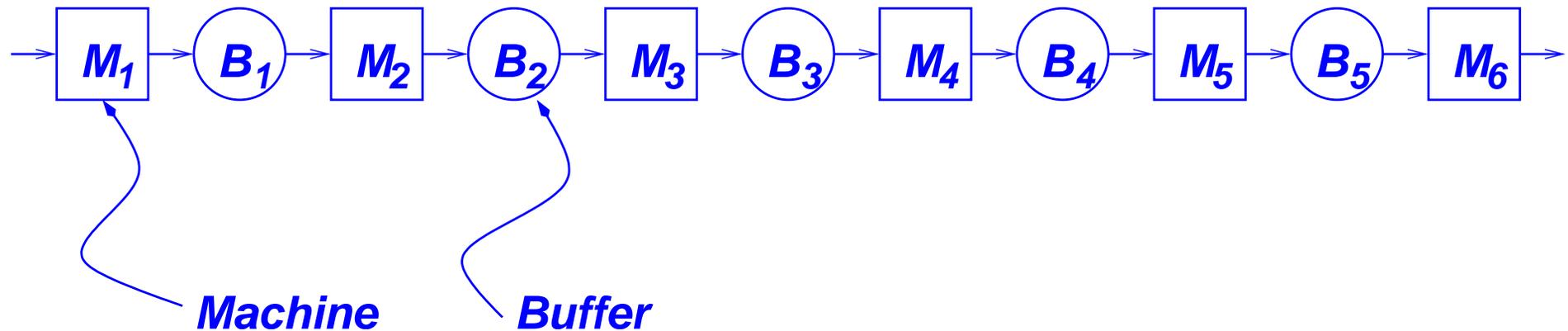


Single-part-type, multiple stage systems

Lecturer: Stanley B. Gershwin

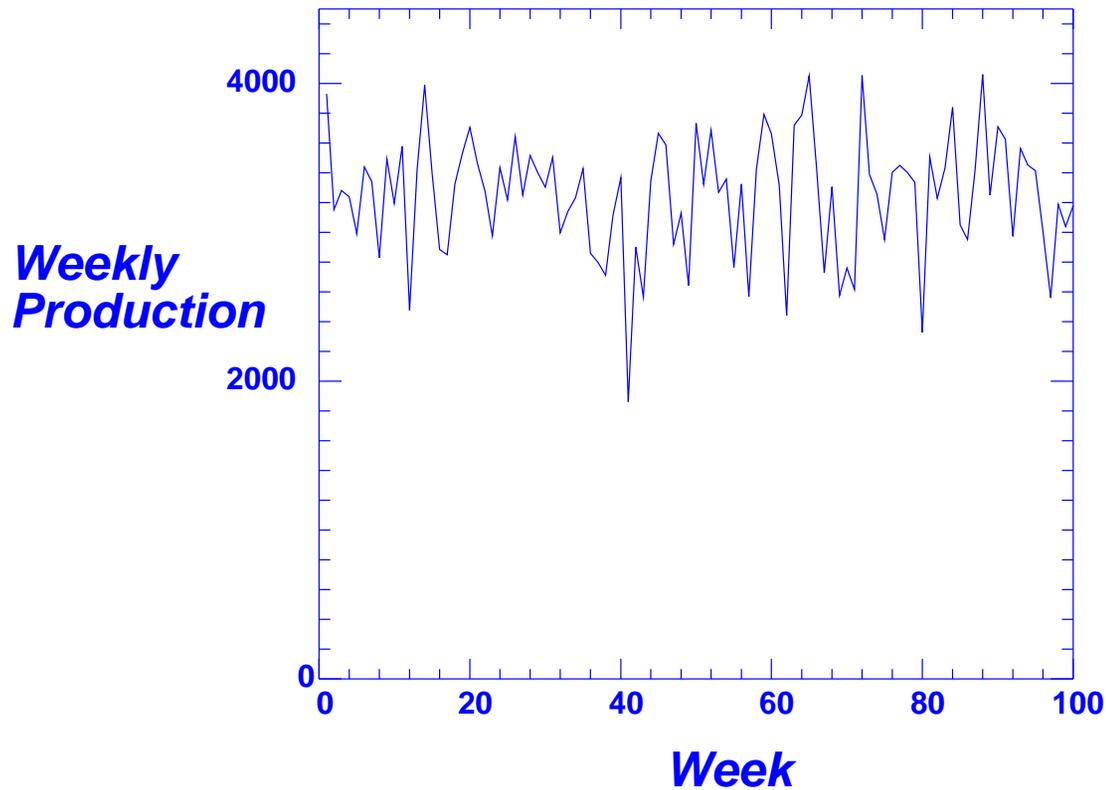
Flow Line

... also known as a Production or Transfer Line.



- Machines are unreliable.
- Buffers are finite.
- In many cases, the operation times are constant and equal for all machines.

Flow Line



Production output from a simulation of a transfer line.

Single Reliable Machine

- If the machine is perfectly reliable, and its average operation time is τ , then its maximum production rate is $1/\tau$.
- *Note:*
 - ★ Sometimes *cycle time* is used instead of *operation time*, but **BEWARE:** cycle time has two meanings!
 - ★ The other meaning is the time a part spends in a system. If the system is a single, reliable machine, the two meanings are the same.

Single Unreliable Machine

Failures and Repairs

- Machine is either *up* or *down* .
- MTTF = mean time to fail.
- MTTR = mean time to repair
- MTBF = MTTF + MTTR

Single Unreliable Machine

Production rate

- If the machine is unreliable, and
 - ★ its average operation time is τ ,
 - ★ its mean time to fail is MTTF,
 - ★ its mean time to repair is MTTR,

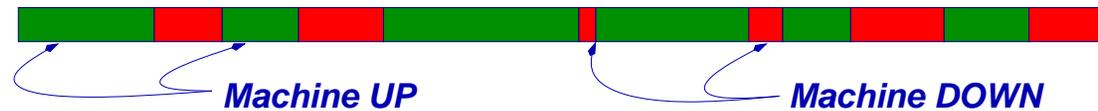
then its maximum production rate is

$$\frac{1}{\tau} \left(\frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \right)$$

Single Unreliable Machine

Production rate

Proof



- Average production rate, while machine is up, is $1/\tau$.
- Average duration of an up period is MTTF.
- Average production during an up period is $MTTF/\tau$.
- Average duration of up-down period: $MTTF + MTTR$.
- Average production during up-down period: $MTTF/\tau$.
- Therefore, average production rate is $(MTTF/\tau)/(MTTF + MTTR)$.

Single Unreliable Machine

Geometric Up- and Down-Times

- *Assumptions:* Operation time is constant (τ). Failure and repair times are *geometrically* distributed.
- Let p be the probability that a machine fails during any given operation. Then $p = \tau / \text{MTTF}$.

Single Unreliable Machine

Geometric Up- and Down-Times

- Let r be the probability that M gets repaired during any operation time when it is down. Then $r = \tau / \text{MTTR}$.

- Then the *average production rate* of M is

$$\frac{1}{\tau} \left(\frac{r}{r + p} \right).$$

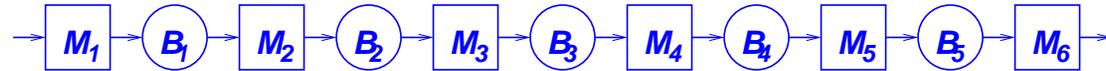
- (*Sometimes we forget to say “average.”*)

Single Unreliable Machine

Production Rates

- So far, the machine really has *three* production rates:
 - ★ $1/\tau$ when it is up (*short-term capacity*) ,
 - ★ 0 when it is down (*short-term capacity*) ,
 - ★ $(1/\tau)(r/(r + p))$ on the average (*long-term capacity*) .

Infinite-Buffer Line



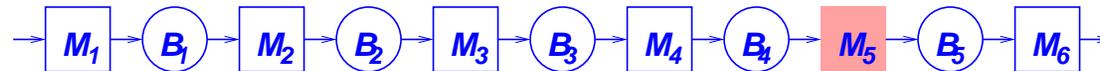
- **Starvation:** Machine M_i is starved at time t if Buffer B_{i-1} is empty at time t .

Assumptions:

- A machine is not idle if it is not starved.
- The first machine is never starved.

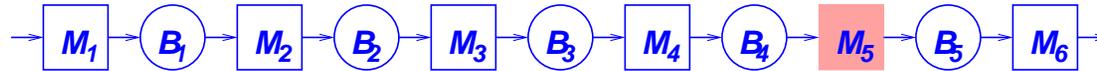
- Operation-Dependent Failures
 - ★ A machine can only fail while it is working — not idle.
 - ★ *(When buffers are finite, idleness also occurs due to blockage.)*
 - ★ **IMPORTANT!** *MTTF must be measured in working time!*
 - ★ This is the usual assumption.

Infinite-Buffer Line



- The production rate of the line is the production rate of the *slowest* machine in the line — called the *bottleneck*.
- **Slowest** means *least average production rate*, where average production rate is calculated from one of the previous formulas.

Infinite-Buffer Line

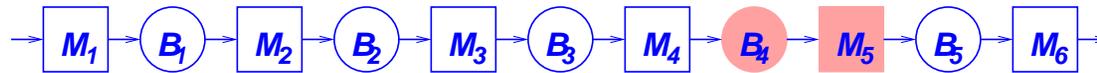


- Production rate is therefore

$$P = \min_i \frac{1}{\tau_i} \left(\frac{\text{MTTF}_i}{\text{MTTF}_i + \text{MTTR}_i} \right)$$

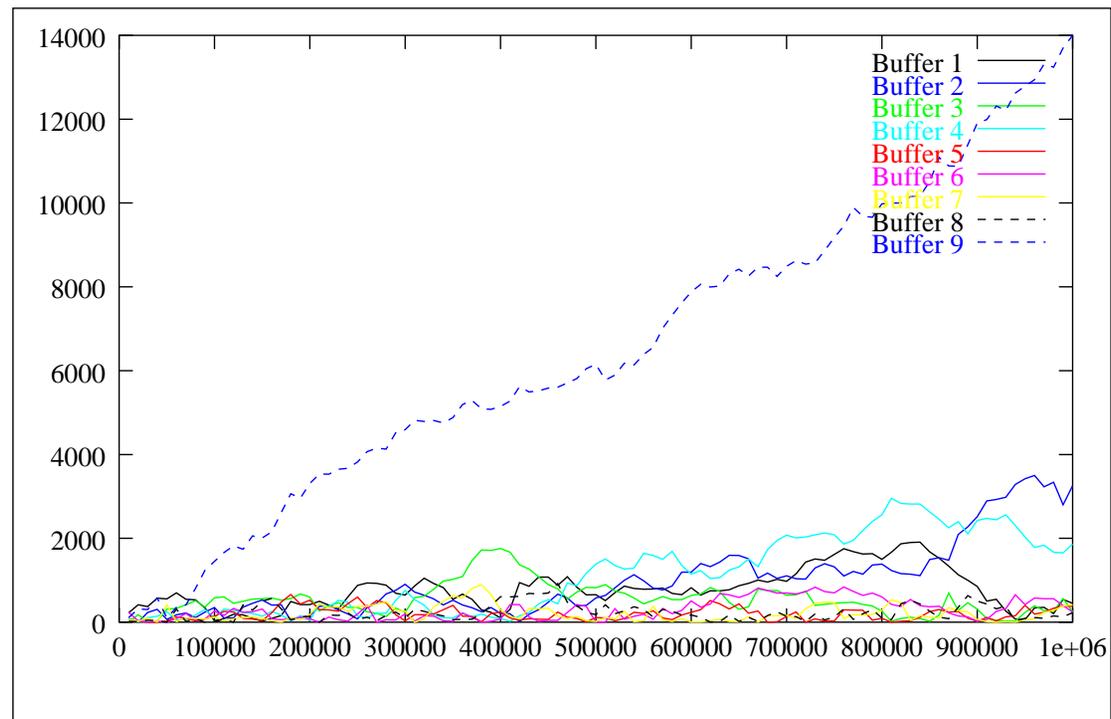
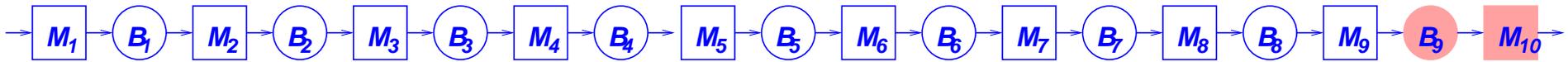
- and M_i is the bottleneck.

Infinite-Buffer Line

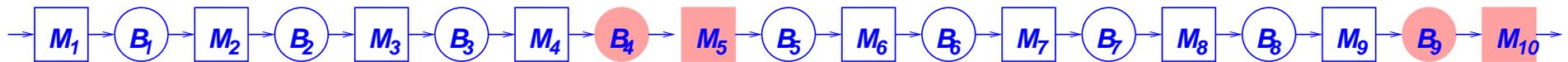


- The system is not in steady state.
- An infinite amount of inventory accumulates in the buffer upstream of the bottleneck.
- A finite amount of inventory appears downstream of the bottleneck.

Infinite-Buffer Line

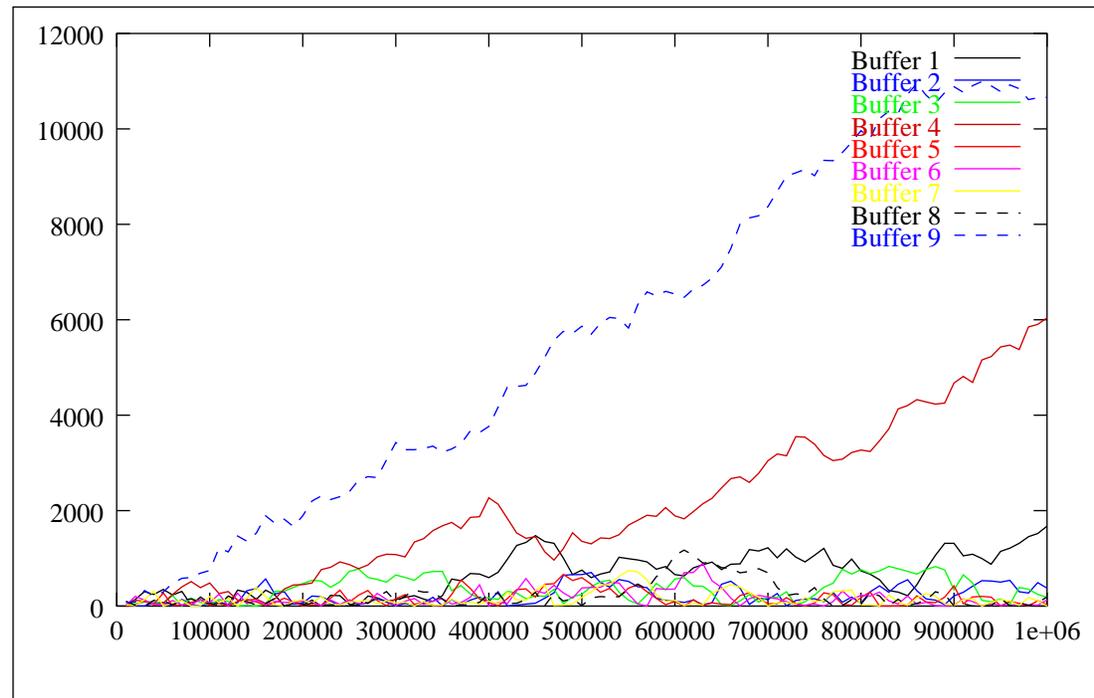
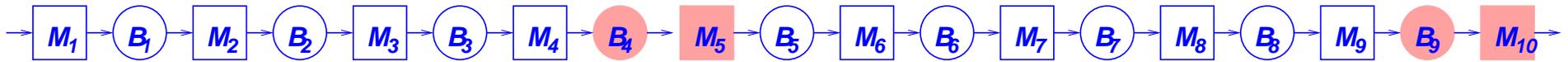


Infinite-Buffer Line



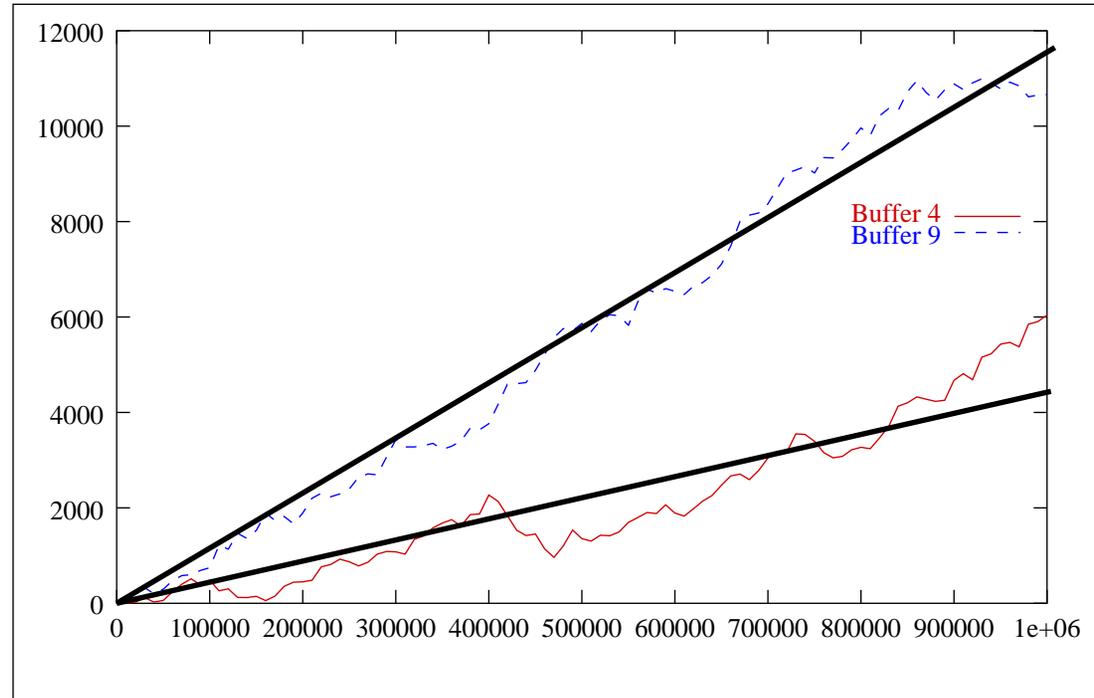
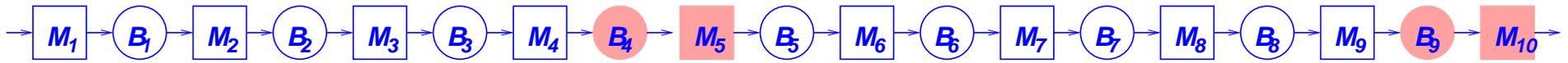
- The *second bottleneck* is the slowest machine upstream of the bottleneck. An infinite amount of inventory accumulates just upstream of it.
- A finite amount of inventory appears between the second bottleneck and the machine upstream of the first bottleneck.
- Et cetera.

Infinite-Buffer Line



A 10-machine line with bottlenecks at Machines 5 and 10.

Infinite-Buffer Line



Question:

- What are the slopes (*roughly!*) of the two indicated graphs?

Infinite-Buffer Line

Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Simulation Note

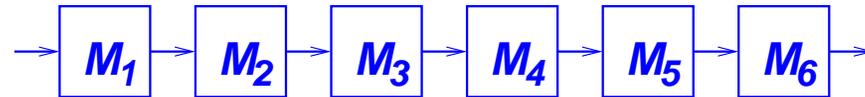
- The simulations shown here were *time-based* rather than *event-based*.
- Time-based simulations are easier to program, but less general, less accurate, and slower, than event-based simulations.
- Primarily for systems where all event times are geometrically distributed.

Simulation Note

Assume that some event occurs according to a geometric probability distribution and it has a mean time to occur of T time steps. Then the probability that it occurs in any time step is $1/T$.

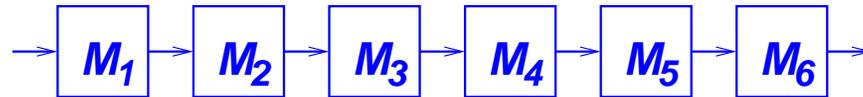
- Discretize time.
- At each time step , choose a $U[0,1]$ random number.
- If the number is less than or equal to $1/T$, the event has occurred. Change the state accordingly.
- If the number is greater than $1/T$, the event has not occurred. Change the state accordingly.

Zero-Buffer Line



- If any one machine fails, or takes a very long time to do an operation, *all* the other machines must wait.
- Therefore the production rate is usually less — possibly much less — than the slowest machine.

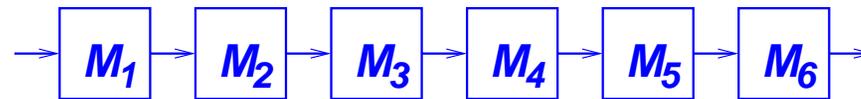
Zero-Buffer Line



- *Example:* Constant, unequal operation times, perfectly reliable machines.
 - ★ The operation time of the line is equal to the operation time of the slowest machine, so the production rate of the line is *equal to* that of the slowest machine.

Zero-Buffer Line

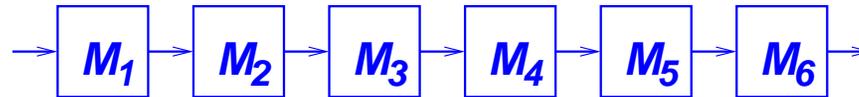
Constant,
equal operation times,
unreliable machines



- *Assumption:* Failure and repair times are *geometrically* distributed.
- Define $p_i = \tau / \text{MTTF}_i$ = probability of failure during an operation.
- Define $r_i = \tau / \text{MTTR}_i$ probability of repair during an interval of length τ when the machine is down.

Zero-Buffer Line

Constant,
equal operation times,
unreliable machines



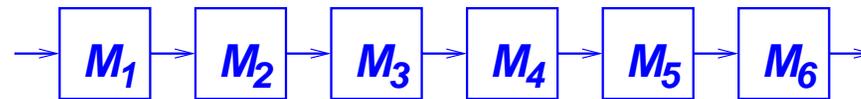
Buzacott's Zero-Buffer Line Formula:

Let k be the number of machines in the line. Then

$$P = \frac{1}{\tau} \frac{1}{1 + \sum_{i=1}^k \frac{p_i}{r_i}}$$

Zero-Buffer Line

Constant,
equal operation times,
unreliable machines

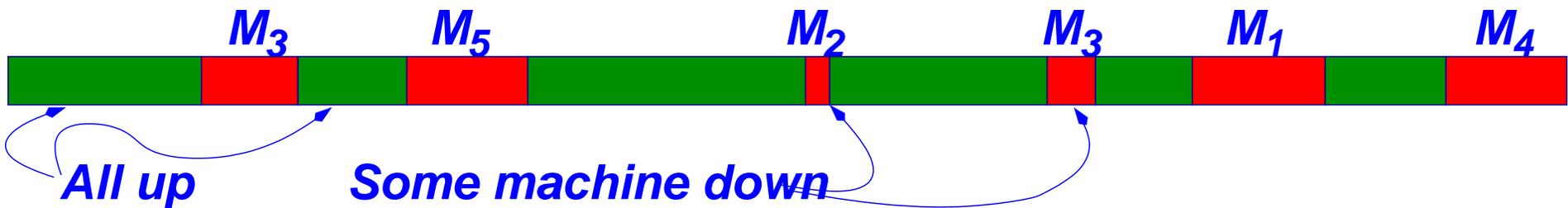


- Same as the earlier formula (page 6, page 9) when $k = 1$. The *isolated production rate* of a single machine M_i is

$$\frac{1}{\tau} \left(\frac{1}{1 + \frac{p_i}{r_i}} \right) = \frac{1}{\tau} \left(\frac{r_i}{r_i + p_i} \right) .$$

Zero-Buffer Line

- Let τ (the operation time) be the time unit.
- *Approximation:* At most, one machine can be down.
- Consider a long time interval of length $T\tau$ during which Machine M_i fails m_i times ($i = 1, \dots, k$).



- Without failures, the line would produce T parts.

Zero-Buffer Line

- The average repair time of M_i is τ / r_i each time it fails, so the total system down time is close to

$$D\tau = \sum_{i=1}^k \frac{m_i\tau}{r_i}$$

where D is the number of operation times in which a machine is down.

Zero-Buffer Line

- The total up time is approximately

$$U_{\tau} = T_{\tau} - \sum_{i=1}^k \frac{m_i \tau}{r_i}$$

- where U is the number of operation times in which all machines are up.

Zero-Buffer Line

- Since the system produces one part per time unit while it is working, it produces U parts during the interval of length $T\tau$.
- Note that, approximately,

$$m_i = p_i U$$

because M_i can only fail while it is operational.

Zero-Buffer Line

- Thus,

$$U_{\tau} = T_{\tau} - U_{\tau} \sum_{i=1}^k \frac{p_i}{r_i},$$

or,

$$\frac{U}{T} = E_{ODF} = \frac{1}{1 + \sum_{i=1}^k \frac{p_i}{r_i}}$$

Zero-Buffer Line

and

$$P = \frac{1}{\tau} \frac{1}{1 + \sum_{i=1}^k \frac{p_i}{r_i}}$$

- Note that P is a function of the *ratio* p_i/r_i and not p_i or r_i separately.
- The same statement is true for the infinite-buffer line.
- However, the same statement is *not* true for a line with finite, non-zero buffers.

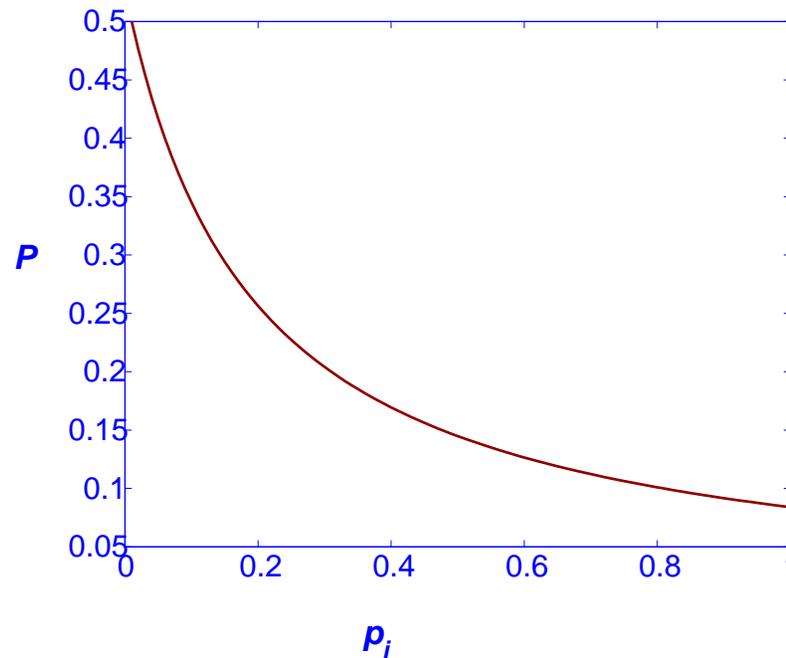
Zero-Buffer Line

Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

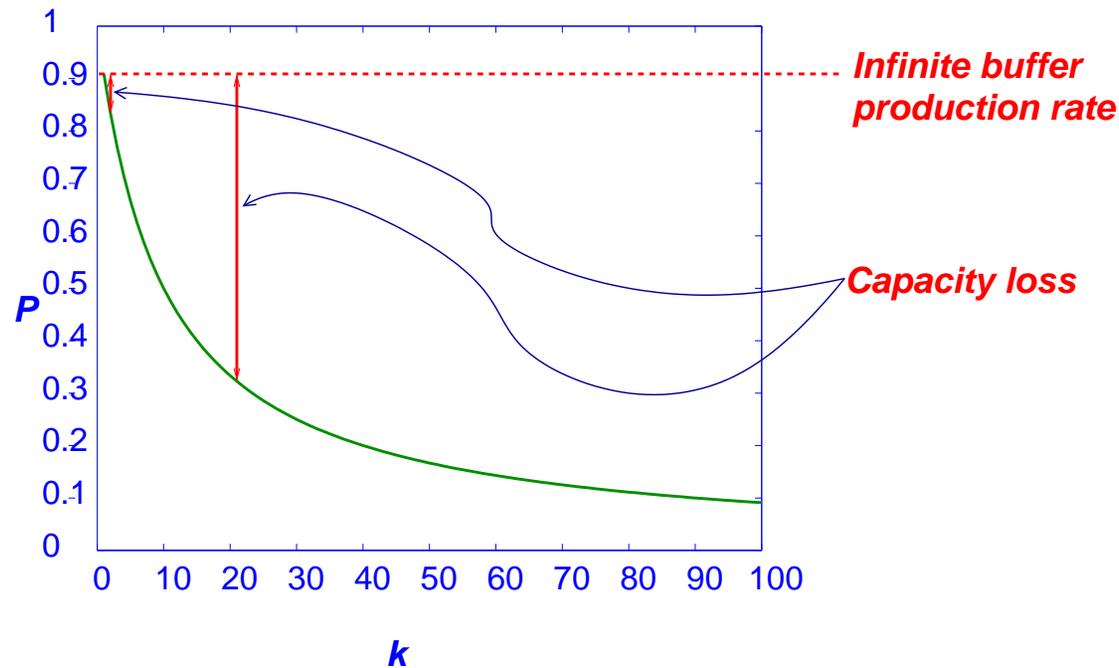
Zero-Buffer Line

All machines are the same except M_i . As p_i increases, the production rate decreases.



Zero-Buffer Line

All machines are the same. As the line gets longer, the production rate decreases.



Finite-Buffer Lines



- Motivation for buffers: recapture some of the lost production rate.
- Cost
 - ★ in-process inventory/lead time
 - ★ floor space
 - ★ material handling mechanism

Finite-Buffer Lines



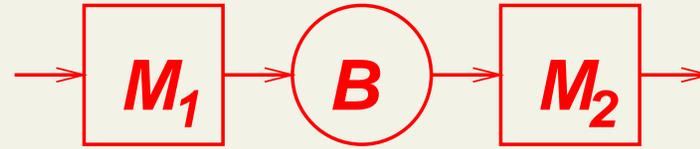
- **Infinite buffers:** delayed downstream propagation of disruptions (*starvation*) and *no* upstream propagation.
- **Zero buffers:** instantaneous propagation in both directions.
- **Finite buffers:** delayed propagation in both directions.
 - ★ New phenomenon: *blockage*.
- **Blockage:** Machine M_i is blocked at time t if Buffer B_i is full at time t .

Finite-Buffer Lines



- Difficulty:
 - ★ No simple formula for calculating production rate or inventory levels.
- Solution:
 - ★ Simulation
 - ★ Analytical approximation
 - ★ *Exact analytical solution for two-machine lines only.*

Two Machine, Finite-Buffer Lines



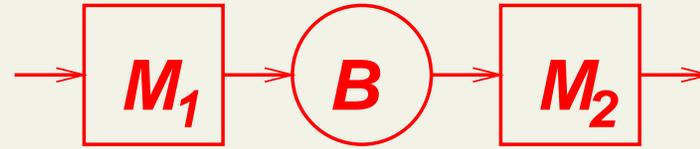
- Exact solution *is* available to Markov process model.
- *Discrete time-discrete state Markov process:*

$$\text{prob}\{X(t+1) = x(t+1) | X(t) = x(t),$$

$$X(t-1) = x(t-1), X(t-2) = x(t-2), \dots\} =$$

$$\text{prob}\{X(t+1) = x(t+1) | X(t) = x(t)\}$$

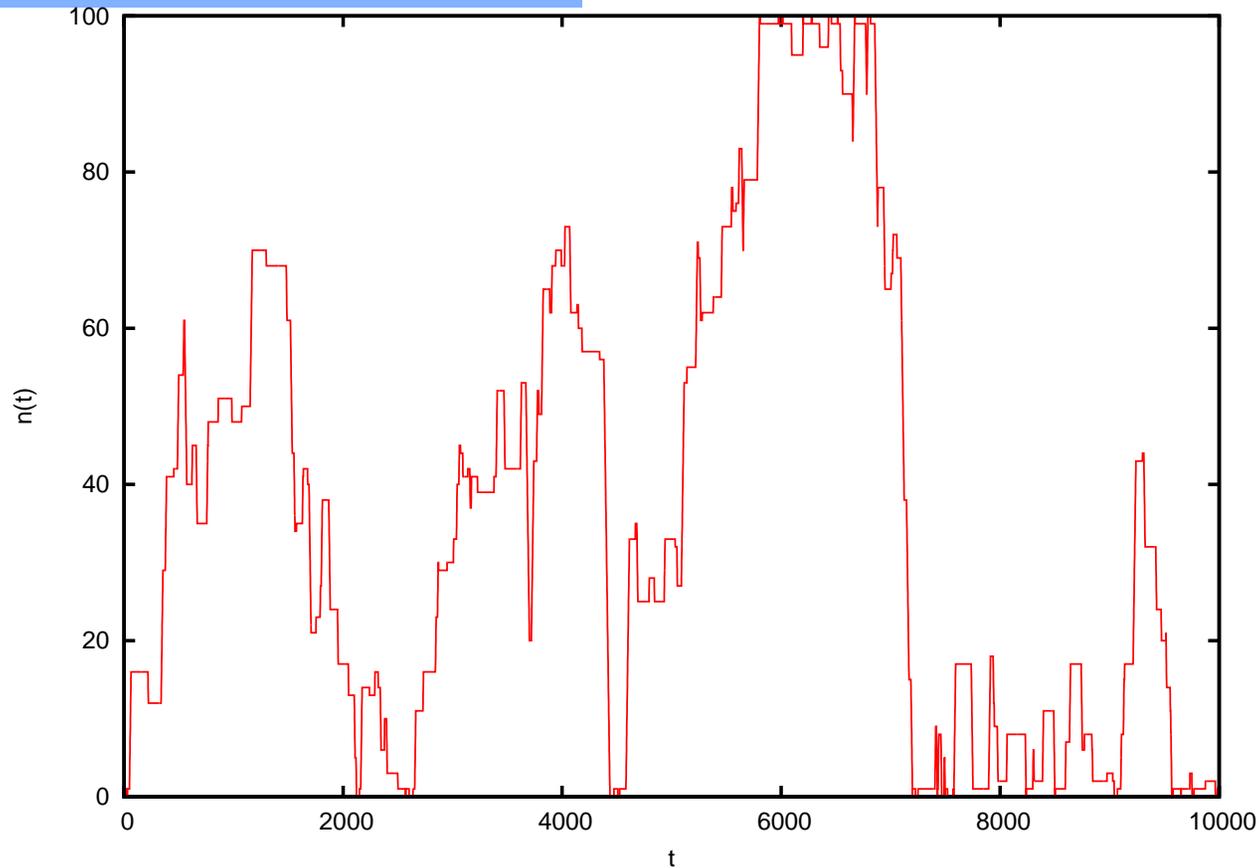
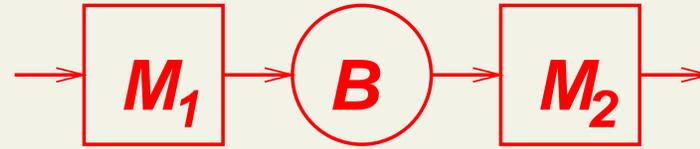
Two Machine, Finite-Buffer Lines



Here, $X(t) = (n(t), \alpha_1(t), \alpha_2(t))$, where

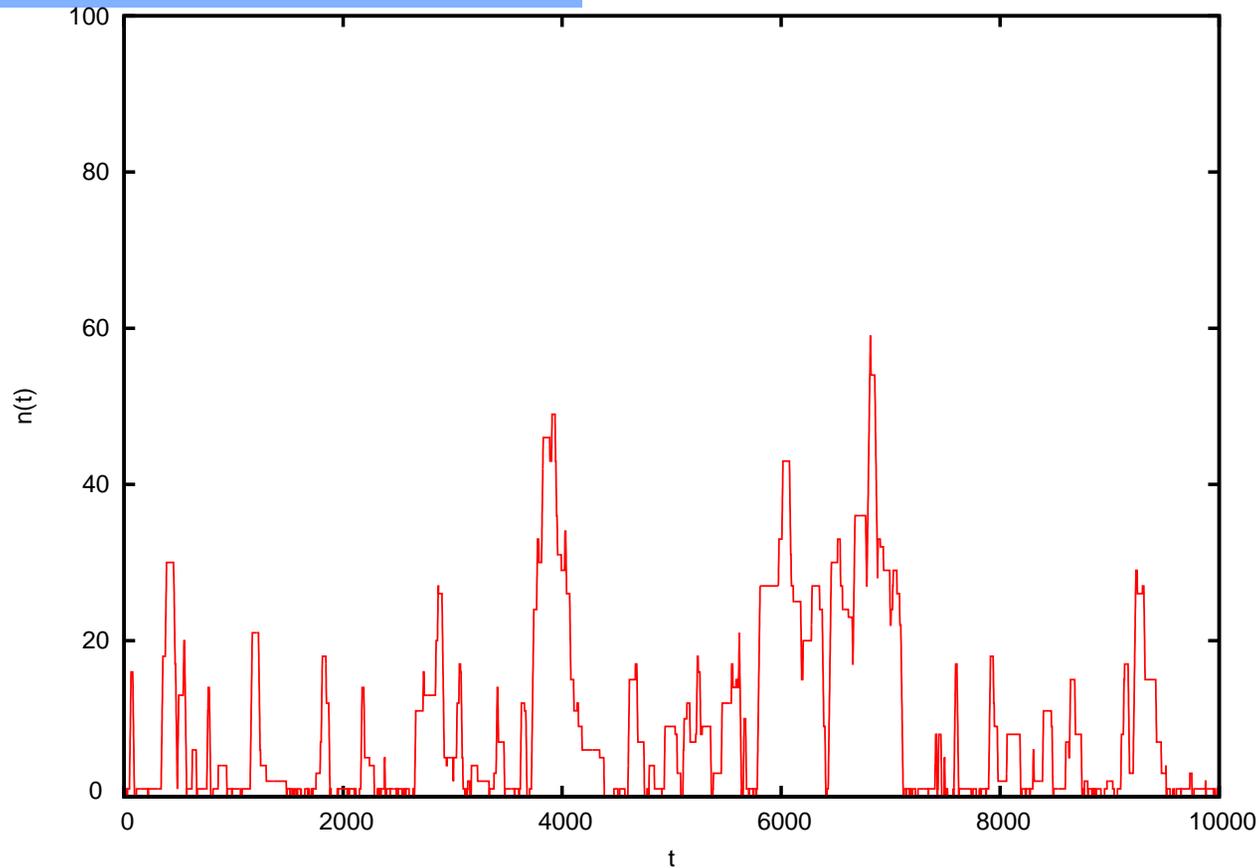
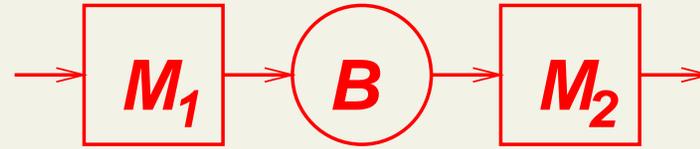
- n is the number of parts in the buffer;
 $n = 0, 1, \dots, N$.
- α_i is the repair state of M_i ; $i = 1, 2$.
 - ★ $\alpha_i = 1$ means the machine is *up* or *operational*;
 - ★ $\alpha_i = 0$ means the machine is *down* or *under repair*.

Two Machine, Finite-Buffer Lines



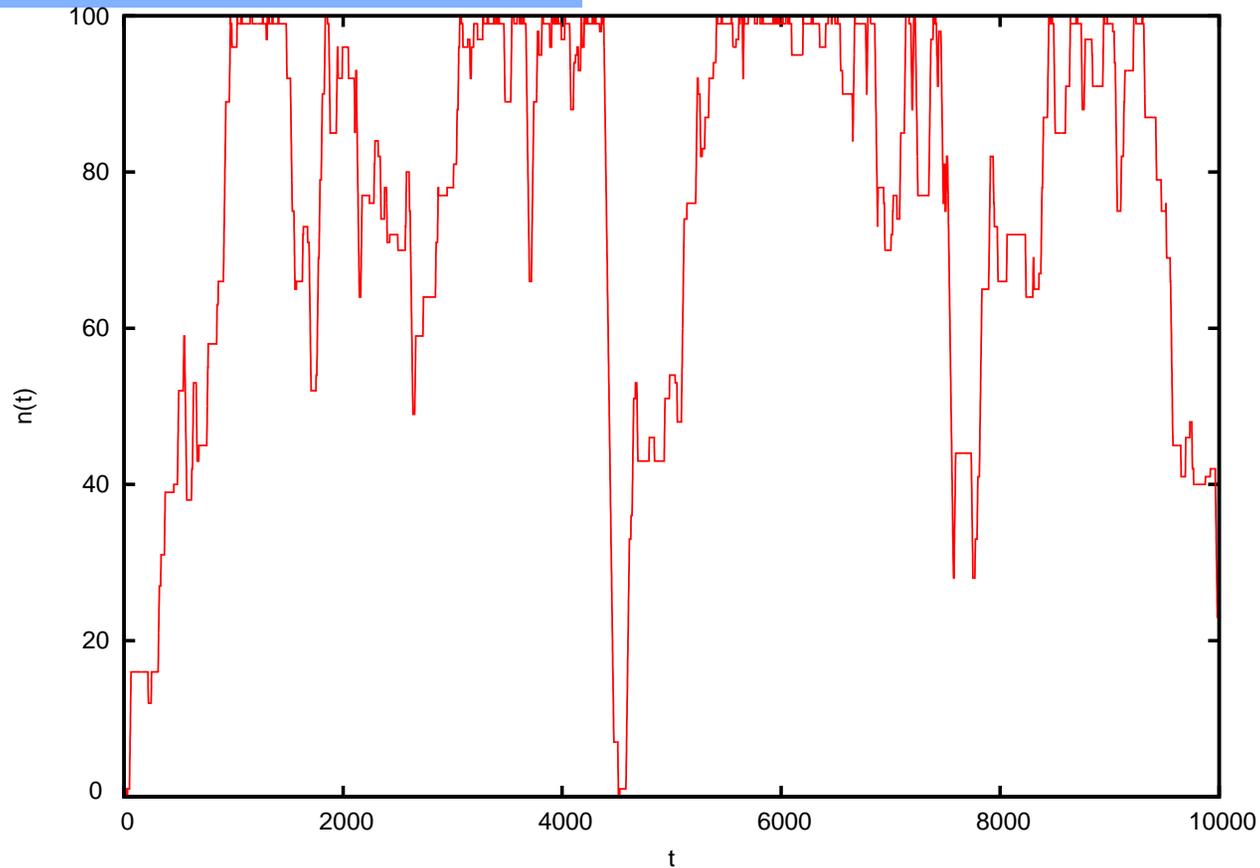
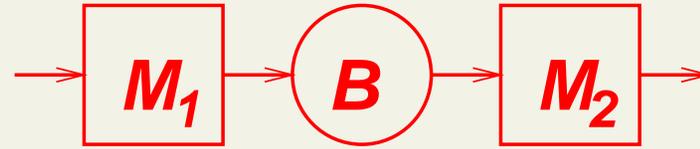
$$r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 100$$

Two Machine, Finite-Buffer Lines



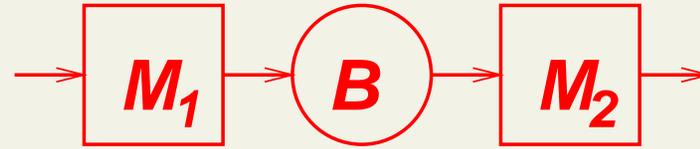
$$r_i = .1, i = 1, 2, p_1 = .02, p_2 = .01, N = 100$$

Two Machine, Finite-Buffer Lines



$$r_i = .1, i = 1, 2, p_1 = .01, p_2 = .02, N = 100$$

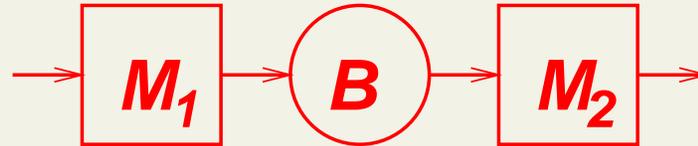
Two Machine, Finite-Buffer Lines



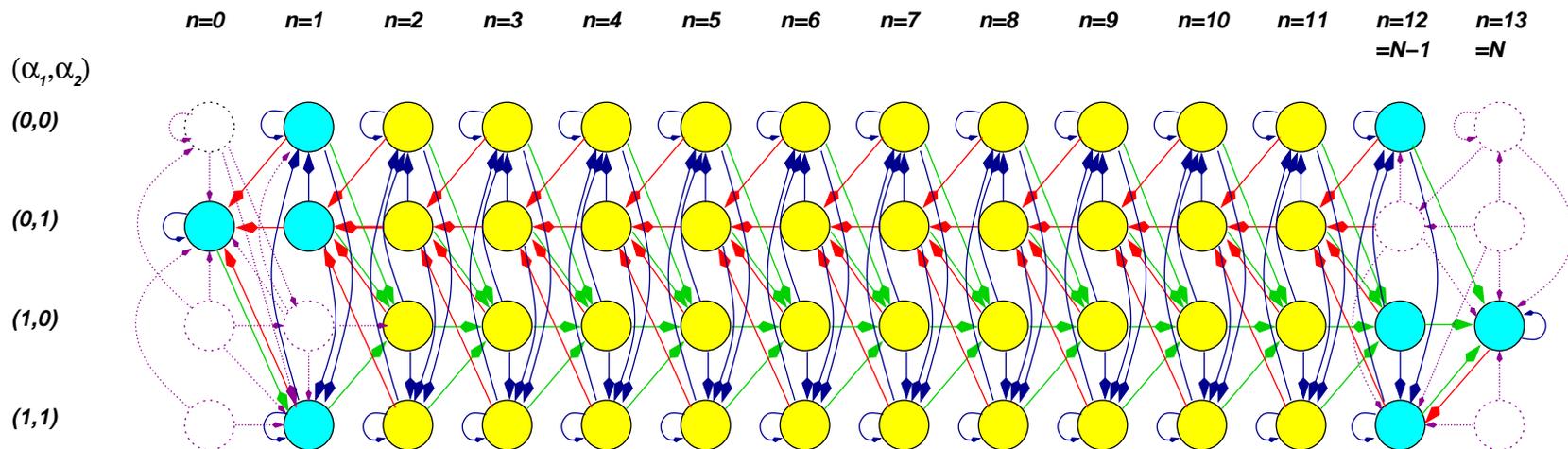
Several models available:

- *Deterministic processing time* , or *Buzacott model*: deterministic processing time, geometric failure and repair times; discrete state, discrete time.

Two Machine, Finite-Buffer Lines



State Transition Graph for Deterministic Processing Time, Two-Machine Line



key

states

transient



non-transient

boundary



internal



transitions

out of transient states



out of non-transient states

to increasing buffer level



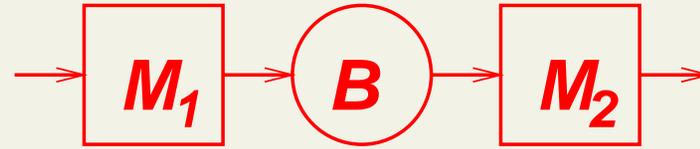
to decreasing buffer level



unchanging buffer level

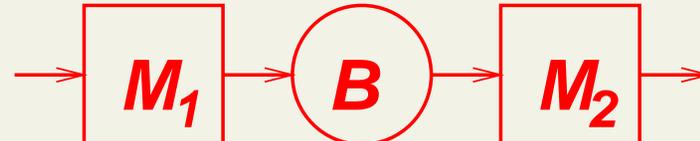


Two Machine, Finite-Buffer Lines



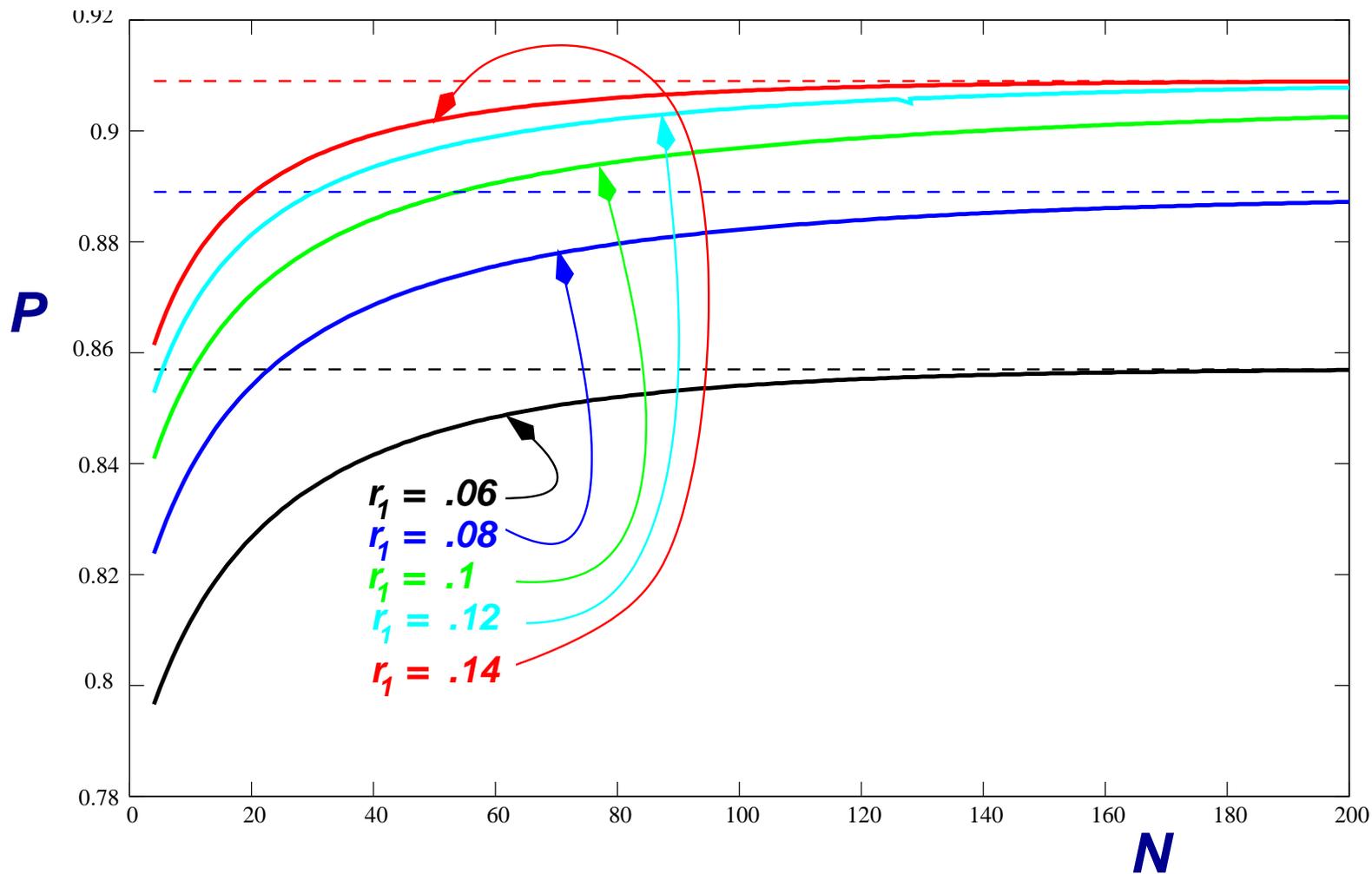
- *Exponential processing time*: exponential processing, failure, and repair time; discrete state, continuous time.
- *Continuous material, or fluid*: deterministic processing, exponential failure and repair time; mixed state, continuous time.

Two Machine, Finite-Buffer Lines

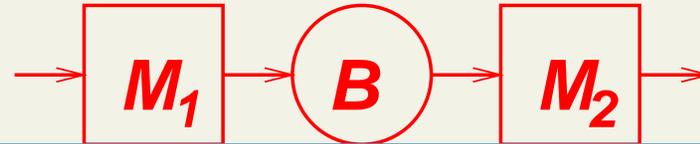


Production rate vs. Buffer Size

$\tau = 1.$
 $p_1 = .01$
 $r_2 = .1$
 $p_2 = .01$



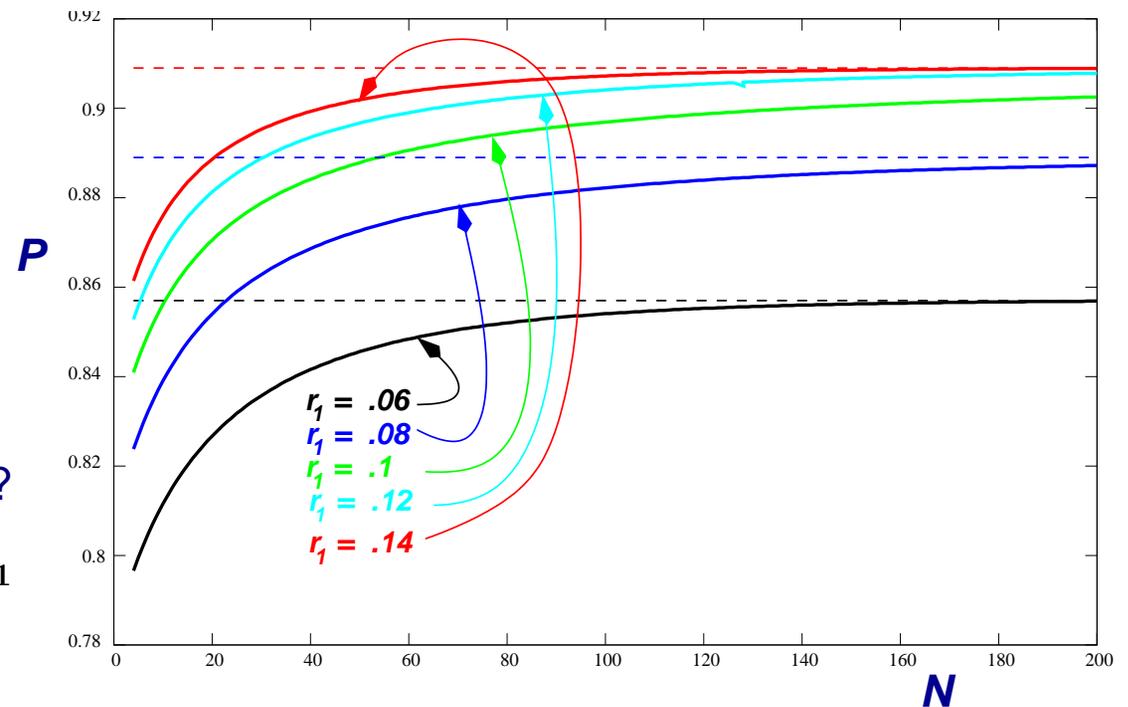
Two Machine, Finite-Buffer Lines



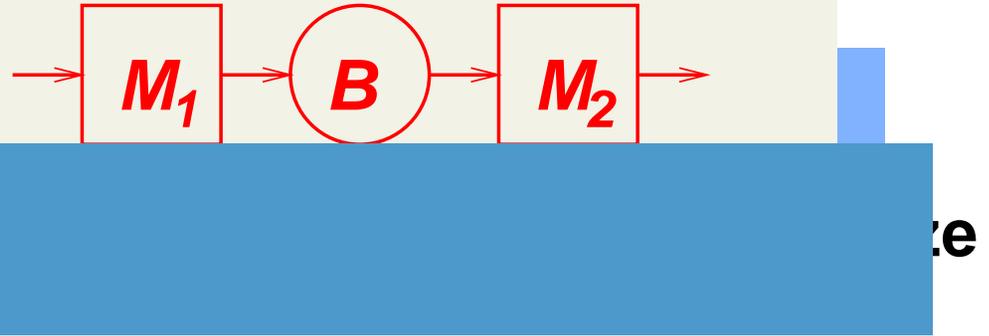
Production rate vs. Buffer Size

Discussion:

- Why are the curves increasing?
- Why do they reach an asymptote?
- What is P when $N = 0$?
- What is the limit of P as $N \rightarrow \infty$?
- Why are the curves with smaller r_1 lower?

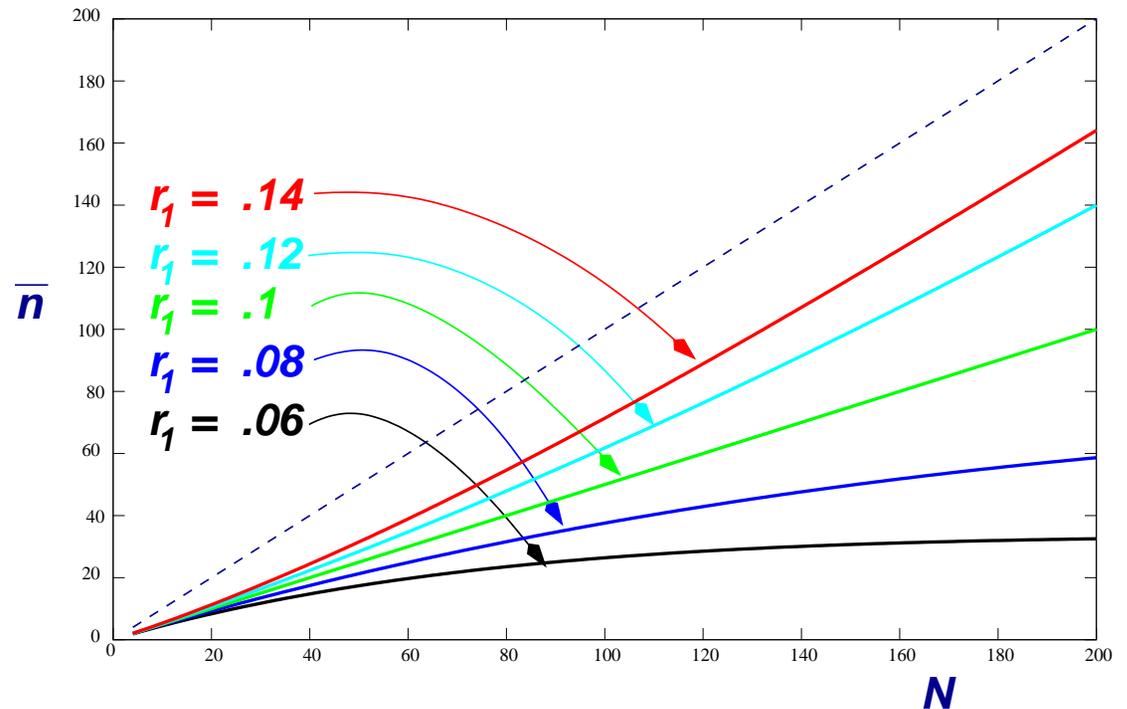


Two Machine, Finite-Buffer Lines

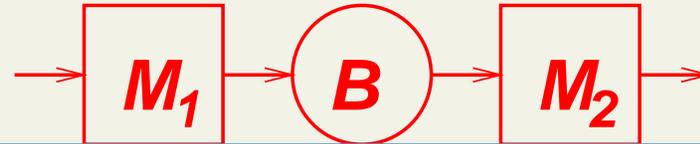


Discussion:

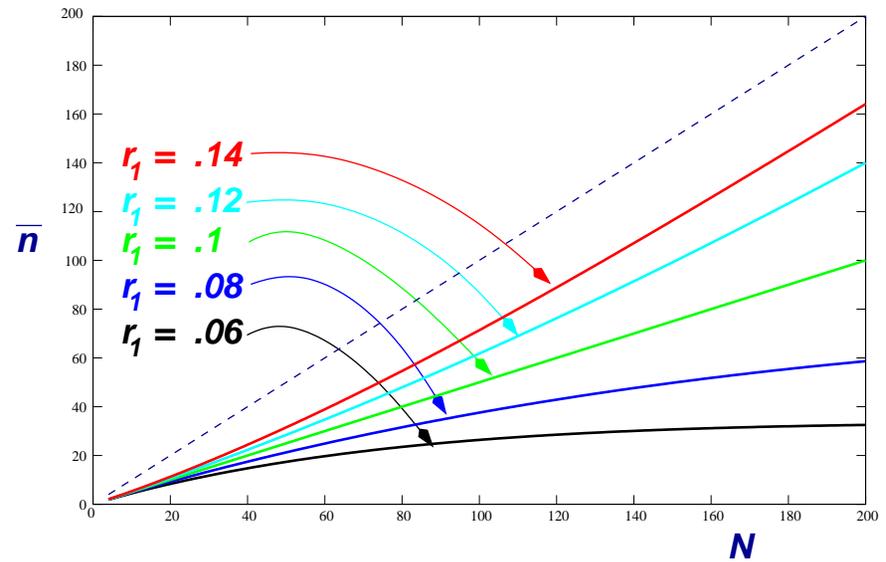
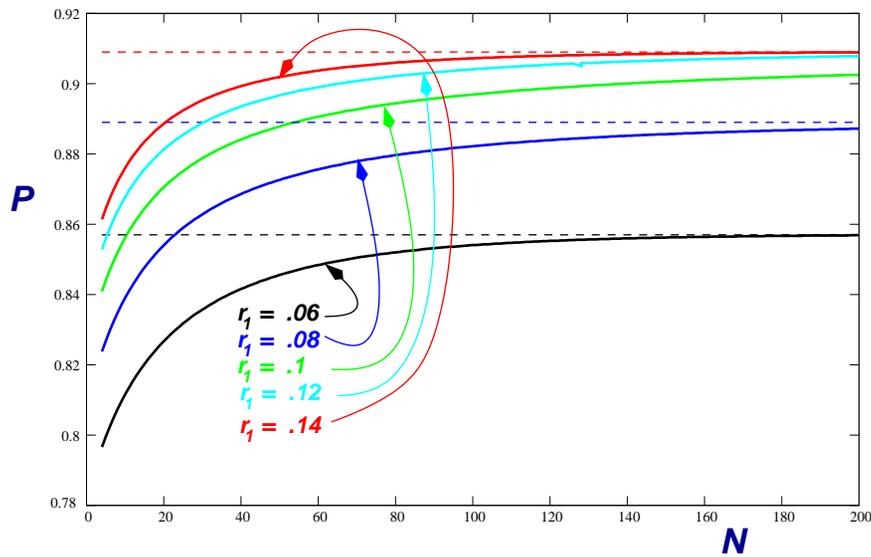
- Why are the curves increasing?
- Why *different* asymptotes?
- What is \bar{n} when $N = 0$?
- What is the limit of \bar{n} as $N \rightarrow \infty$?
- Why are the curves with smaller r_1 lower?



Two Machine, Finite-Buffer Lines

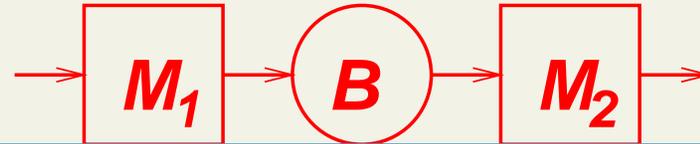


Discussion



- *What can you say about the optimal buffer size?*
- *How should it be related to r_i, p_i ?*

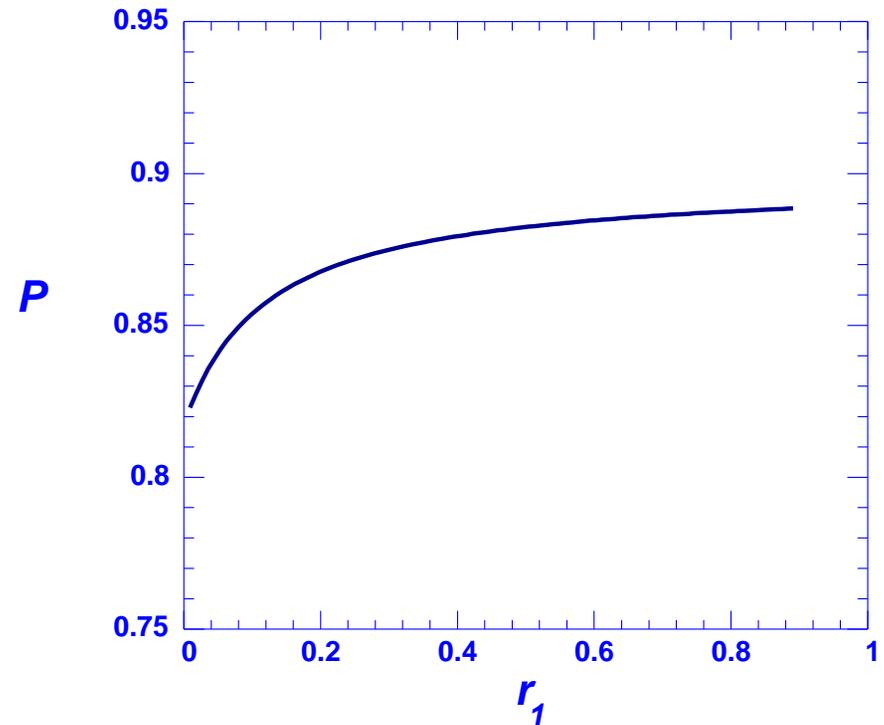
Two Machine, Finite-Buffer Lines



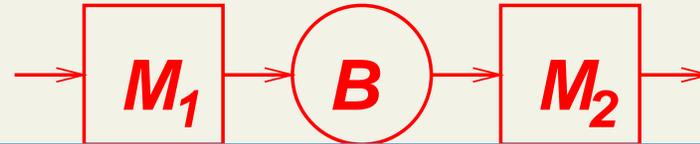
Discussion

Should we prefer short, frequent, disruptions or long, infrequent, disruptions?

- $r_2 = 0.8$, $p_2 = 0.09$, $N = 10$
- r_1 and p_1 vary together and $\frac{r_1}{r_1+p_1} = .9$
- *Answer:* evidently, short, frequent failures.
- *Why?*



Two Machine, Finite-Buffer Lines

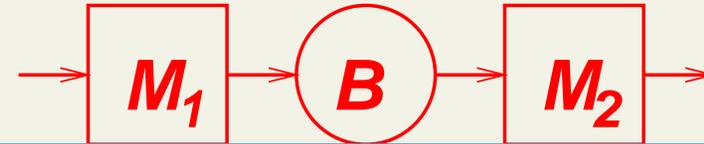


Discussion

Questions:

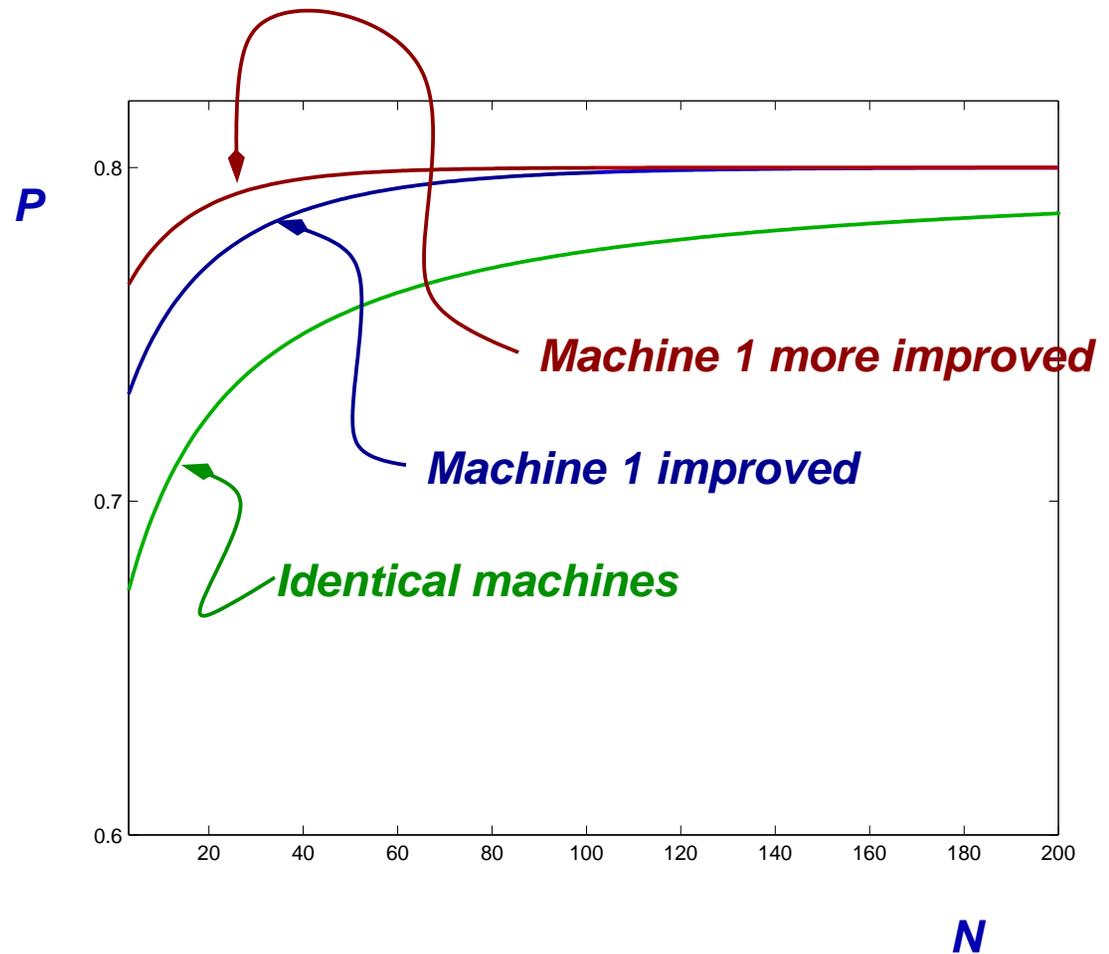
- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Two Machine, Finite-Buffer Lines

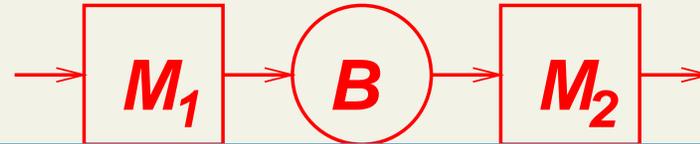


Production rate vs. storage space

Improvements to
non-bottleneck
machine.

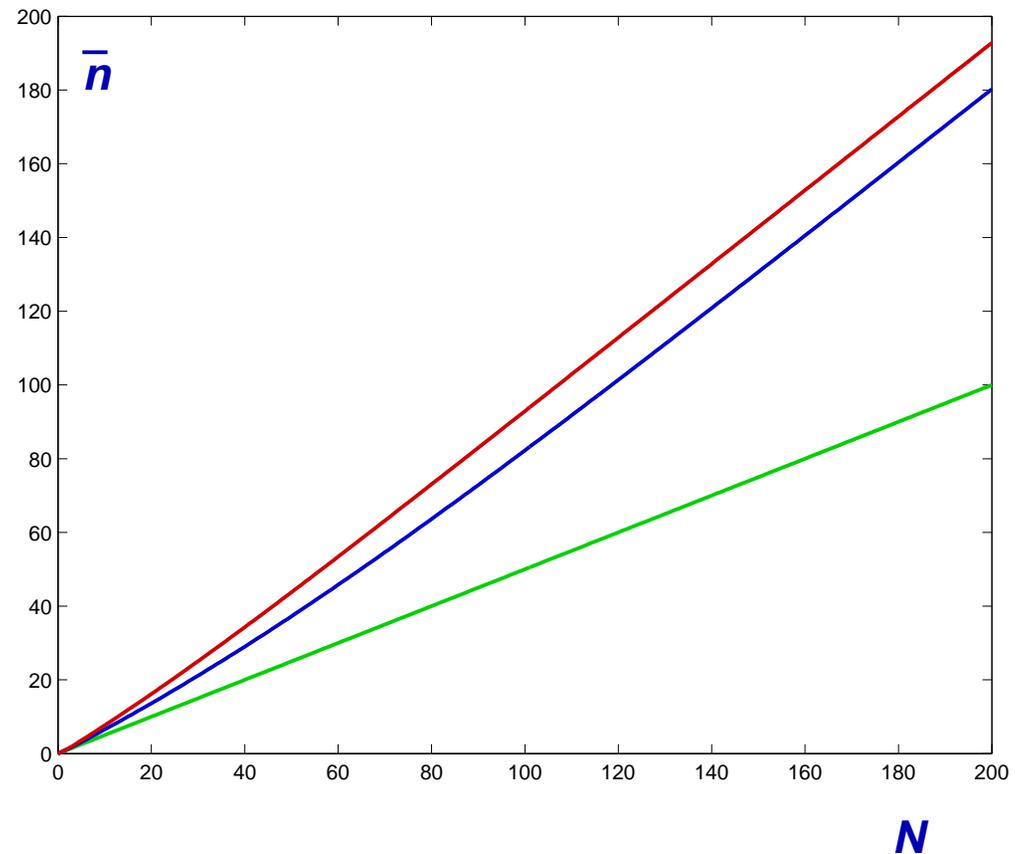


Two Machine, Finite-Buffer Lines

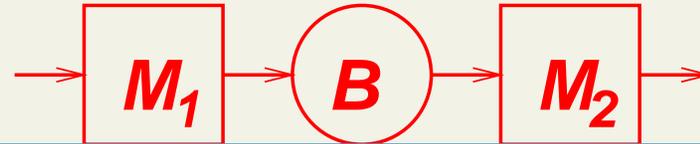


Avg. inventory vs. storage space

- Inventory increases as the (non-bottleneck) *upstream* machine is improved and as the buffer space is increased.
- If the *downstream* machine were improved, the inventory would be less and it would increase much less as the space increases.



Two Machine, Finite-Buffer Lines

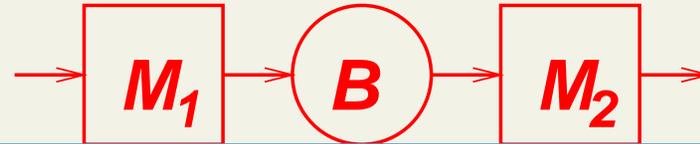


Other models

Exponential — discrete material, continuous time

- $\mu_i \delta t =$ the probability that M_i completes an operation in $(t, t + \delta t)$;
- $p_i \delta t =$ the probability that M_i fails during an operation in $(t, t + \delta t)$;
- $r_i \delta t =$ the probability that M_i is repaired, while it is down, in $(t, t + \delta t)$;

Two Machine, Finite-Buffer Lines

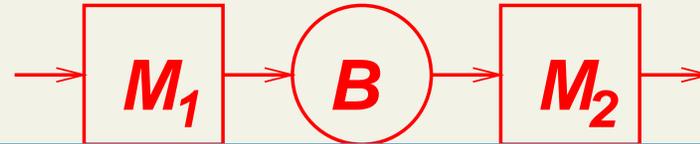


Other models

Continuous — continuous material, continuous time

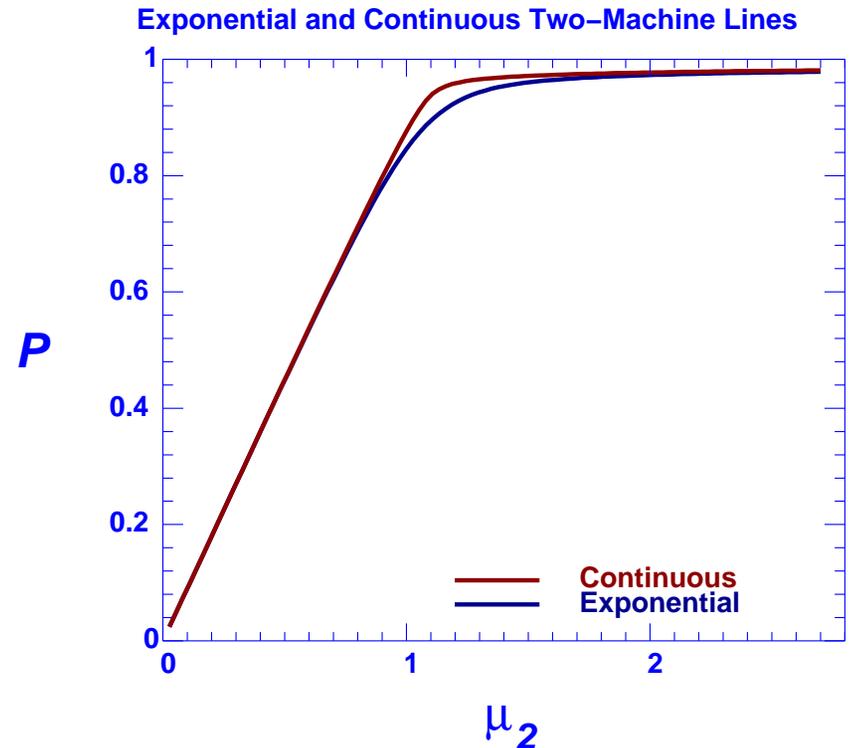
- $\mu_i \delta t$ = the amount of material that M_i processes, while it is up, in $(t, t + \delta t)$;
- $p_i \delta t$ = the probability that M_i fails, while it is up, in $(t, t + \delta t)$;
- $r_i \delta t$ = the probability that M_i is repaired, while it is down, in $(t, t + \delta t)$;

Two Machine, Finite-Buffer Lines

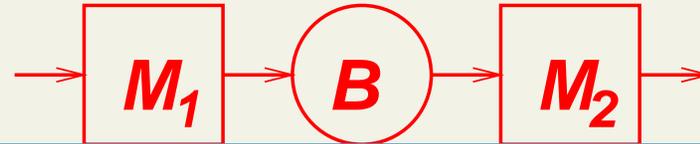


Other models

- $r_1 = 0.09$, $p_1 = 0.01$, $\mu_1 = 1.1$
- $r_2 = 0.08$, $p_2 = 0.009$
- $N = 20$
- *Explain the shapes of the graphs.*

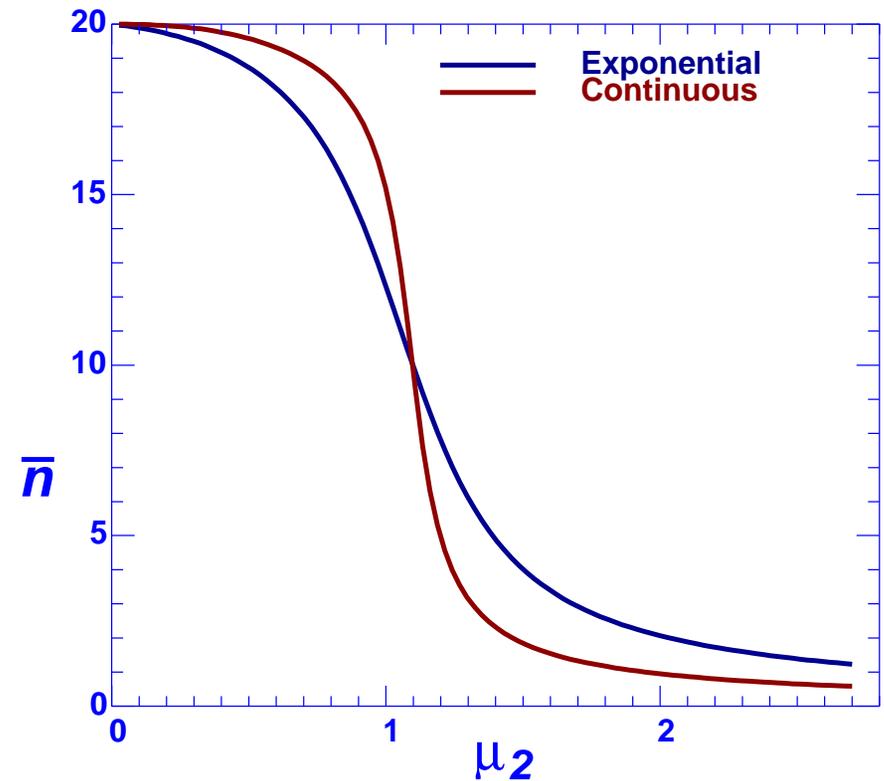


Two Machine, Finite-Buffer Lines

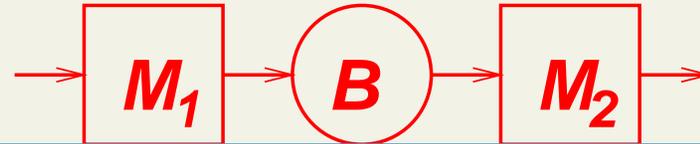


Other models

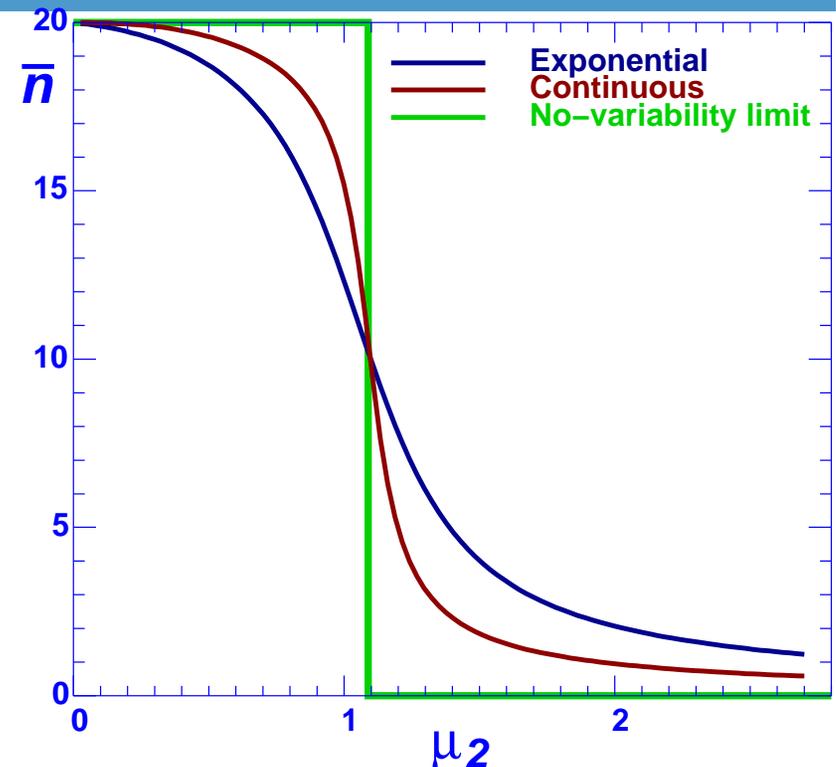
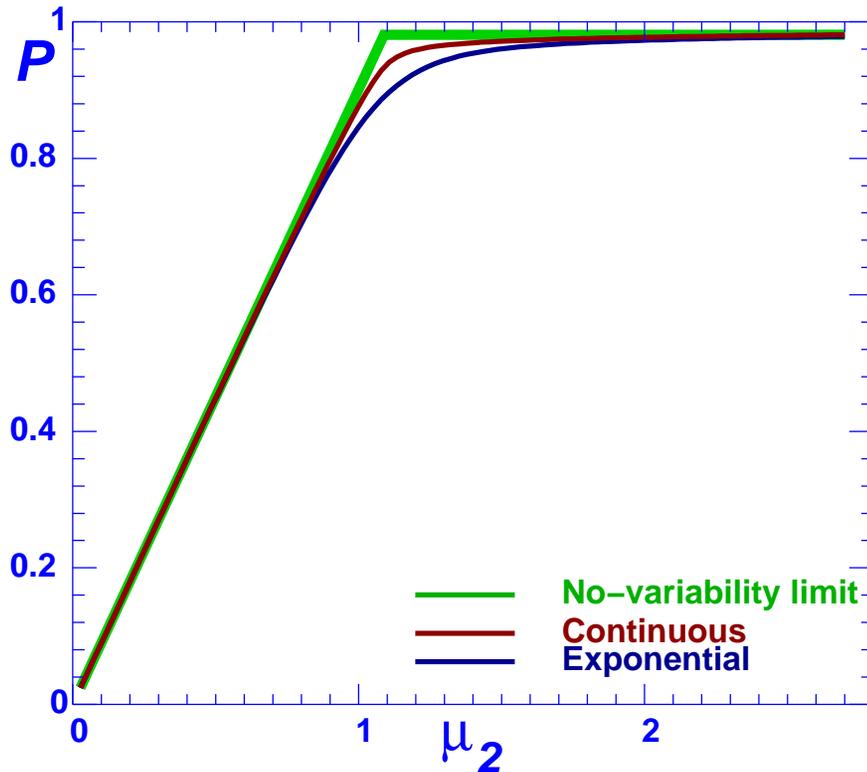
- *Explain the shapes of the graphs.*



Two Machine, Finite-Buffer Lines



Other models



No-variability limit: a continuous model where both machines are reliable, and processing rate μ'_i of machine i in the no-variability is the same as the isolated production rate of machine i in the other cases. That is, $\mu'_i = \mu_i r_i / (r_i + p_i)$.

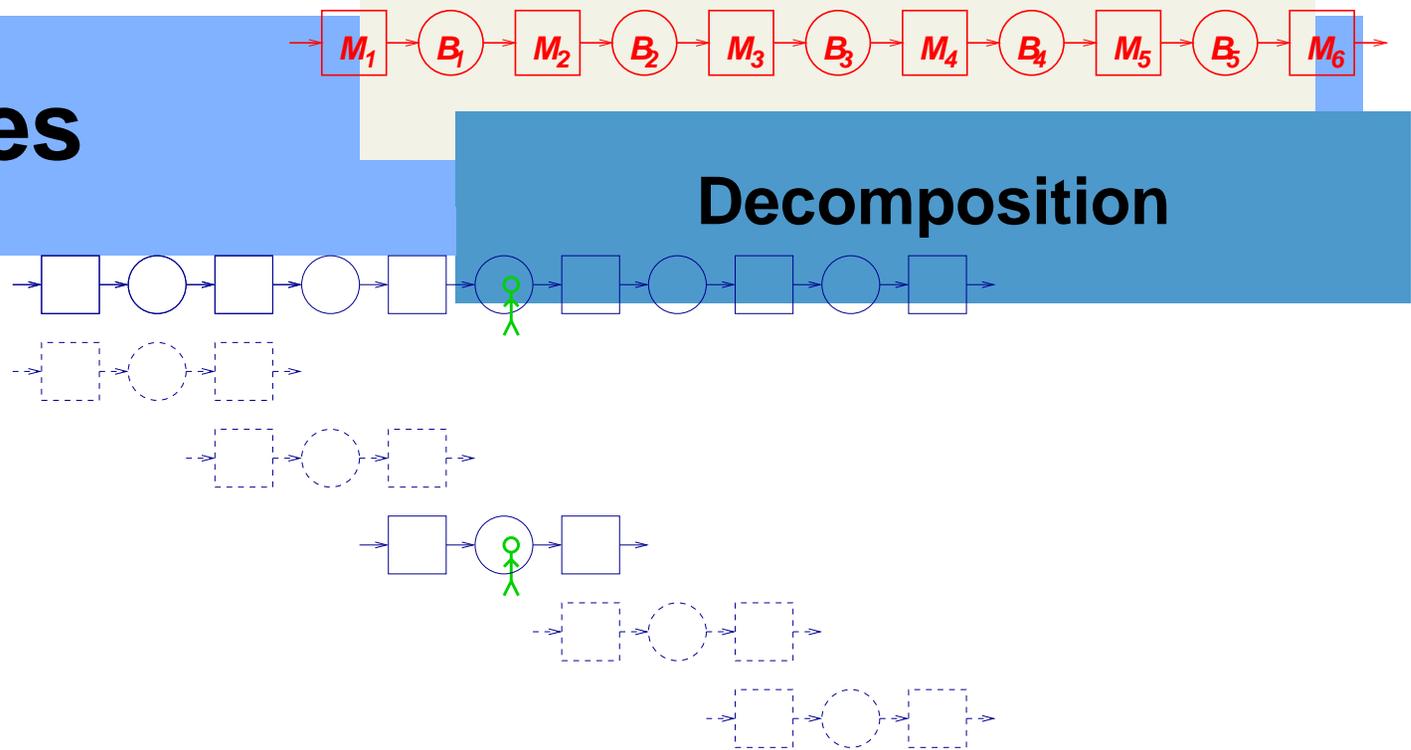
Long Lines



- Difficulty:

- ★ No simple formula for calculating production rate or inventory levels.
- ★ State space is too large for exact numerical solution.
 - * If all buffer sizes are N and the length of the line is k , the number of states is $S = 2^k (N + 1)^{k-1}$.
 - * if $N = 10$ and $k = 20$, $S = 6.41 \times 10^{25}$.
- ★ *Decomposition* seems to work successfully.

Long Lines



- Decomposition breaks up systems and then reunites them.
- Conceptually: put an observer in a buffer, and tell him that he is in the buffer of a two-machine line.
- Question: *What would the observer see, and how can he be convinced he is in a two-machine line? Construct the two-machine line. Construct all the two-machine lines.*

Long Lines



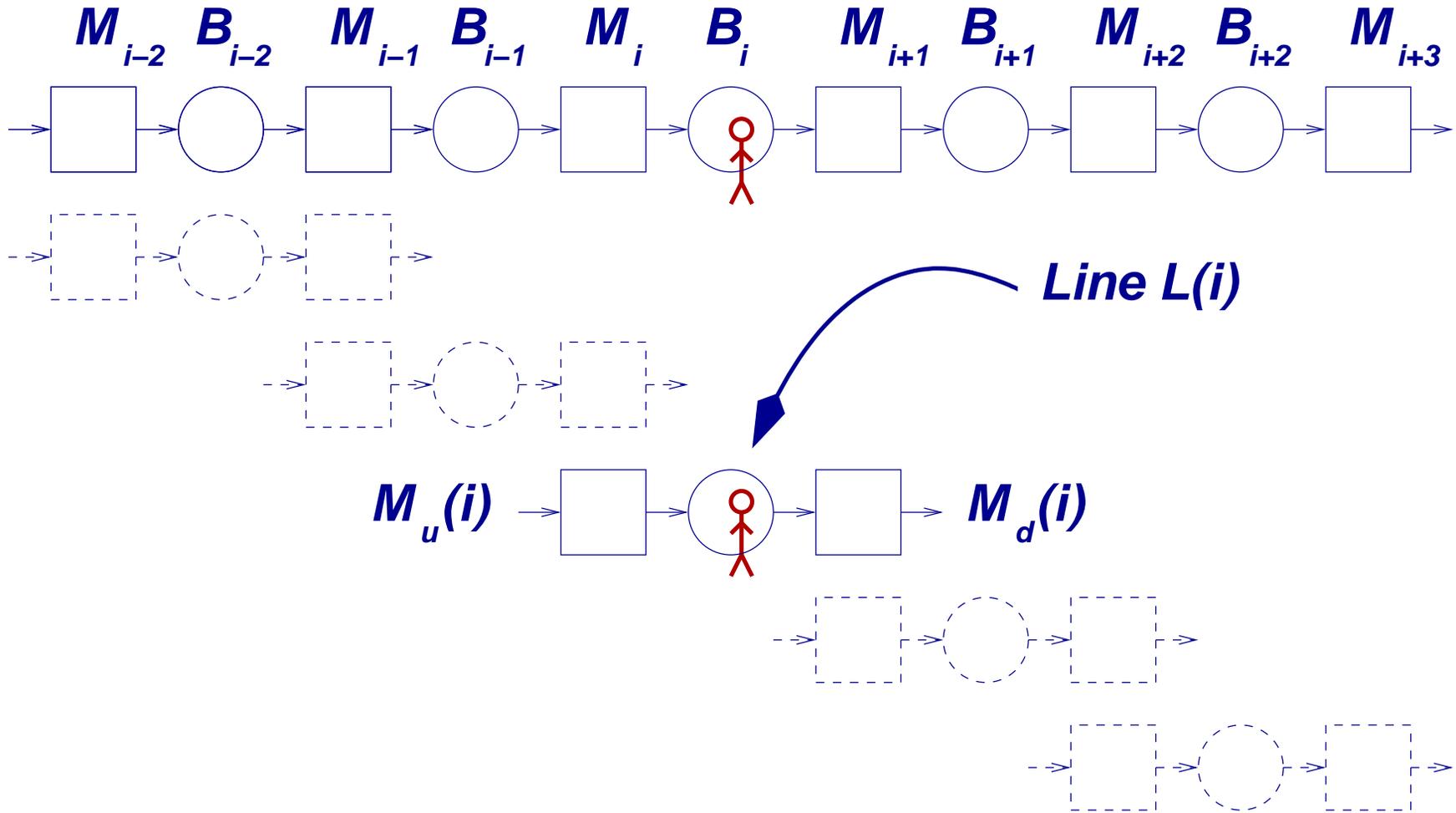
Decomposition

- Consider an observer in Buffer B_i .
 - ★ Imagine the material flow process that the observer sees *entering* and the material flow process that the observer sees *leaving* the buffer.
- We construct a two-machine line $L(i)$
 - ★ ie, we find machines $M_u(i)$ and $M_d(i)$ with parameters $r_u(i)$, $p_u(i)$, $r_d(i)$, $p_d(i)$, and $N(i) = N_i$such that an observer in its buffer will see almost the same processes.
- The parameters are chosen as functions of the behaviors of the *other* two-machine lines.

Long Lines



Decomposition



Long Lines



Decomposition

There are $4(k - 1)$ unknowns. Therefore, we need

- $4(k - 1)$ equations, and
- an algorithm for solving those equations.

Long Lines

- *Conservation of flow*, equating all production rates.
- *Flow rate/idle time*, relating production rate to probabilities of starvation and blockage.
- *Resumption of flow*, relating $r_u(i)$ to upstream events and $r_d(i)$ to downstream events.
- *Boundary conditions*, for parameters of $M_u(\mathbf{1})$ and $M_d(k - \mathbf{1})$.

Long Lines

- All the quantities in these equations are
 - ★ specified parameters, or
 - ★ unknowns, or
 - ★ functions of parameters or unknowns derived from the two-machine line analysis.
- This is a set of $4(k - 1)$ equations.

Long Lines

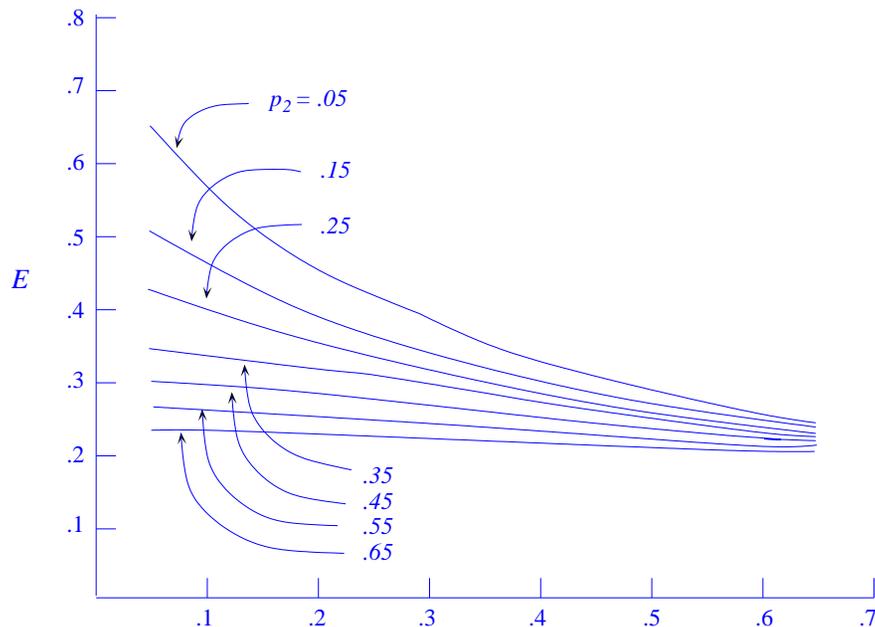
DDX algorithm : due to Dallery, David, and Xie (1988).

1. Guess the downstream parameters of $L(1)$ ($r_d(1), p_d(1)$). Set $i = 2$.
2. Use the equations to obtain the upstream parameters of $L(i)$ ($r_u(i), p_u(i)$). Increment i .
3. Continue in this way until $L(k - 1)$. Set $i = k - 2$.
4. Use the equations to obtain the downstream parameters of $L(i)$. Decrement i .
5. Continue in this way until $L(1)$.
6. Go to Step 2 or terminate.

Long Lines

Examples

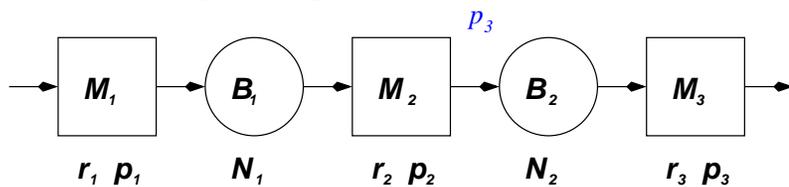
Three-machine line – production rate.



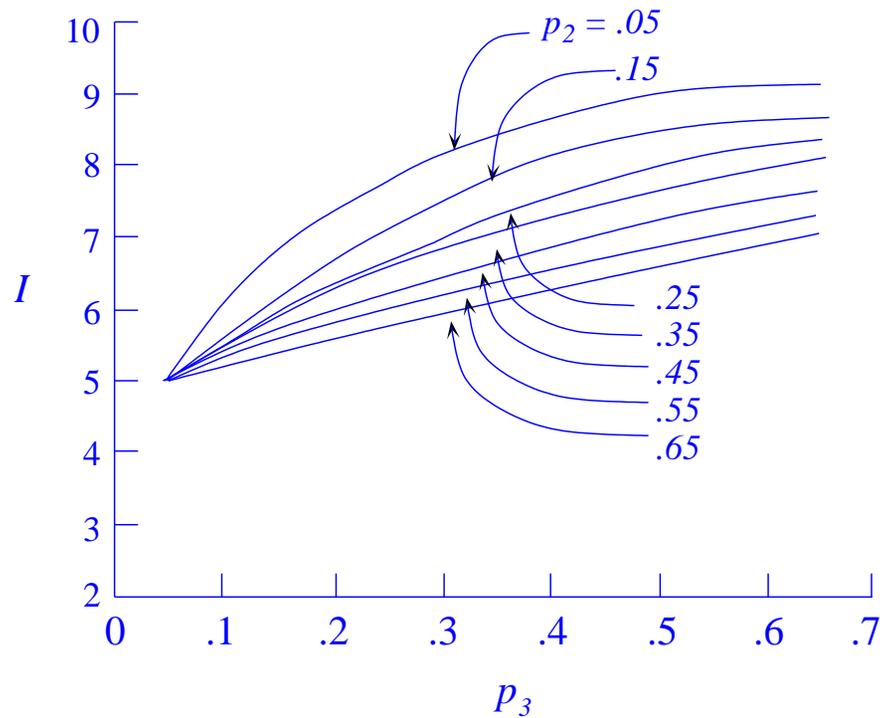
$$r_1 = r_2 = r_3 = .2$$

$$p_1 = .05$$

$$N_1 = N_2 = 5$$



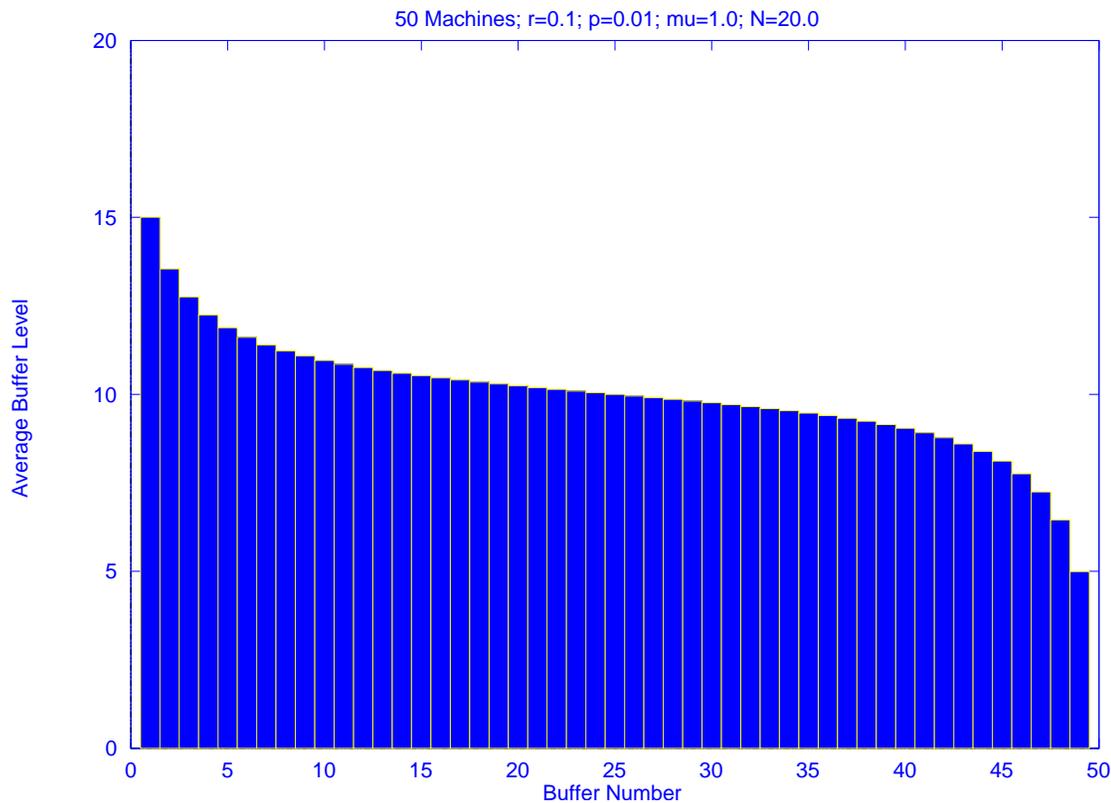
Three-machine line – total average inventory



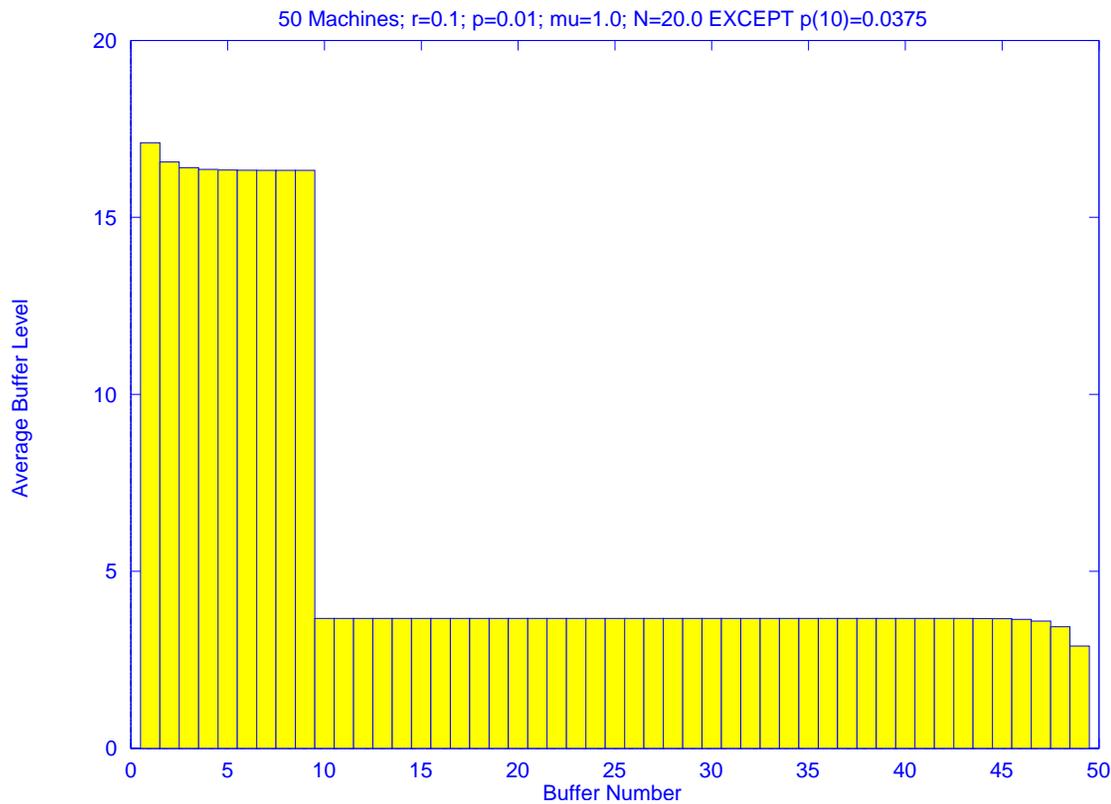
$$r_1 = r_2 = r_3 = .2$$

$$p_1 = .05$$

$$N_1 = N_2 = 5$$



Distribution of material in a line with identical machines and buffers. *Explain the shape.*



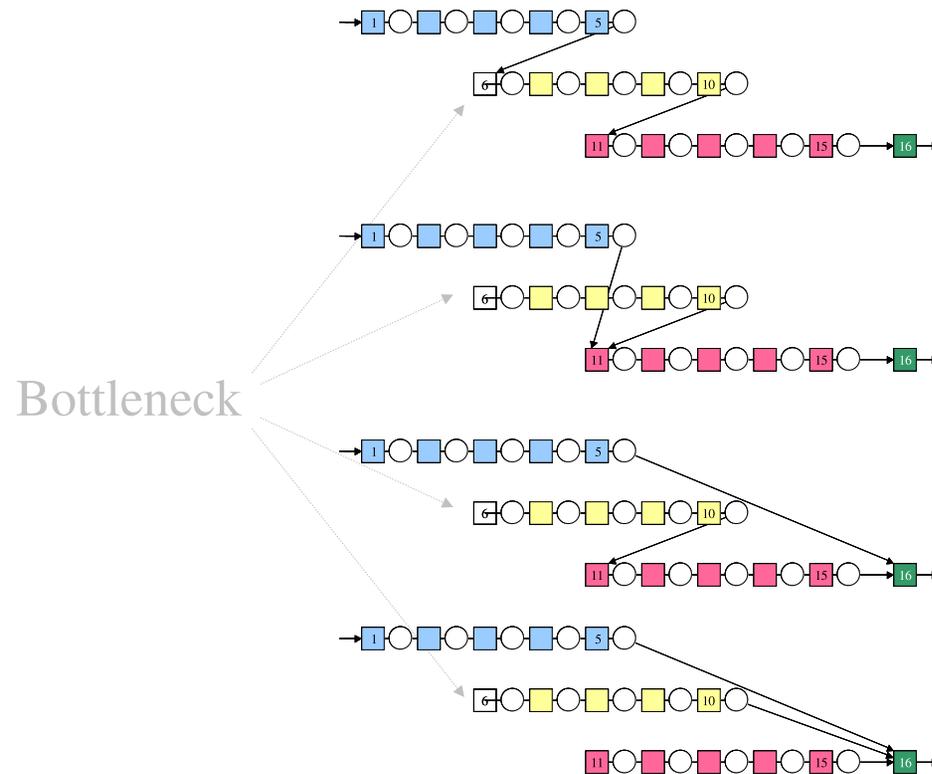
Effect of a bottleneck. Identical machines and buffers, except for M_{10} .

Long Lines

- Decomposition can be extended to assembly systems.
- *Question:* How should an assembly system be structured?
 - ★ Add parts to a growing assembly *or* form subassemblies and then assemble them?
 - ★ Production rates are roughly the same, but inventories can be affected.

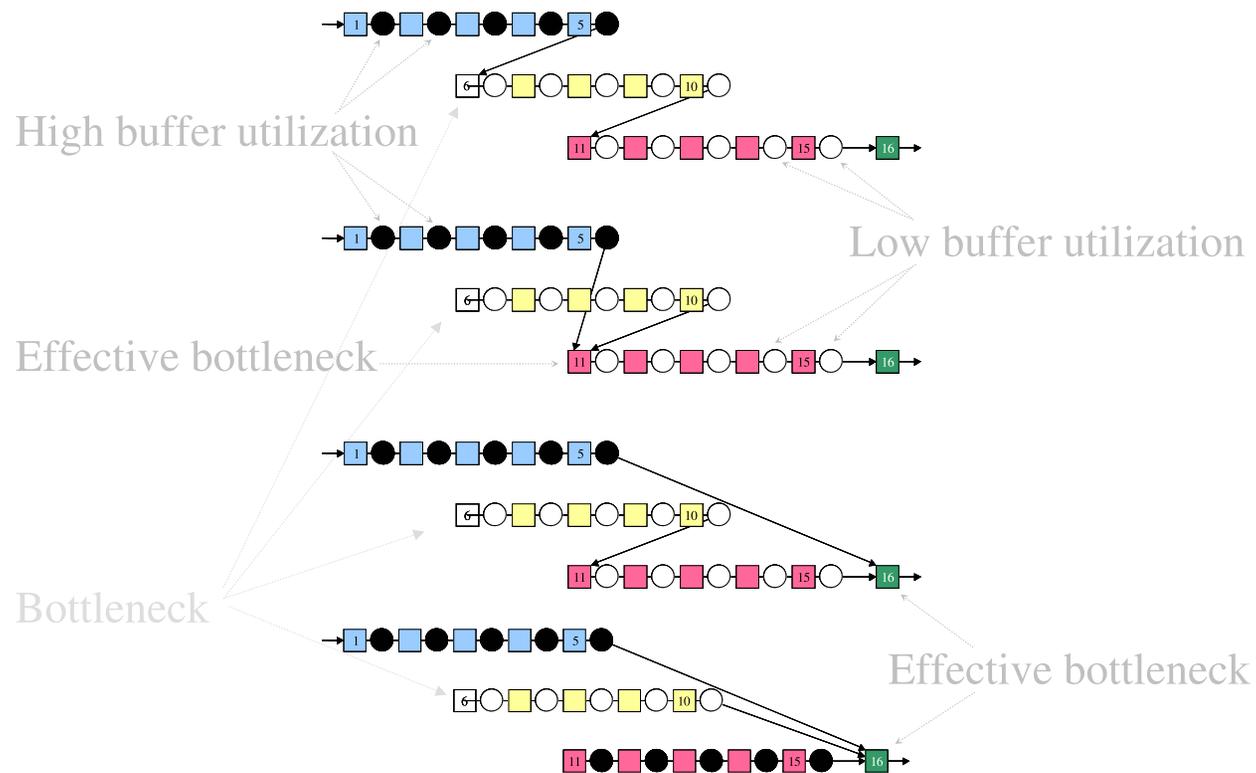
Long Lines

Assembly



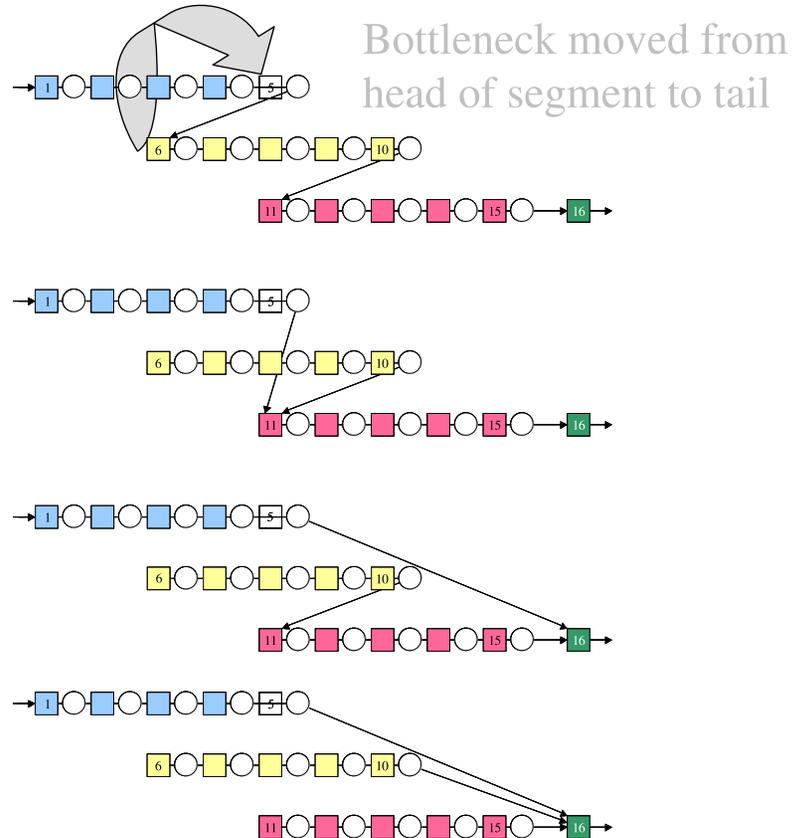
Long Lines

Assembly



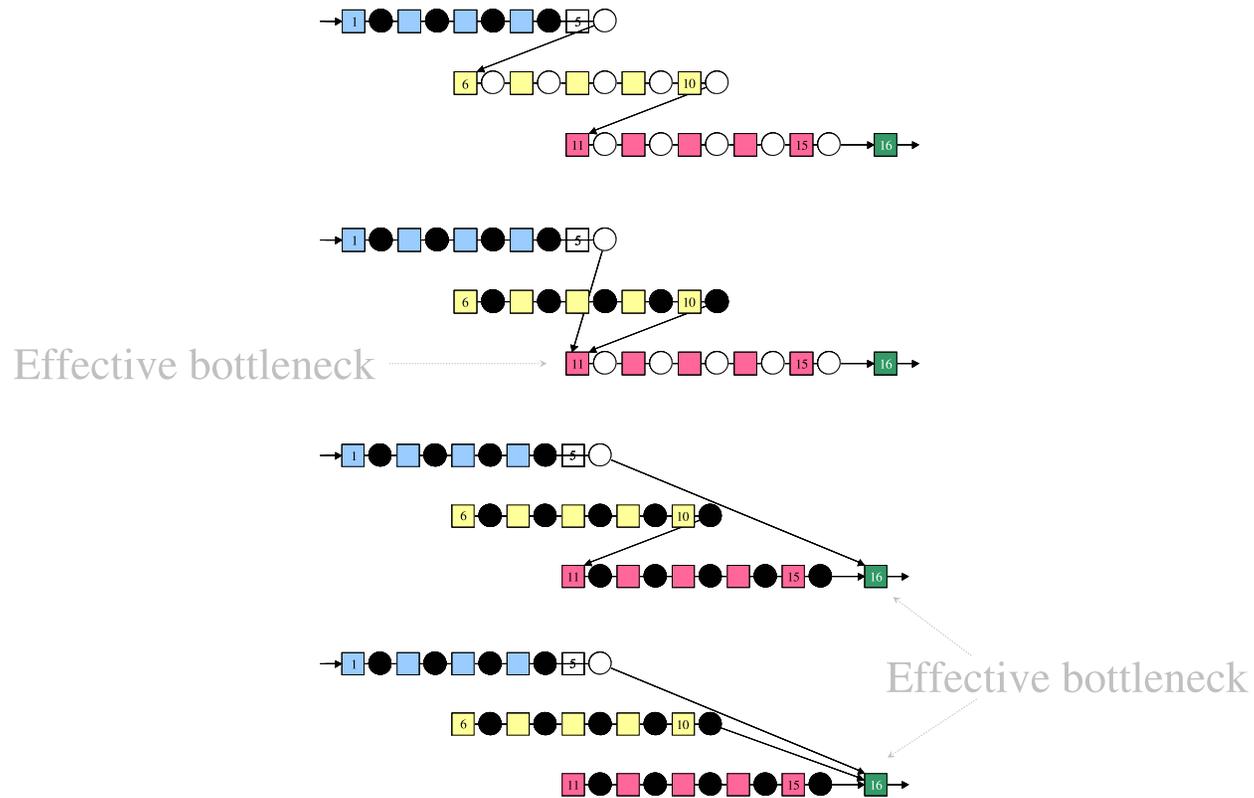
Long Lines

Assembly



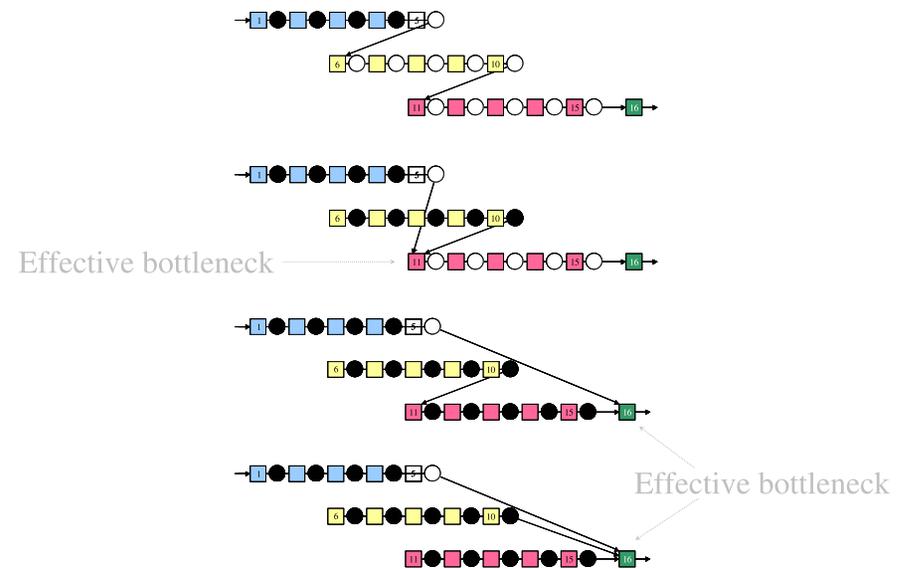
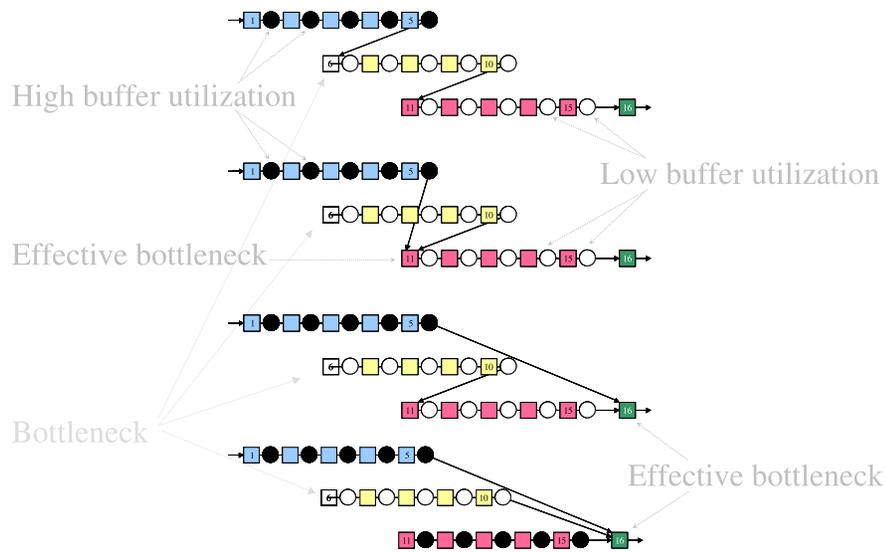
Long Lines

Assembly



Long Lines

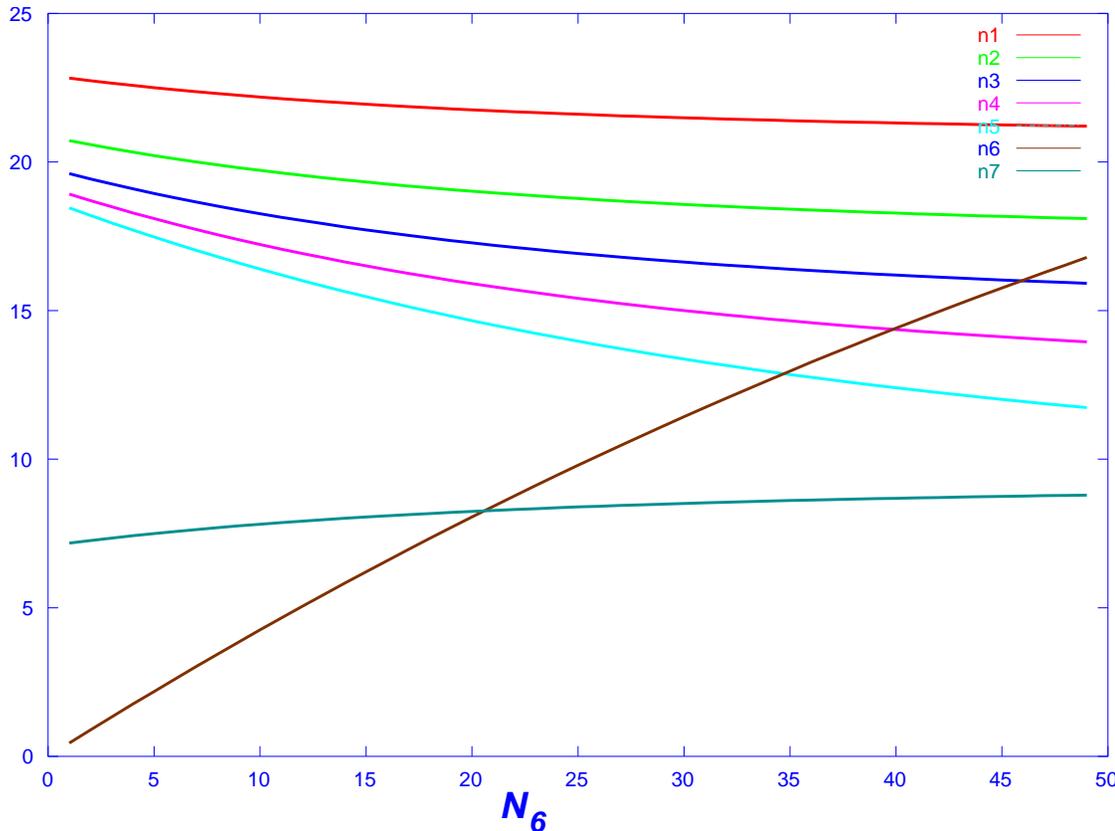
Assembly



Long Lines



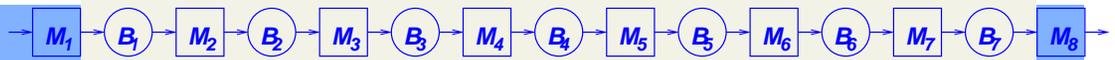
Examples



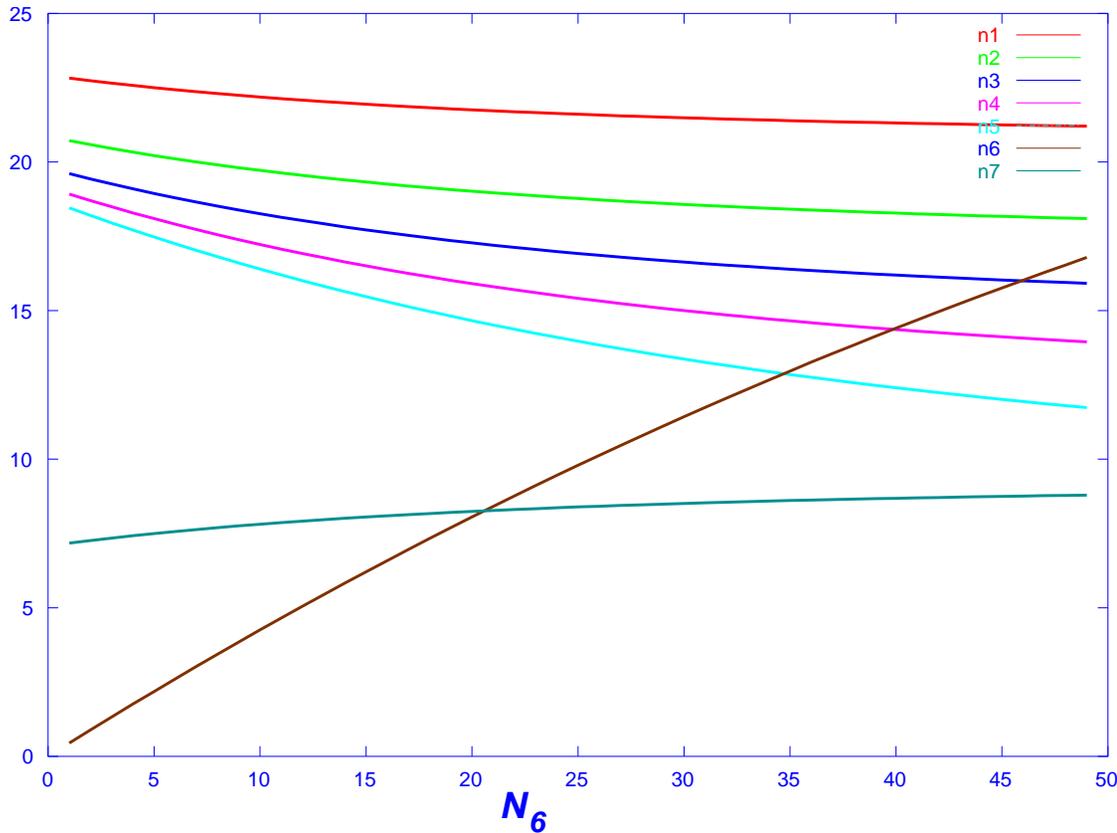
Continuous material model.

- Eight-machine, seven-buffer line.
- For each machine, $r = .075$, $p = .009$, $\mu = 1.2$.
- For each buffer (*except Buffer 6*), $N = 30$.

Long Lines



Examples



- Which \bar{n}_i are decreasing and which are increasing?
- Why?

Long Lines

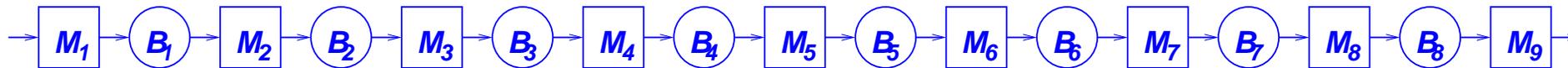


Examples

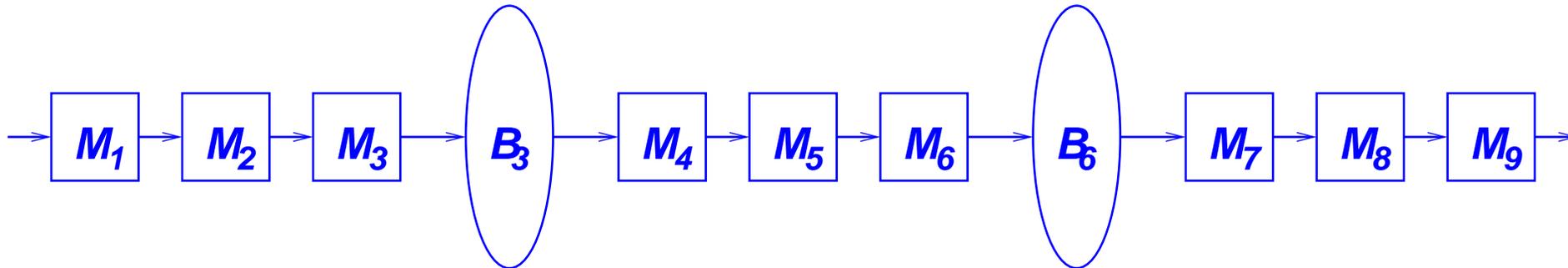
Which has a higher production rate?

- 9-Machine line with two buffering options:

★ 8 buffers equally sized; and



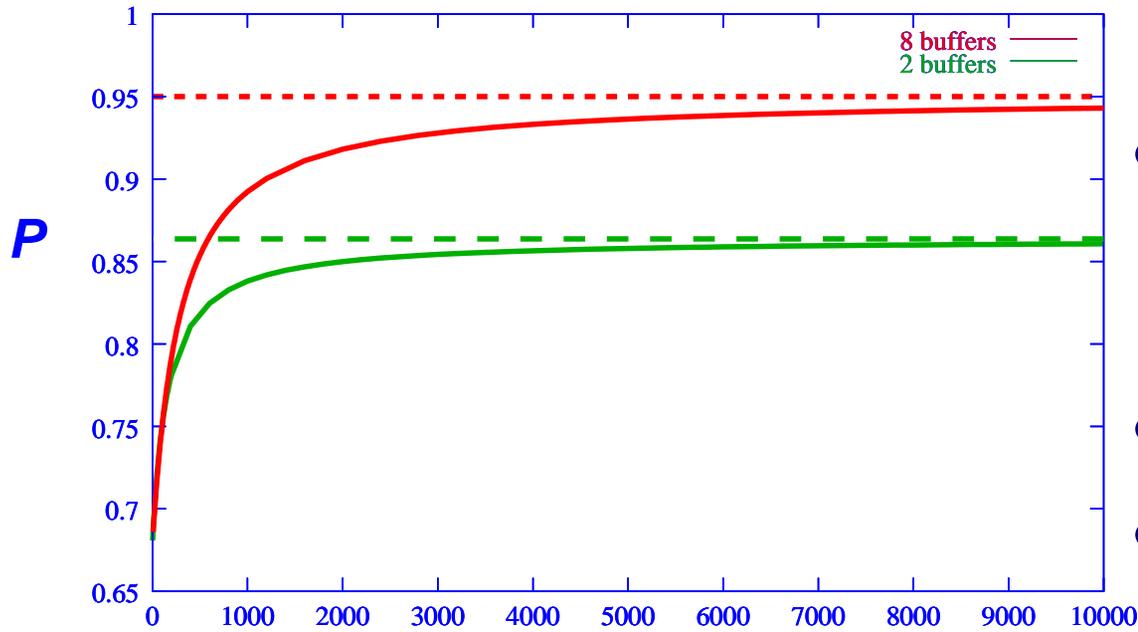
★ 2 buffers equally sized.



Long Lines



Examples



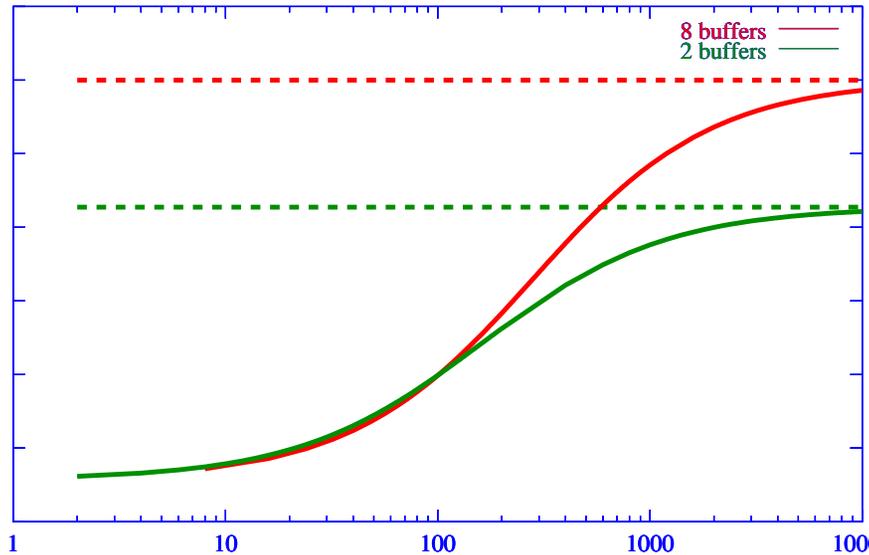
Total Buffer Space

- Continuous model; all machines have $r = .019$, $p = .001$, $\mu = 1$.
- What are the asymptotes?
- Is 8 buffers *always* faster?

Long Lines



Examples



- *Is 8 buffers always faster?*
- Perhaps not, but difference is not significant in systems with very small buffers.

Long Lines



Optimal buffer space distribution

- Design the buffers for a 20-machine production line.
- The machines have been selected, and the only decision remaining is the amount of space to allocate for in-process inventory.
- *The goal is to determine the smallest amount of in-process inventory space so that the line meets a production rate target.*



Long Lines

Optimal buffer space distribution

- The common operation time is one operation per minute.
- The target production rate is .88 parts per minute.



Long Lines

Optimal buffer space distribution

- *Case 1* MTTF= 200 minutes and MTTR = 10.5 minutes for all machines ($P = .95$ parts per minute).



Long Lines

Optimal buffer space distribution

- *Case 1* MTTF= 200 minutes and MTTR = 10.5 minutes for all machines ($P = .95$ parts per minute).
- *Case 2* Like Case 1 except Machine 5. For Machine 5, MTTF = 100 and MTTR = 10.5 minutes ($P = .905$ parts per minute).



Long Lines

Optimal buffer space distribution

- *Case 1* MTTF= 200 minutes and MTTR = 10.5 minutes for all machines ($P = .95$ parts per minute).
- *Case 2* Like Case 1 except Machine 5. For Machine 5, MTTF = 100 and MTTR = 10.5 minutes ($P = .905$ parts per minute).
- *Case 3* Like Case 1 except Machine 5. For Machine 5, MTTF = 200 and MTTR = 21 minutes ($P = .905$ parts per minute).

Long Lines



C

n

Are buffers really needed?

Line	Production rate with no buffers, parts per minute
Case 1	.487
Case 2	.475
Case 3	.475

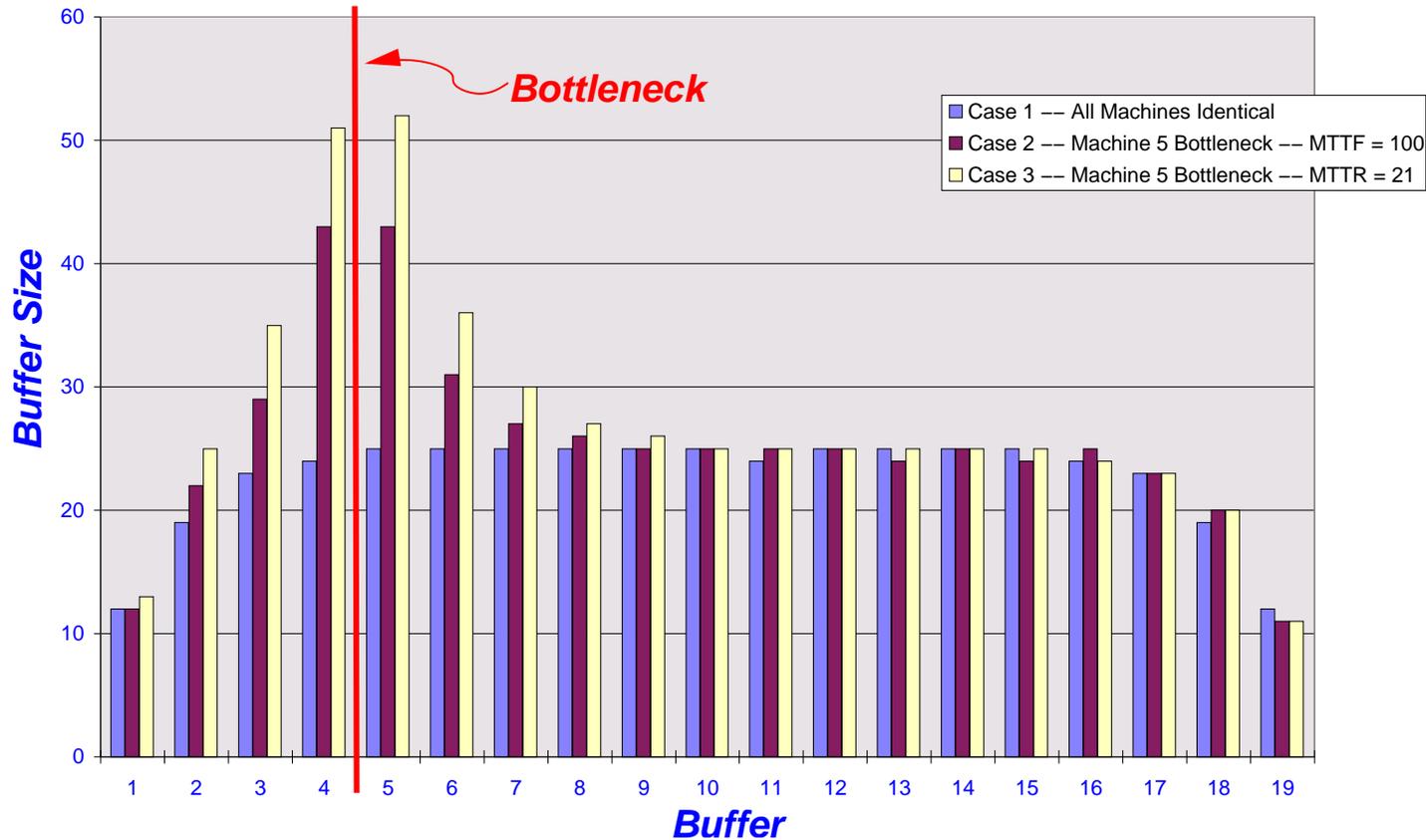
Yes. *How were these numbers calculated?*

Long Lines



Control

Solution



Line	Space
Case 1	430
Case 2	485
Case 3	523



Long Lines

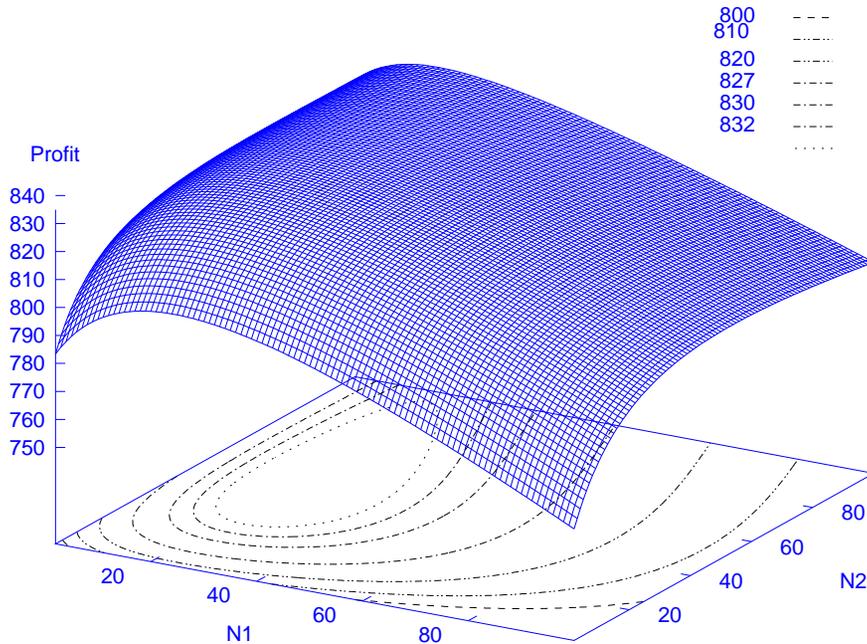
Optimal buffer space distribution

- Observation from studying buffer space allocation problems:
 - ★ *Buffer space is needed most where buffer level variability is greatest!*

Long Lines



Profit as a function of buffer sizes



- Three-machine, continuous material line.
- $r_i = .1, p_i = .01, \mu_i = 1.$
- $\Pi = 1000P(N_1, N_2) - (\bar{n}_1 + \bar{n}_2).$

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