

MAS.836 Sensor Systems for Interactive Environments

Distributed: Tuesday February 2nd, 2010 Due: Tuesday February 9th, 2010

Problem Set #1

Simple Circuit Analysis

The purpose of this problem set is to familiarize yourself with Kirchoff's voltage and current laws, and the operation of simple op-amp circuits. To accomplish this, you will need to use the following:

Standard Circuit Nomenclature: Discrete components, named node voltages and input/output signals are in upper case. For example V_{in} , the voltage into a circuit, R_3 is resistor number 3, V_1 is the voltage a node 1. Properties of named components, such as resistors, are in lower case. For example, a resistor R_3 would have a voltage of v_3 across it, and a current of i_3 through it. The standard convention is to always have positive current flow from the positive voltage terminal to the negative voltage terminal. The current through a circuit may be represented by an arrow above, or an arrow on the symbol. They are used interchangeably for clarity.

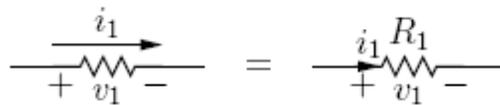


Figure 1: Equivalent circuit element representations.

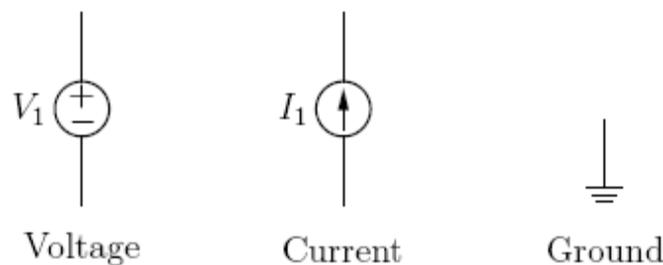
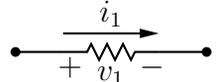


Figure 2: Voltage and Current circuit source symbols, which supply power to circuits. The +, -, and arrow denote voltage polarity and current direction. The ground symbols are defined at the point in the circuit where there is 0V.

Ohm's Law : The voltage drop across a resistor is proportional to the current flowing through it and is modeled by "Ohm's Law."

$$v_1 = i_1 \cdot R_1 \quad (1)$$


The diagram shows a resistor symbol on a horizontal wire. An arrow labeled i_1 points to the right above the wire, indicating current flow. Below the resistor symbol, a '+' sign is on the left and a '-' sign is on the right, with v_1 written between them, indicating the voltage drop across the resistor.

Figure 3: Resistor notation showing v_1 , the voltage across the resistor, and i_1 , the current through the resistor.

For example, if $5mA$ of current is flowing through a resistor of $10k\Omega$, then the voltage drop across the resistor will be $5mA \cdot 10k\Omega = 50Volts$. It is very important to remember that the voltage drops in the direction of the current flow, as the electrons move from a higher potential to a lower potential. To ensure that you do not get your voltages and currents backwards, you should always label them before performing any calculations.

Kirchoff's Current Law : To satisfy the conservation of charge (think flow of electrons), all currents flowing into a node must sum to zero, [1] page 3.

$$i_1 = \sum_{m=2}^n i_m \quad (2)$$

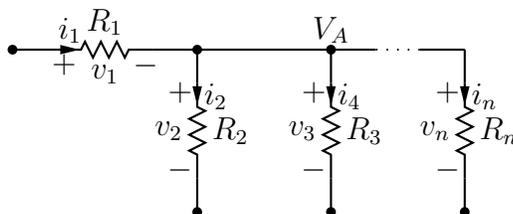


Figure 4: A series of n resistors connected together at a single node V_A .

This property is known as "Kirchoff's Current Law." A node is any place in a circuit where two or more elements touch. Since the number of electrons at any point must be conserved, the current flowing into the node must equal the current flowing out of that node, regardless of the number of circuit elements that meet at that node.

Kirchoff's Voltage Law : The sum of all voltages around a loop of circuit elements is zero. If the voltage is raised by a certain amount due to a circuit element, it must be lowered by that same amount somewhere else in the circuit to compensate. In many circuit diagrams, symbols are used to represent certain fixed voltages, rather than drawing a line to complete the loop. When a voltage is referred to in a circuit, it is the voltage drop from that point to ground, unless the voltage drop across a specific component is asked for. For example, in the circuit below, the voltage drop across R_1 is v_1 , but the voltage at the R_1 and R_2 juncture is v_2 . This law applies to loops regardless of other connected circuits.

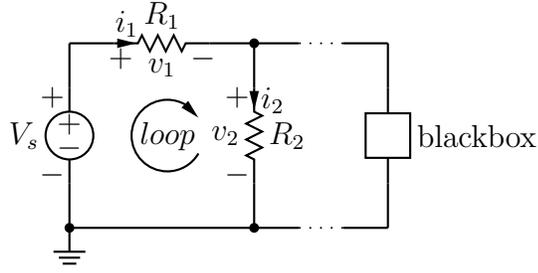


Figure 5: Given a continuous loop of circuit elements, the sum of the voltages must be zero. In this example $V_s = v_1 + v_2$.

See [2] section 4.4 or [3] page 35.

Ideal Op-amp Model : The ideal op-amp can be viewed as a device which

- draws no current into its input pins v_{in+} and v_{in-} .
- is able to source an infinite amount of current at its output pin (v_{out}), i.e. zero output resistance.
- has infinite gain A_v .

A slightly more accurate model on an op-amp includes a finite gain factor A_v . The op-amp amplifies the difference between its input pins and presents this voltage at the output $v_{out} = A_v \cdot (v_{in+} - v_{in-})$. This gain factor will vary with frequency in a non-ideal op-amp. As the frequency of the input signals increase, the gain of the op-amp decreases. This gain/frequency relationship is known as the gain-bandwidth product (GBW), since the gain times the frequency is fixed. For example, if you have a signal at $10kHz$ which is being amplified by an op-amp with a GBW of $2MHz$, the maximum voltage gain you would be able to achieve would be $2MHz/10kHz = 200$.

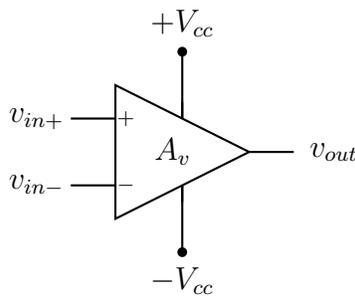


Figure 6: Diagram of an ideal Op-amp where $v_{out} = A_v \cdot (v_{in+} - v_{in-})$. The positive and negative V_{cc} 's supply the power to the Op-amp.

Problem 1

The following two resistor circuit is assembled in a series connection, as one resistor follows the other in a series. For this circuit, answer the following questions.

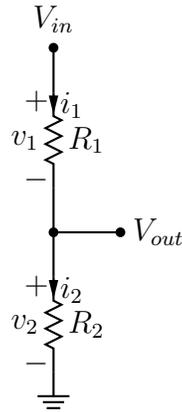


Figure 7: Circuit for problem 1. i_1 is the current flowing into R_1 and i_2 is the current flowing into R_2 . V_{out} is unconnected.

- What is the relationship between i_1 and i_2 ?
- What is the voltage drop across R_1 in terms of i_1 and R_1 ?
- What is the voltage drop across R_2 in terms of i_2 and R_2 ?
- What is the voltage at V_{out} in terms of v_2 ?
- What is the voltage at V_{in} in terms of v_2 and v_1 ?
- What is the relationship between V_{in} and V_{out} in terms of R_1 and R_2 ? (Note that this is the voltage divider equation)
- What is V_{in}/i_1 in terms of R_1 , and R_2 ? (Note that this is the formula for summing resistors in series)

Problem 2

The following two resistor circuit is assembled in a parallel connection, as one resistor runs parallel to the other. For this circuit, answer the following questions.

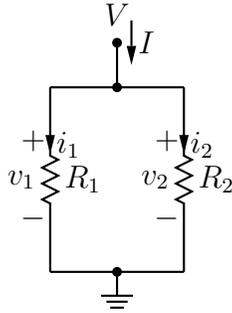


Figure 8: Circuit for problem 2.

- What is the voltage drop across R_1 in terms of i_1 and R_1 ?
- What is the voltage drop across R_2 in terms of i_2 and R_2 ?
- What is the relationship between v_1 , v_2 , and V ?
- What is the relationship between i_1 , V , and R_1 ?
- What is the relationship between i_2 , V , and R_2 ?
- What is the relationship between I , i_1 , and i_2 ?
- What is $\frac{V}{I}$ in terms of R_1 and R_2 ? (Note that this is the formula for summing two resistors in parallel)

Problem 3

The following op-amp circuit is an inverting amplifier, so called because it inverts and amplifies an input signal. Assume the ideal op-amp relationships given at the beginning of the problem set and answer the following questions.

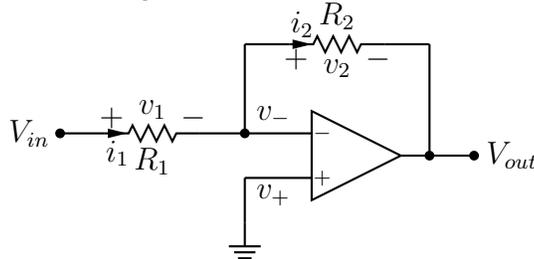


Figure 9: Circuit for problem 3.

- What is the relationship between i_1 and i_2 ?
- What is the voltage at (v_-) in terms of V_{in} , i_1 , and R_1 ?
- What is the voltage at (v_-) in terms of V_{out} , i_2 , and R_2 ?
- Using solutions to part a, b, and c, What is i_2 in terms of V_{out} , V_{in} , R_1 , and R_2 ?
- Using the ideal op-amp equation, what is the voltage at V_{out} in terms of the op-amp gain A_v , (v_-), and (v_+)?
- What is the voltage at (v_+)?
- Using c, e, and f, what is i_2 in terms of V_{out} , A_v , and R_2 ?
- Using d and g, what is V_{out}/V_{in} in terms of A_v , R_1 , and R_2 ?
- Assuming A_v is much greater than one, such that $(1 + 1/A_v) \approx 1$, what does your result from h simplify to? (Note that this is the gain of an inverting amplifier under ideal conditions)
- If you were using an op-amp with a gain-bandwidth-product of $2MHz$, what would be the highest frequency at which your answer to i would hold true? (Assume A_v is high enough to neglect its effect)
- Using the equations in a, b, and c, what is V_{out} in terms of (v_-), V_{in} , R_1 , and R_2 ?
- Using the results of i and k, what is the value of (v_-)? (Note that this gives us our third ideal op-amp law, that an op-amp with negative feedback (i.e. a connection between its output and the inverting input terminal) will always work to make $(v_+) = (v_-)$)

Problem 4

The following op-amp configuration is called a summing amplifier. To analyze this configuration, note that it has negative feedback, and therefore the third op-amp rule of $(v_+) = (v_-)$ will hold. Answer the following questions.

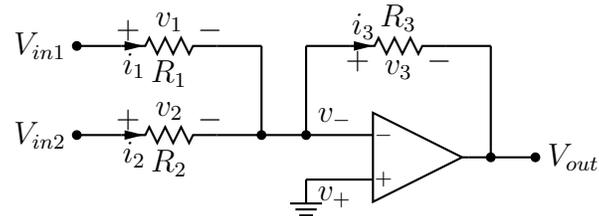


Figure 10: Circuit for problem 4.

- What is the voltage at (v_+) and (v_-) ?
- What is i_1 in terms of v_1 and R_1 ?
- What is i_2 in terms of v_2 and R_2 ?
- What is i_3 in terms of V_{out} and R_3 ?
- What is the relationship between i_1 , i_2 , and i_3 ?
- What is V_{out} in terms of v_1 , v_2 , R_1 , R_2 , and R_3 ? (Note that this performs the mathematical sum of the two inputs with scaling factors inversely proportional to the input resistor values)

Problem 5

The following op-amp circuit is a non-inverting amplifier configuration, so called because it amplifies the input signal without inverting it. Again, assume all three ideal op-amp rules, and answer the following questions.

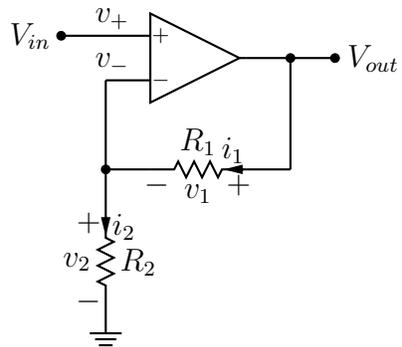


Figure 11: Circuit for problem 5

- What is the relationship between (v_-) , (v_+) and V_{in} ?
- What is the relationship between i_1 and i_2 ?
- Using your answer to Problem One, part f, what is V_{out} in terms of (v_-) , R_1 , and R_2 ?
- Using your result from a and c, what is the gain (V_{out}/V_{in}) of this circuit? (Note that this is the gain for an ideal non-inverting amplifier)
- What is the smallest gain value attainable with this configuration?

Problem 6

Design the following circuits using typical resistance values available. Choose resistors available at (digikey.com that are $1/4W$, 1%, and metal film).

- a. Design the *simplest* circuit that takes a $V_{in} = 9V$ and makes has a $V_{out} = 3.3V$.
- b. You have a power supply that is $9V$, and a sensor that requires $2.5V$ bias and nominally draws $100mA$ of current. Design a single resistor circuit to bias this sensor. Design an op-amp based circuit to bias this sensor. How will the bias voltage change if the sensor draws $100 \pm 20mA$?
- c. Design the *simplest* circuit that makes an inverting amplifier with a gain of 10.
- d. Design the *simplest* circuit that adds one voltage input with a gain of 5, and another input with a gain of 8, *without* inverting the output. Optimize your design for maximum band-width.
- e. Design the *simplest* circuit that makes a non-inverting amplifier with a gain of 12. Calculate the band-width given a GBW of $200MHz$.

References

- [1] P. Horowitz and W. Hill, *The Art of Electronics*. New York: Cambridge University Press, 1989.
- [2] *The ARRL handbook for radio communications*. Newington, CT: ARRL - the national association for Amateur Radio, 2006.
- [3] R. E. Thomas and A. J. Rosa, *The analysis and design of linear circuits*. New Jersey: Prentice Hall, 1994.

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