22.01 Fall 2015, Problem Set 2

Due: September 28, 11:59PM on Stellar

August 26, 2015

Complete all the assigned problems, and do make sure to show your intermediate work. Please upload your full problem set in PDF form on the Stellar site. Make sure to upload your work at least 15 minutes early, to account for computer/network issues.

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1 Predicting Nuclear Stability

Using the Table of Nuclides, answer the following questions about sulfur (Z=16):

1. Using the excess mass from the table of nuclides, calculate the binding energy and the binding energy per nucleon of ^{32}S. Use the table of nuclides to check your answer.
2. Graph the excess mass of each isotope of sulfur as a function of mass number (A), and as a function of binding energy per nucleon. What trends do you see?
3. How do you explain the deviation in the smooth-ish curve for ^{33}S?
4. For each region where an increase is seen from the most stable isotope(s) (left third, middle third, right third), briefly say why the nuclei in each region are most unstable. (Hint: What do you know about the relative number of protons and neutrons in a nucleus, and how does that help determine stability?)

2 Liquid-Drop Nuclear Models

For these questions, consider the liquid drop model of nuclear mass, which states that the mass of a nucleus can be empirically calculated as in Eq. 4.10 (p. 59) of *Nuclear Radiation Interactions* by S. Yip.

1. Explain the origin of each additive term in this expression. Pay particular attention to the exponents in each one, and explain why they are what they are.
2. Why does the δ term in this expression change sign for odd/even nuclei?
3. Modify equation 4.10 to empirically calculate the total rest mass of a given nucleus.
4. Graph the empirically calculated mass per nucleon for each isotope of sulfur, and compare it directly to your answer to problem 1.2. What differences do you observe?
5. Derive an expression for the most stable number of neutrons for a given nucleus with Z protons. Graph this expression as a function of Z. How does your prediction compare with the isotopes of sulfur?

3 Q-Values and Nuclear Power

For these questions, consider equations 4.2 - 4.6 (pp. 54-55) in *Nuclear Radiation Interactions*.

1. Show that the Q-value for a reaction can be expressed solely in terms of nuclear binding energies (derive equation 4.6).

2. Using a data extraction program like DataThief to get data points from Figure 4.4 (p. 58), produce an empirical expression for the optimum number of neutrons (N) for a given number of protons (Z) in a nucleus. Comment on the quality of that fit, and explain in which regions the fit is the best, and in which the fit is the worst.

3. Using a data extraction program to get data from Figure 4.5 (p. 61) or a similar curve, produce a graph of the Q value for the fusion reaction of a given nucleus with a ^{4}He nucleus as a function of the nucleus' mass number (A):

$$M\,(A,\,Z) +^4 He \rightarrow M\,(A+4,\,Z+2) + Q \tag{1}$$

   What noticeable features do you see? At which mass number (A) does the curve cross the x-axis? What does this say about the energetics of light vs. heavy nuclei for fusion (in other words, when is it exothermic, and when is it endothermic)?

Note: For the last two sub-problems, the Web-based tool WebPlotDigitizer lets you easily digitize scatter plots by color. Using the X-Step algorithm, and by defining axes much like in DataThief, it automatically extracted values of binding energy per nucleon for integer values of A, for example, in the last problem.