Useful Distributions

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Suppose we have a population $X \sim N(\mu, \sigma^2)$, and x_1, x_2, $\cdots$, x_n is a n size sample of the population. The mean and variance of the sample are given as:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \text{ and } s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{1}$$

The following distributions constructed with $\bar{x}$ and s^2 (or s) are very useful in Statistics. They could be helpful for your homework.

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \text{ or } \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1) \tag{2}$$

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \sim t(n-1) \tag{3}$$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1) \tag{4}$$

Suppose there are two populations $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$, and a n_1 size sample of X, and a n_2 size sample of Y. Means of the two samples are $\bar{x}$ and $\bar{y}$, respectively. Variances of the two samples are s_1^2 and s_2^2, respectively. Then,

$$\frac{(\bar{x} - \mu_1) - (\bar{y} - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1) \tag{5}$$

$$\frac{(\bar{x} - \mu_1) - (\bar{y} - \mu_2)}{\sqrt{\frac{s^2}{n_1} + \frac{s^2}{n_2}}} \sim t(n_1 + n_2 - 2) \tag{6}$$

where

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The distribution (6) above can be used in the Hypothesis Test of means of two populations, in case that σ_1 and σ_2 are unknown, but can be assumed equal.

$$\frac{\left(\frac{s_1^2}{\sigma_1^2}\right)}{\left(\frac{s_2^2}{\sigma_2^2}\right)} \sim F(n_1 - 1, n_2 - 1) \tag{7}$$

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