22.01 Fall 2015, Problem Set 4 (Analytical Version)

Due: October 21, 11:59PM on Stellar

October 14, 2015

Complete all the assigned problems, and do make sure to show your intermediate work. Please upload your full problem set in PDF form on the Stellar site. Make sure to upload your work at least 15 minutes early, to account for computer/network issues.

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1 Other Worldly Decay

Suppose a parallel universe exists, where for an ensemble of radioactive particles in a box, the probability of one particle decaying in a fixed amount of time Δt is proportional by a constant η to the number of unstable particles remaining in the box. Derive a new expression for the half life of a given other-worldly isotope, as a function of the number of particles in a box (N_0) at time t_0. Graph this expression compared to the normal half-life equation, and comment on the reasons for the differences in the shapes of the curves.

2 Activity and Half Lives

Back to earth for now... suppose a given smoke detector is rated at an activity of 0.8μCi of ^{241}Am upon manufacture. It is powered by a betavoltaic power module, which encases a tritium source $\left(^3H\right)$ rated at an activity of 2Ci upon its manufacture.

1. Which component of the smoke detector will fail to provide exactly 1nA of current first, and after how long? Assume that the equation for allowable current through an ionization chamber is as follows:

$$I_{max} = e_c \left(\frac{E_m}{E_i}\right) \left[\frac{3.7 \cdot 10^{10}\, Bq}{Ci}\right] S\left(t\right) \tag{1}$$

   where I_{max} is the saturation current through the ionization chamber, e_c is the charge of an electron in Coulombs, E_m is the energy of the ionizing particle (you will have to look this up), E_i is the ion formation pair energy in air ($14eV$), and $S\left(t\right)$ is the activity of the ^{241}Am source.

2. Suppose this first component were to be inexhaustible. How long would it take for the second component to drop to a supplied/allowable current of exactly 1nA? For this question, make sure to take the uncertainty due to significant figures into account, and give a maximum, minimum, and expected value.

3 Radioactive Food

Two of the most radioactive foods we eat are bananas and brazil nuts.

1. Look up the specific activities of each of these foods, and write the decay equations for the isotope(s) responsible for each one.

2. Despite being more radioactive, why do Brazil nuts give the body less of a radiation dose compared to bananas?

3. How long would you have to keep these two foods before eating to ensure that their radioactivity has dropped to 10% of its original level? When would you consider the "original level" to be established (in other words, when is t_0)?

4 Successive Decay Chains and Radioactive Dating

1. For the smoke detector problem above, repeat your analysis in parts 1 and 2, but this time take successive decays of ^{241}Am into account. You may (and should) make an assumption that half-lives below a certain time constant (which you will have to choose) can be considered instantaneous, and write your total ionization chamber saturation current as a sum of all potentially present isotopes.

2. One major source of radioactivity intake for some people is smoking. This is because soils with elevated radon levels create daughter products, which stick to tiny hairs on the tobacco leaves.

   (a) What is the source of these radioactive daughter products? Write as complete of a decay chain as you can for the creation of radioactive substances found in tobacco.

   (b) Many of these radioactive products are alpha emitters, which are particularly damaging to tissue. Estimate the additional radiation activity to someone's lungs if they smoke one pack of cigarettes a day.

   (c) How long would you have to keep these cigarettes in storage for this radioactivity to reach 1% of its original level, from the day the tobacco was picked? You will need to consider simultaneous production and decay of radioactive isotopes in your answer.

5 Statistics and Certainty

For these problems, consider that you have just measured the activity of a source stamped at $1\mu Ci$ to be $0.70\mu Ci$, by measuring a count rate of 15 counts per second.

1. Taking uncertainty into account, how long would you have to count to ensure 95% confidence in your measurement? What would the standard deviation of your measurement be in this case?

2. Taking uncertainty into account, when could this source have been calibrated at $1\mu Ci$?

6 Radioactive Dating with Confidence

For this problem, consider the methods used to radioactively date the Shroud of Turin, thought to be the burial cloth of Jesus of Nazareth.

1. Why did the investigators use carbon as the dating isotope? Consider what other isotopes could have been present, and give at least three reasons that carbon was chosen.

2. How did the investigators use statistics to prove beyond a reasonable doubt that the Shroud of Turin was *not* the burial cloth? What is a χ^2 test, and how did they arrive at the χ^2 values in the paper?

3. Why did the investigators send so many unknown control samples to so many laboratories, and why did they use different cleaning procedures?

References

[1] C. Litton, "Optimizing Ionization-Type Smoke Detectors," *Fire Technol.*, 15(1) (1979).

22.01 Introduction to Nuclear Engineering and Ionizing Radiation
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