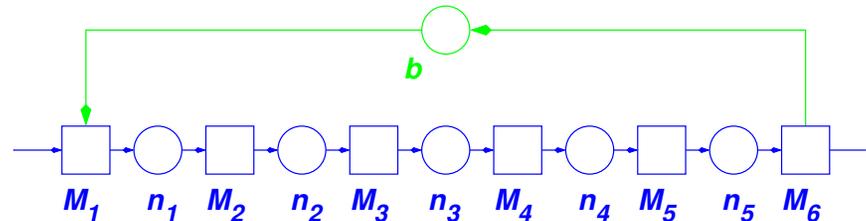


- Define  $C = n_1 + n_2 + \dots + n_5 + b$ .
- Whenever  $M_j$  does an operation,  $C$  is unchanged,  $j = 2, \dots, 5$ .
  - ★ ... because  $n_{j-1}$  goes down by 1 and  $n_j$  goes up by 1, and nothing else changes.
- Whenever  $M_1$  does an operation,  $C$  is unchanged.
  - ★ ... because  $b$  goes down by 1 and  $n_1$  goes up by 1, and nothing else changes.

# Heuristics

## Material/token policies

### CONWIP Proof

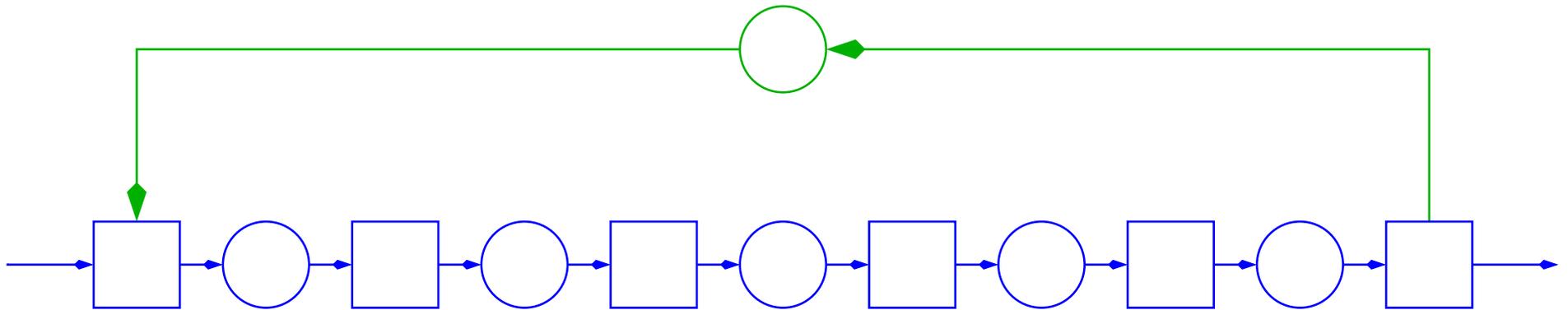


- Whenever  $M_6$  does an operation,  $C$  is unchanged.
  - ★ ... because  $n_5$  goes down by 1 and  $b$  goes up by 1, and nothing else changes.
- Similarly for  $M_1$ .
- That is, whenever *anything* happens,  
 $C = n_1 + n_2 + \dots + n_5 + b$  is unchanged.
- $C$  is an *invariant*.
- $C$  is the maximum population of the material in the system.

# Heuristics

## Material/token policies

### CONWIP/Kanban Hybrid

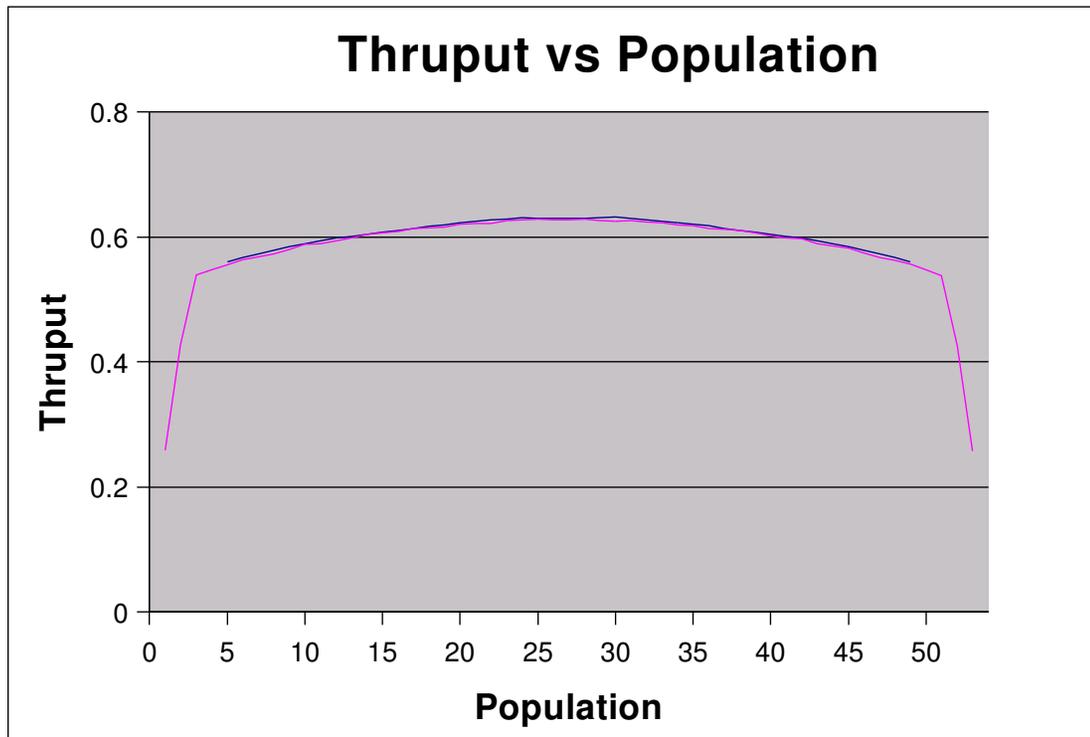


- Finite buffers
- Finite material population
- Limited material population at all times.
- Population and sizes of buffers should be related to magnitude of disruptions.

# Heuristics

## Material/token policies

### CONWIP/Kanban Hybrid

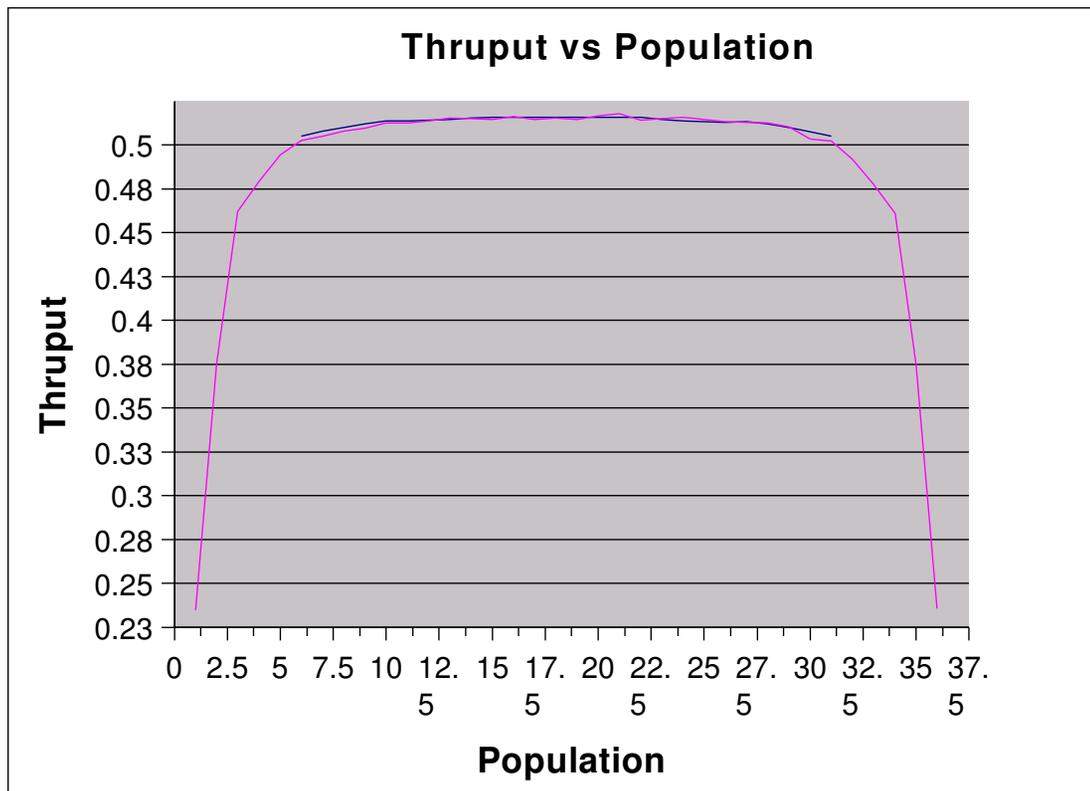


- Production rate as a function of CONWIP population.
- In these graphs, total buffer space (including for tokens) is finite.

# Heuristics

## Material/token policies

### CONWIP/Kanban Hybrid

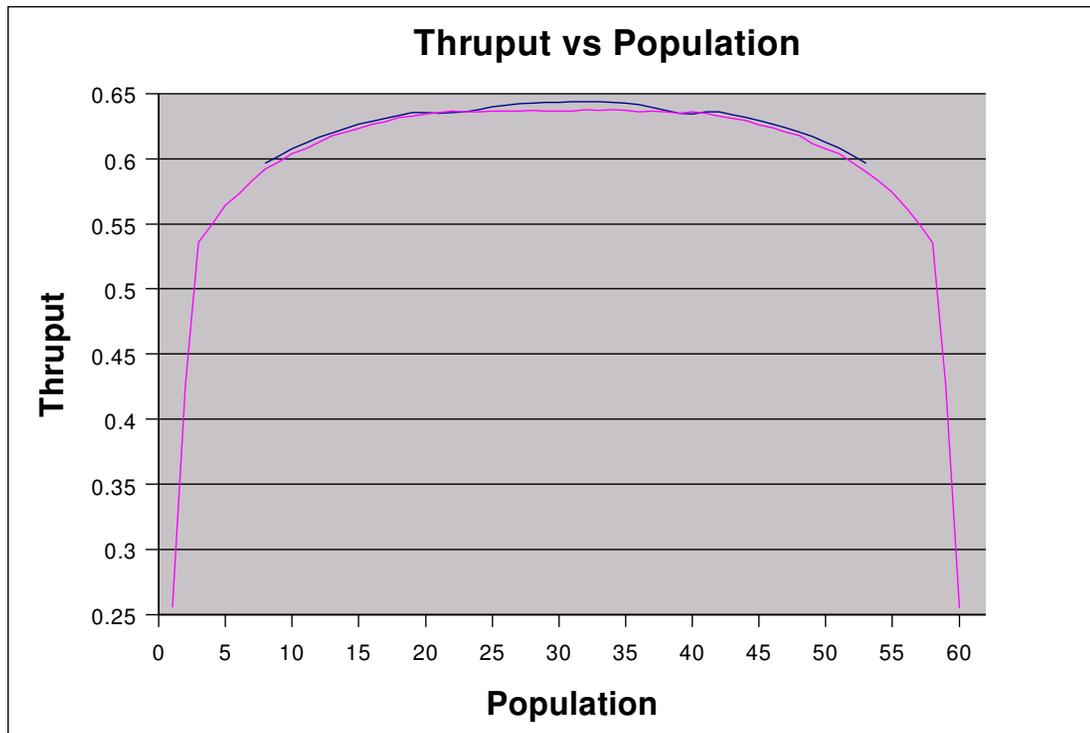


- Maximum production rate occurs when population is half of total space.

# Heuristics

## Material/token policies

### CONWIP/Kanban Hybrid

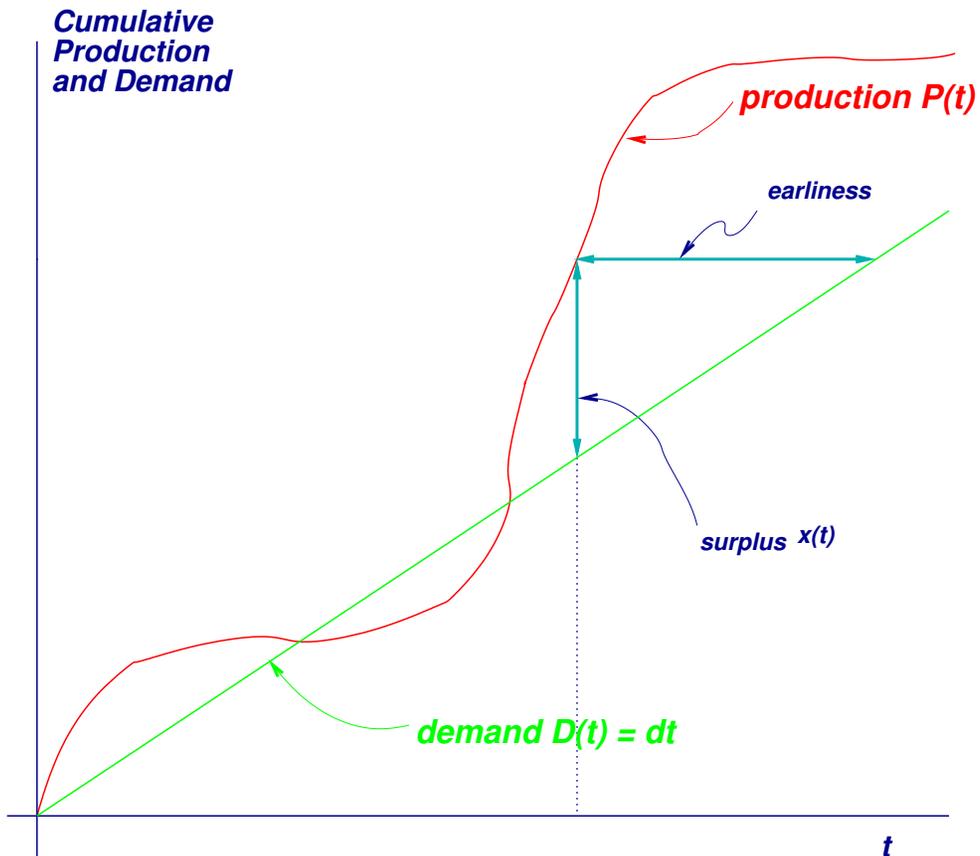


- When total space is infinite, production rate increases only.

# Simple Policies

## Material/token policies

### Hedging point



- *State:*  $(x, \alpha)$
- $x$  = surplus = difference between cumulative production and demand.
- $\alpha$  = machine state.  
 $\alpha = 1$  means machine is up;  $\alpha = 0$  means machine is down.

# Simple Policies

## Material/token policies

### Hedging point

- *Control:*  $u$
- $u$  = short term production rate.
  - ★ if  $\alpha = 1$ ,  $0 \leq u \leq \mu$ ;
  - ★ if  $\alpha = 0$ ,  $u = 0$ .

# Simple Policies

## Material/token policies

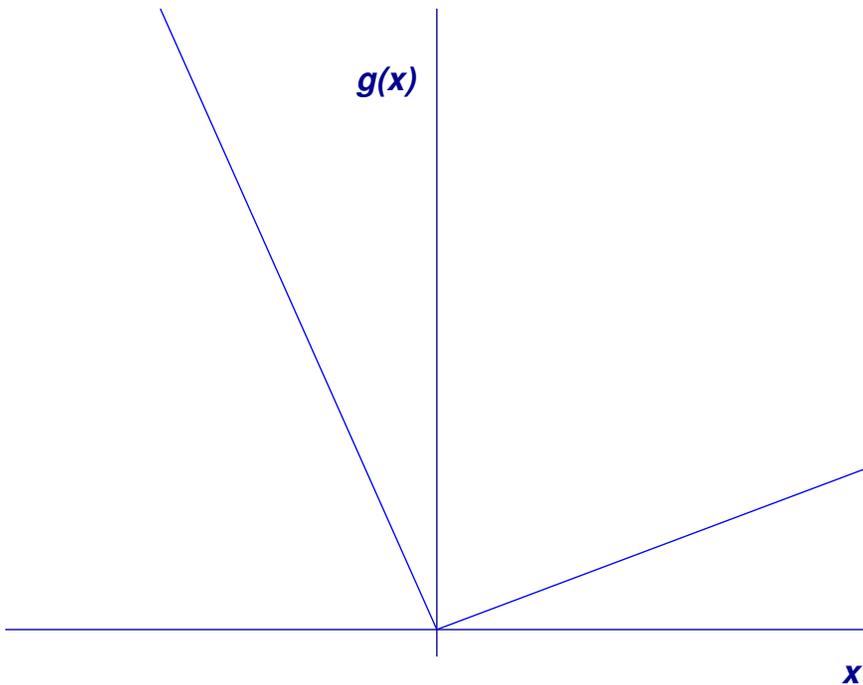
### Hedging point

- Objective function:

$$\min E \int_0^T g(x(t)) dt$$

- where

$$g(x) = \begin{cases} g_+ x, & \text{if } x \geq 0 \\ -g_- x, & \text{if } x < 0 \end{cases}$$



# Simple Policies

## Material/token policies

### Hedging point

- Dynamics:

- ★  $\frac{dx}{dt} = u - d$

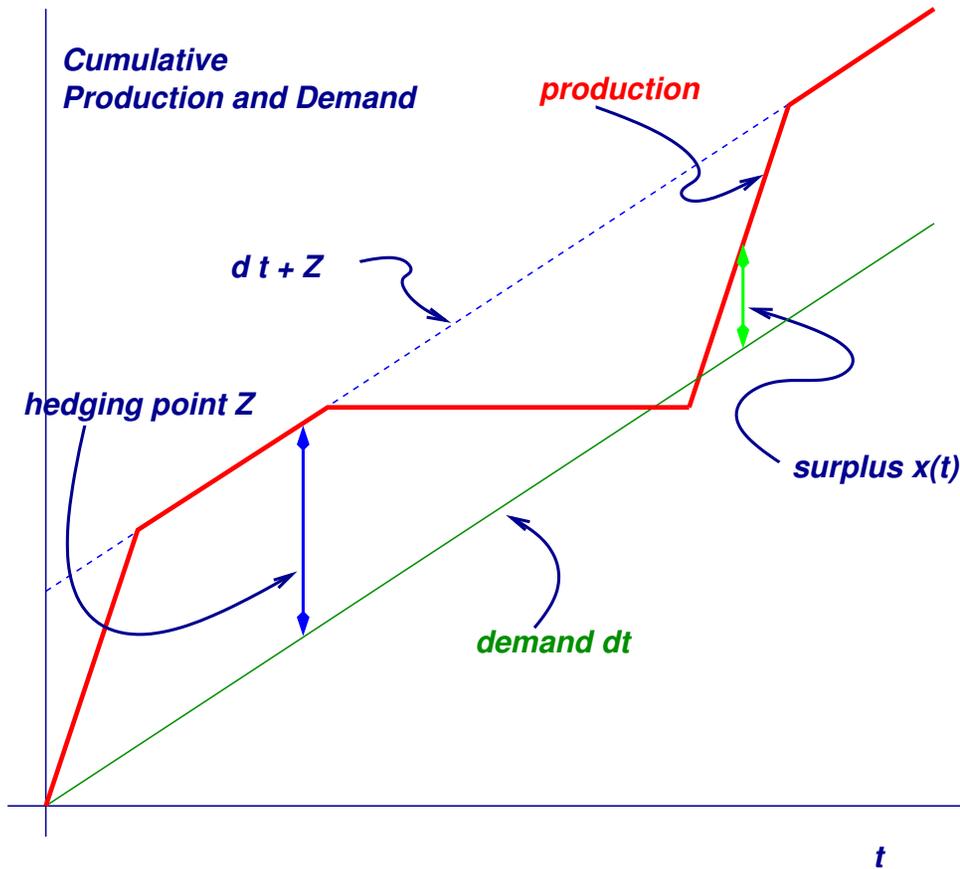
- ★  $\alpha$  goes from 0 to 1 according to an exponential distribution with parameter  $r$ .

- ★  $\alpha$  goes from 1 to 0 according to an exponential distribution with parameter  $p$ .

# Simple Policies

## Material/token policies

### Hedging point



### Solution:

- if  $x(t) > Z$ , wait;
- if  $x(t) = Z$ , operate at demand rate  $d$ ;
- if  $x(t) < Z$ , operate at maximum rate  $\mu$ .

# Simple Policies

## Material/token policies

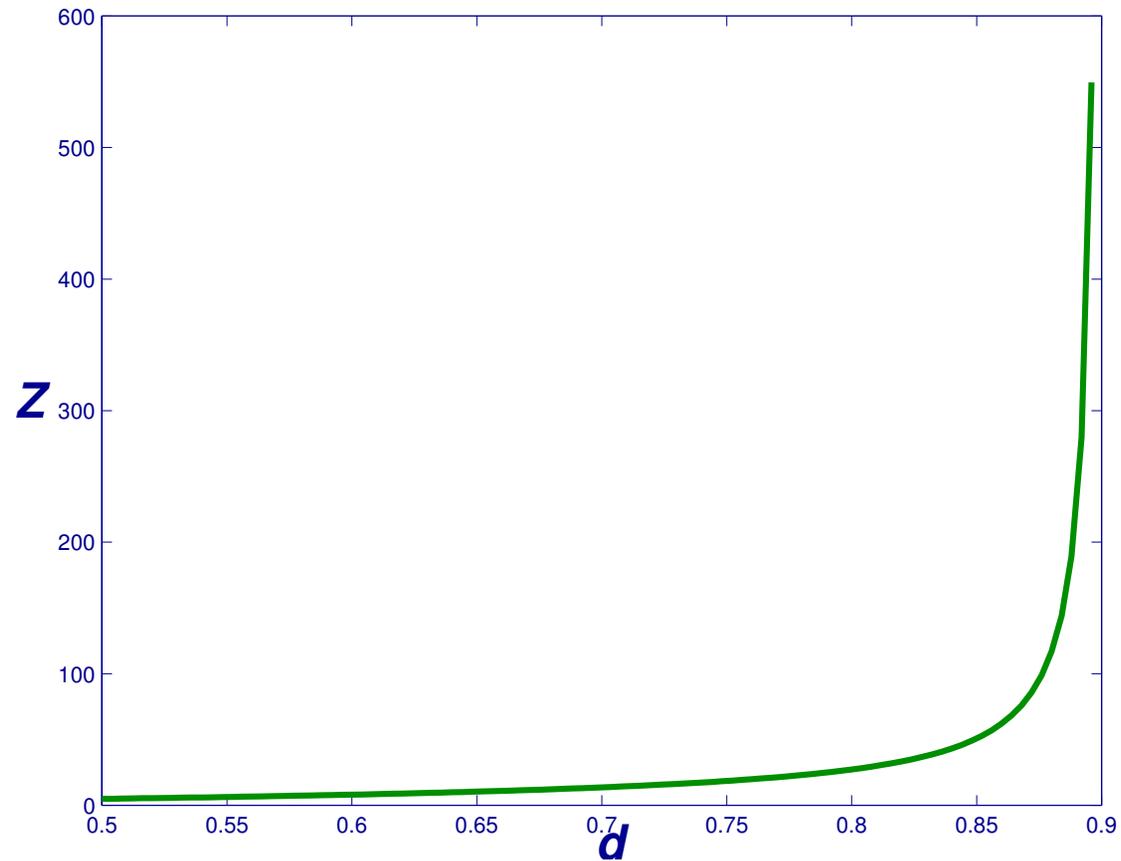
### Hedging point

- The hedging point  $Z$  is the single parameter.
- It represents a trade-off between costs of inventory and risk of disappointing customers.
- It is a function of  $d, \mu, r, p, g_+, g_-$ .

# Simple Policies

## Material/token policies

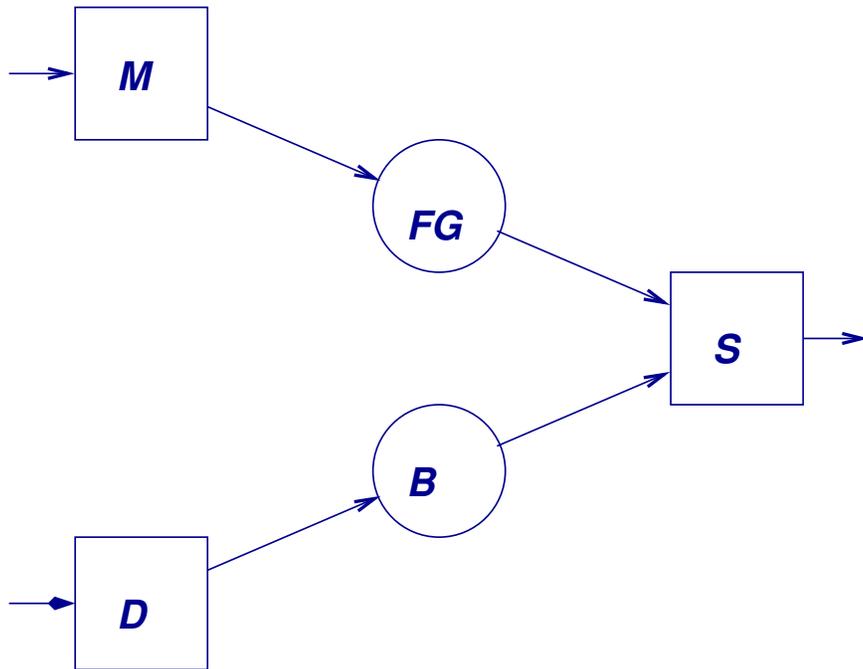
### Hedging point



# Simple Policies

## Material/token policies

### Hedging point



- Operating Machine  $M$  according to the hedging point policy is equivalent to operating this assembly system according to a finite buffer policy.

# Simple Policies

## Material/token policies

### Hedging point

- $D$  is a *demand generator* .
  - ★ Whenever a demand arrives,  $D$  sends a token to  $B$ .
- $S$  is a synchronization machine.
  - ★  $S$  is perfectly reliable and infinitely fast.
- $FG$  is a finite finished goods buffer.
- $B$  is an infinite backlog buffer.

# Simple Policies

## Material/token policies

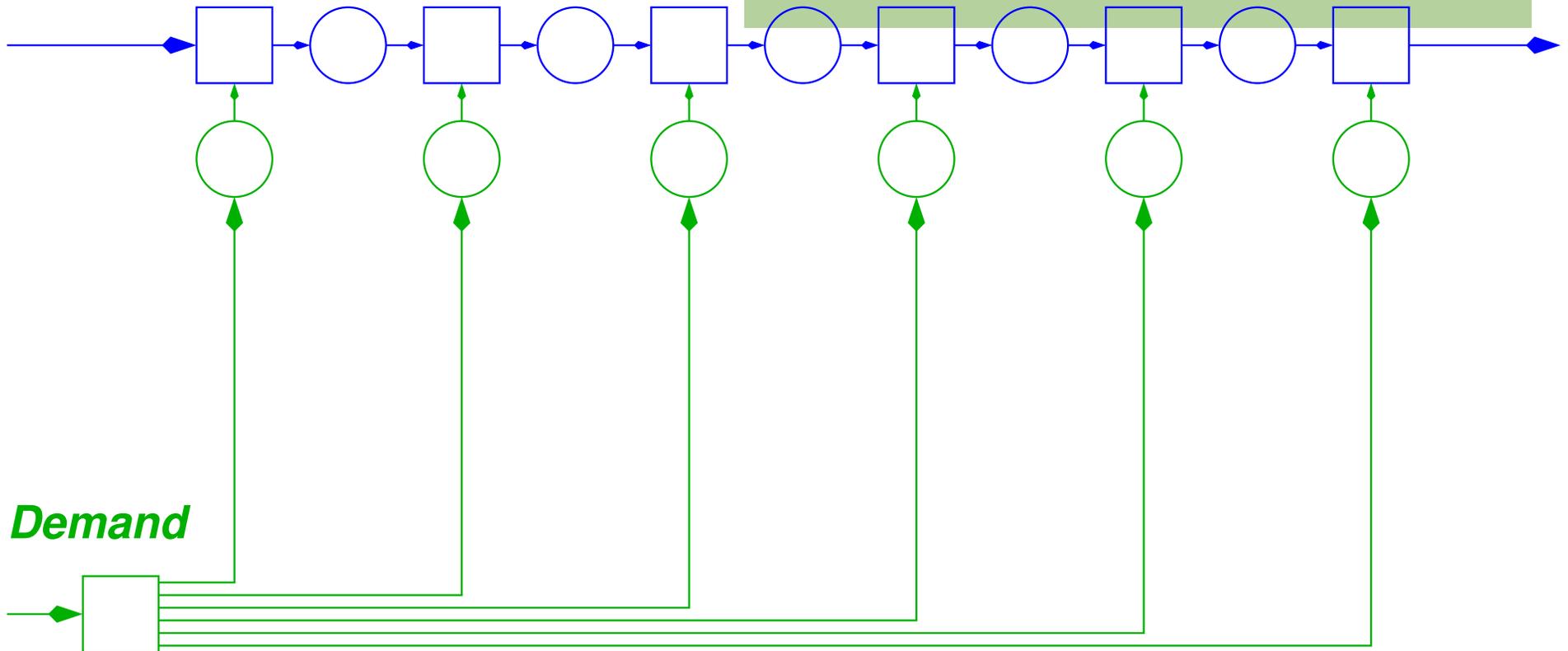
### Basestock

- *Base Stock*: the amount of material and backlog between each machine and the customer is limited.
- Deviations from targets are adjusted locally.

# Simple Policies

## Material/token policies

### Basestock

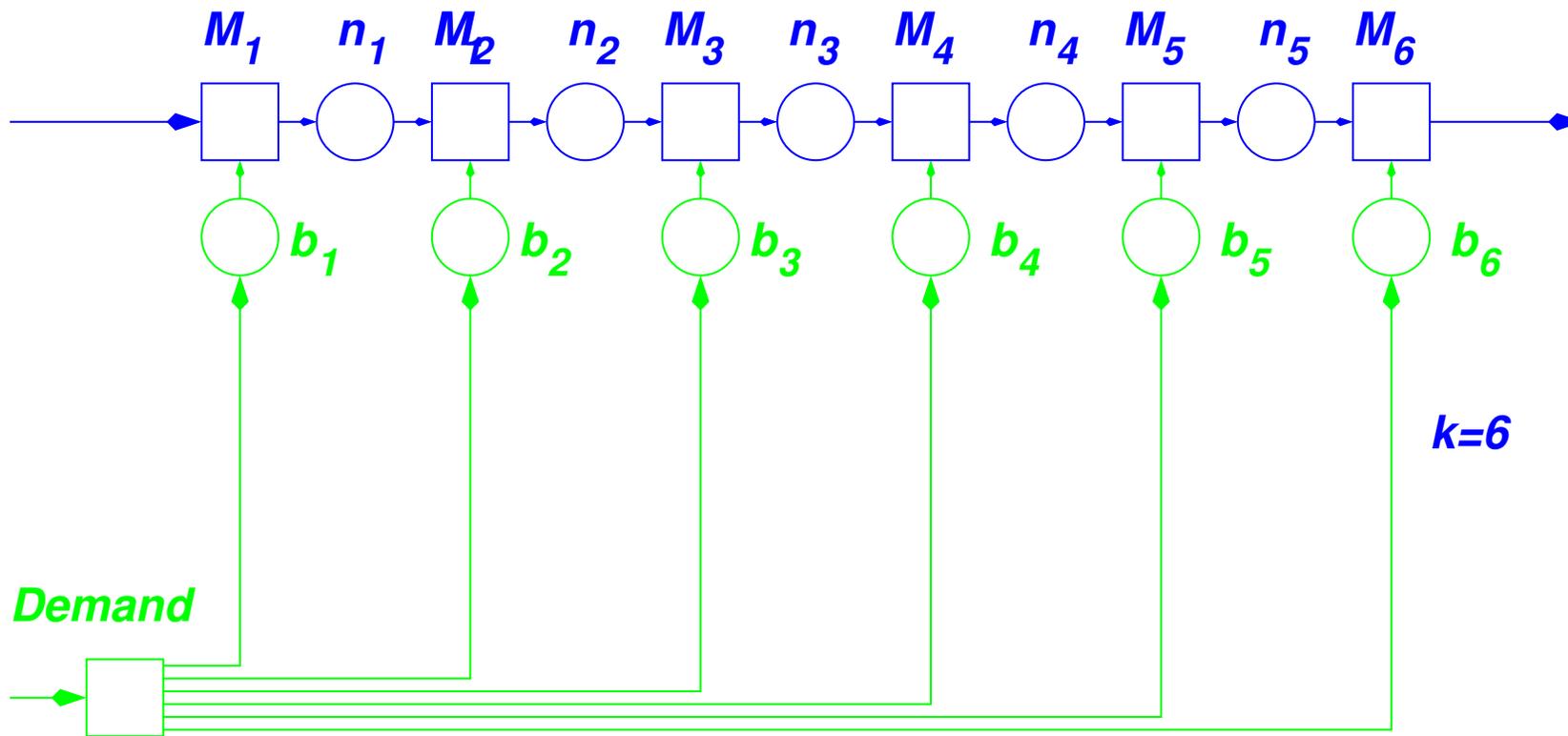


- Infinite buffers.
- Finite initial levels of material and token buffers.

# Simple Policies

## Material/token policies

### Basestock Proof



**Claim:**  $b_j + n_j + n_{j+1} + \dots + n_{k-1} - b_k, 1 \geq j \geq k$   
remains constant at all times.

# Simple Policies

## Material/token policies

### Basestock Proof

- Consider  $b_1 + n_1 + n_2 + \dots + n_{k-1} - b_k$
- When  $M_i$  does an operation ( $1 < i < k$ ),
  - ★  $n_{i-1}$  goes down by 1,  $b_i$  goes down by 1,  $n_i$  goes up by 1, and all other  $b_j$  and  $n_j$  are unchanged.
  - ★ That is,  $n_{i-1} + n_i$  is constant, and  $b_i + n_i$  is constant.
  - ★ Therefore  $b_1 + n_1 + n_2 + \dots + n_{k-1} - b_k$  stays constant.
- When  $M_1$  does an operation,  $b_1 + n_1$  is constant.
- When  $M_k$  does an operation,  $n_{i-1} - b_k$  is constant.
- Therefore, when *any* machine does an operation,  $b_1 + n_1 + n_2 + \dots + n_{k-1} - b_k$  remains constant.

# Simple Policies

## Material/token policies

### Basestock Proof

- Now consider  $b_j + n_j + n_{j+1} + \dots + n_{k-1} - b_k, 1 < j < k$
- When  $M_i$  does an operation,  $i \geq j$ ,  
 $b_j + n_j + n_{j+1} + \dots + n_{k-1} - b_k$  remains constant, from the same reasoning as for  $j = 1$ .
- When  $M_i$  does an operation,  $i < j$ ,  
 $b_j + n_j + n_{j+1} + \dots + n_{k-1} - b_k$  remains constant, because it is unaffected.

# Simple Policies

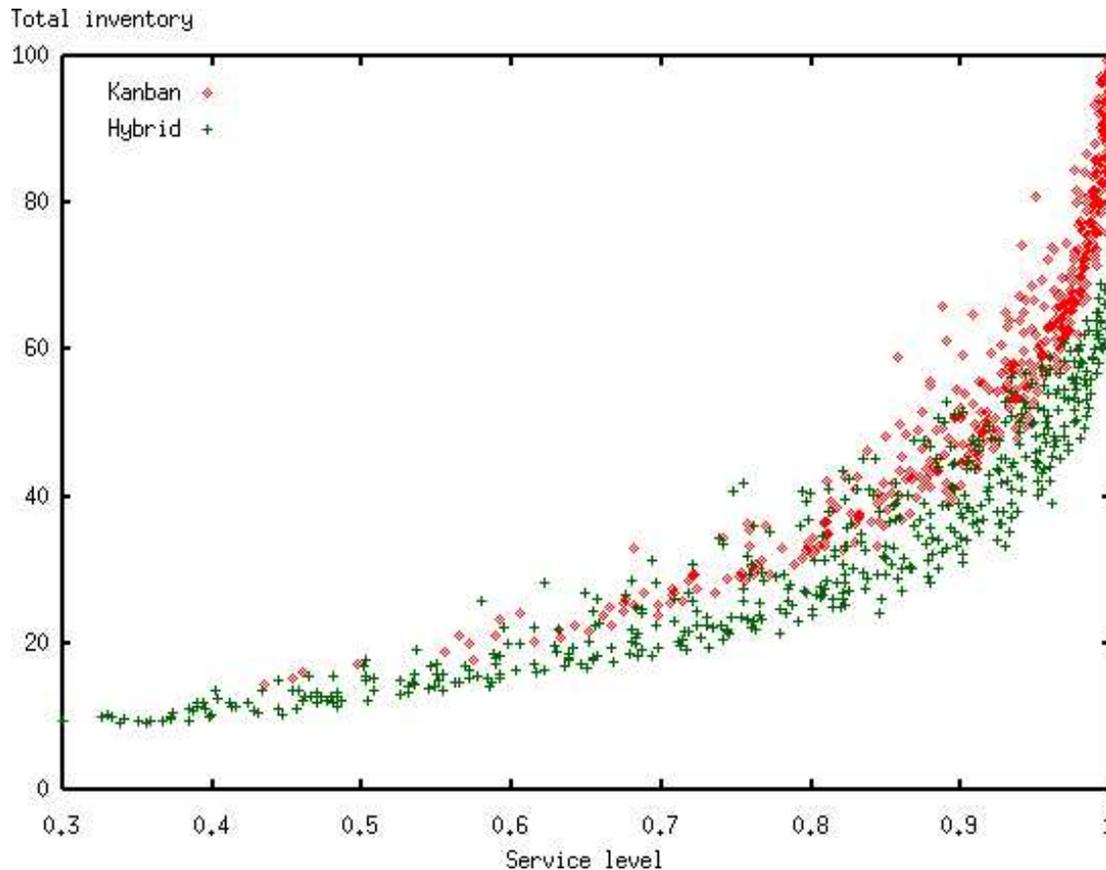
## Basestock Proof

- When a demand arrives,
  - ★  $n_j$  stays constant, for all  $j$ , and all  $b_j$  increase by one.
  - ★ Therefore  $b_j + n_j + n_{j+1} + \dots + n_{k-1} - b_k$  remains constant for all  $j$ .
- *Conclusion:* whenever *any* event occurs,  $b_j + n_j + n_{j+1} + \dots + n_{k-1} - b_k$  remains constant, for all  $j$ .

# Simple Policies

## Material/token policies

### Comparisons

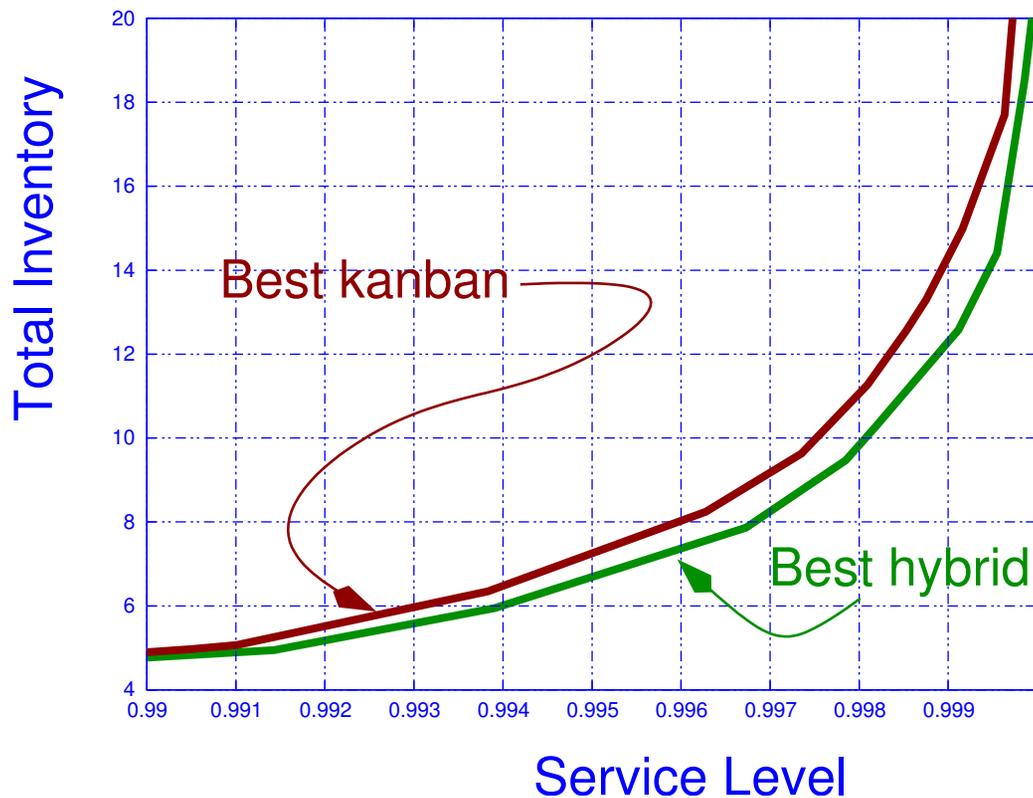


- Simulation of simple Toyota feeder line.
- We simulated *all* possible kanban policies and *all* possible kanban/CONWIP hybrids.

# Simple Policies

## Material/token policies

### Comparisons



- The graph indicates the *best* of all kanbans and all hybrids.

# Simple Policies

## Material/token policies

### Comparisons

More results of the comparison experiment: best parameters for service rate =.999.

Policy	Buffer sizes				Base stocks			
	2	2	4	10	—	—	—	—
Finite buffer	2	2	4	10	—	—	—	—
Kanban	2	2	4	9	—	—	—	—
Basestock	$\infty$	$\infty$	$\infty$	$\infty$	1	1	1	12
CONWIP	$\infty$	$\infty$	$\infty$	$\infty$	—	—	—	15
Hybrid	2	3	5	15	—	—	—	15

# Simple Policies

## Material/token policies

### Comparisons

More results of the comparison experiment:  
performance.

Policy	Service level	Inventory
Finite buffer	0.99916 $\pm$ .00006	15.82 $\pm$ .05
Kanban	0.99909 $\pm$ .00005	15.62 $\pm$ .05
Basestock	0.99918 $\pm$ .00006	14.60 $\pm$ .02
CONWIP	0.99922 $\pm$ .00005	14.59 $\pm$ .02
Hybrid	0.99907 $\pm$ .00007	13.93 $\pm$ .03

## Other policies

# Simple Policies

## FIFO

- First-In, First Out.
- Simple conceptually, but you have to keep track of arrival times.
- Leaves out much important information:
  - ★ due date, value of part, current surplus/backlog state, etc.

## Other policies

# Simple Policies

EDD

- Earliest due date.
- Easy to implement.
- Does not consider work remaining on the item, value of the item, etc..

## Other policies

## Simple Policies

### SRPT

- *Shortest Remaining Processing Time*
- Whenever there is a choice of parts, load the one with least remaining work before it is finished.
- Variations: include waiting time with the work time. Use expected time if it is random.

## Other policies

# Simple Policies

## Critical ratio

- Widely used, but many variations. One version:
  - ★ Define  $CR = \frac{\text{Processing time remaining until completion}}{\text{Due date} - \text{Current time}}$
  - ★ Choose the job with the highest ratio (provided it is positive).
  - ★ If a job is late, the ratio will be negative, or the denominator will be zero, and that job should be given highest priority.
  - ★ If there is more than one late job, schedule the late jobs in SRPT order.

## Other policies

## Simple Policies

### Least Slack

- This policy considers a part's due date.
- Define *slack* = due date - remaining work time
- When there is a choice, select the part with the least slack.
- Variations involve different ways of estimating remaining time.

## Other policies

# Simple Policies

## Drum-Buffer-Rope

- Due to Eli Goldratt.
- Based on the idea that every system has a bottleneck.
- *Drum*: the common production rate that the system operates at, which is the rate of flow of the bottleneck.
- *Buffer*: DBR establishes a CONWIP policy between the entrance of the system and the bottleneck. The buffer is the CONWIP population.
- *Rope*: the limit on the difference in production between different stages in the system.
- But: What if bottleneck is not well-defined?

# Conclusions

- Many policies and approaches.
- No simple statement telling which is better.
- Policies are not all well-defined in the literature or in practice.
- My opinion:
  - ★ This is because policies are not *derived* from first principles.
  - ★ Instead, they are tested and compared.
  - ★ Currently, we have little intuition to guide policy development and choice.

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