22.01 Fall 2015, Problem Set 8 Solutions (Normal Version)

Due: November 25, 11:59PM on Stellar

December 13, 2015

Complete all the assigned problems, and do make sure to show your intermediate work. Please upload your full problem set in PDF form on the Stellar site. Make sure to upload your work at least 15 minutes early, to account for computer/network issues.

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1 Radiation in the Environment

1. List the five largest *natural* sources of background radiation from living in Cambridge, MA, and what percent of your yearly background dose they comprise.
   The first largest source is always radon inhalation, and Cambridge, MA is in the highest of the three EPA zones at >4pCi/L of air[1]. It is estimated at about 2mSv per year, or a little over 50% of your yearly dose.
   The next largest source is almost always internal radiation, from the K-40, C-14, and other isotopes in the body. It comprises about 0.4mSv per year, or around 11% of your yearly dose.
   The third largest source is likely cosmic radiation, consisting mostly of the high energy gamma rays produced by neutral pion decay resulting from protons striking the upper atmosphere, and showers of particles from muon production in the atmosphere. It constitutes about 0.27mSv per year, or around 8% of your yearly dose.
   The fourth largest source is radiation from the building materials around us, like wood (which contains much K-40), and brick & granite, which can contain plenty of uranium & radium, especially if sourced locally (Conway granite from New Hampshire is particularly radioactive). It constitutes about the same dose as cosmic radiation.
   The fifth source doesn't occur for everyone. If you're a smoker (technically a natural source, as it's just the dust on the surfaces of tobacco leaves), your background dose per year can be as high as 100x[2] the normal background dose in Boston. If the person is a smoker, then the above sources get shifted down by one level in importance.

2. Estimate the increase in background dose per hour from flying from Boston to Japan, over the North Pole.
   A direct flight from Boston to Japan takes 14 hours when traveling East to West. The Federal Aviation Administration (FAA) actually has this nifty radiation calculator[3] for exactly this purpose! Assuming the flight is from BOS (Logan International) to NRT (Narita International), and it takes 30 minutes each to reach and descend from a cruising altitude of 38,000 feet, a dose of 0.09239mSv will be incurred, or about 1/3rd of the normal *yearly* cosmic radiation dose!

[1] https://www.epa.gov/radon/find-information-about-local-radon-zones-and-radon-programs#stateradon
[2] http://www.rmeswi.com/36.html
[3] http://jag.cami.jccbi.gov/cariprofile.asp

2 Analytical Environmental Questions

Given a EPA-reported radon activity concentration of 0.4 $\frac{\mathrm{pCi}}{\mathrm{L}}$ in normal air, estimate your increase in background dose if a rain cloud 1km above the ground washes all the radon down to the ground. You may approach this problem in one of two ways: External exposure, or Internal exposure.

For the external exposure approach, consider the shielding of the air, compared to the water (Hint: solve this as an integral problem). You may assume that photon energy loss mechanisms do not apply; that is, attenuated photons are absorbed. You may find it helpful to break up the problem into the following steps:

1. Estimate the radon dose to you, from a volume element of air with activity A at a distance D from you. Account for the number of photons attenuated by air between you and the volume element of air. **Let us start by considering a volume element dV of air at any distance D from the human. Let us also assume that the human is approximately spherical (I think this was done for cows in 8.01, I seem to remember the phrase "assume a spherical cow"), with a radius r. The total specific activity A of the air without the storm is:**

$$A = \left(0.4\frac{pCi}{L}\right)\left(10^{-12}\frac{Ci}{pCi}\right)\left(3.7\cdot 10^{10}\frac{Bq}{Ci}\right)\left(10^{3}\frac{L}{m^3}\right)\left(dV\,m^3\right) = 14.8dV\,Bq \tag{1}$$

Now take into account the solid angle subtended by the human (see p. 9 of http://www.geo.mtu.edu/˜scarn/teaching/GE4250/radiation_lecture_slides.pdf):

$$\theta_{human} = 2\,sin^{-1}\left(\frac{r}{D}\right) \tag{2}$$

$$\Omega_{human} = 2\pi\left(1 - cos\left(\theta_{human}\right)\right) \tag{3}$$

We then use the solid angle to get the fraction of emitted gamma rays from each dV that go in the direction of the spherical human:

$$A_{towards\,human\,per\,dV} = (14.8dV)\left(\frac{2\pi\left(1 - cos\left(2\,sin^{-1}\left(\frac{r}{D}\right)\right)\right)}{4\pi}\right) \tag{4}$$

Here we have divided the solid angle by 4π to account for the fraction of gamma rays isotropically emitted that actually head towards the human. Finally, we account for the fraction of gammas which are attenuated before they reach the human:

$$A_{towards\,human\,per\,dV} = \underbrace{(14.8dV)}_{Activity\,per\,dV}\underbrace{\left(\frac{\cancel{2\pi}\left(1 - cos\left(2\,sin^{-1}\left(\frac{r}{D}\right)\right)\right)}{\underset{2}{\cancel{4\pi}}}\right)}_{Solid\,angle\,fraction}\underbrace{e^{-\left(\frac{\mu}{\rho}\right)\rho D}}_{Attenuation} \tag{5}$$

Using the NIST tables for dry air, we get a $\left(\frac{\mu}{\rho}\right)$ value of about 0.01 $\frac{m^2}{kg}$ at 510 keV, the energy of the gamma ray emitted by direct Rn-222 decay only 0.08% of the time. Using an air density of 1.225 $\frac{kg}{m^3}$ for air at sea level, this yields the following equation for the total gamma activity per dV of air to a human of radius r at distance D:

$$\Phi_{received\,by\,human\,per\,dV} = (0.0008)\,(14.8dV)\left(\frac{\left(1 - cos\left(2\,sin^{-1}\left(\frac{r}{D}\right)\right)\right)}{2}\right)e^{-0.01225D} \tag{6}$$

Now we must account for how many of these gammas, which we will assume all interact by the photoelectric effect (depositing their full energy), actually interact inside the human of radius r. For soft tissue, we find using the NIST tables that $\left(\frac{\mu}{\rho}\right)$ is also about 0.01 $\frac{m^2}{kg}$

at 510 keV, and the density of a human is 985 $\frac{kg}{m^3}$ on average:

$$\Phi_{interacting\,in\,human\,per\,dV} = \underbrace{\left[(0.0008)\,(7.4dV)\left(1 - cos\left(2\,sin^{-1}\frac{r}{D}\right)\right)e^{-0.01225D}\right]}_{Gamma\,flux\,reaching\,human\,per\,dV}\quad\underbrace{\left(1 - e^{-9.85r}\right)}_{Fraction\,interacting\,in\,human} \tag{7}$$

Finally, we multiply by the energy of each gamma ray, divide by the mass of a human, and we note that this constitutes a full body dose (tissue factor of 1) by gamma rays (quality factor of 1), so the dose is Gy is equal to the effective dose in Sv:

$$\left.\frac{dSv}{dt}\right._{per\,dV} = \frac{\left(5.1 \cdot 10^5\,eV\right)\left(1.6 \cdot 10^{-19}\,\frac{J}{eV}\right)}{75\,kg}\underbrace{\left[(0.0008)\,(7.4dV)\left(1 - cos\left(2\,sin^{-1}\frac{r}{D}\right)\right)e^{-0.01225D}\right]}_{Gamma\,flux\,reaching\,human\,per\,dV}\quad\underbrace{\left(1 - e^{-9.85r}\right)}_{Fraction\,interacting\,in\,human} \tag{8}$$

$$\left.\frac{dSv}{dt}\right._{per\,dV} = dV\left(6.44 \cdot 10^{-18}\right)\left[\left(1 - cos\left(2\,sin^{-1}\frac{r}{D}\right)\right)e^{-0.01225D}\right]\left(1 - e^{-9.85r}\right) \tag{9}$$

2. Integrate this dose in a hemisphere surrounding you to get the total dose. You may want to define a cutoff radius, beyond which effectively no radiation reaches you.
Here, we integrate over all possible distances and angles, noting that we only integrate over the hemisphere of air above ground:

$$Dose\,rate = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{r_{cutoff}} \left.\frac{dSv}{dt}\right._{per\,dV} D^2 sin\phi dD d\phi d\theta \tag{10}$$

$$Dose\,rate = \int_0^{2\pi}\int_0^{r_{cutoff}} \left.\frac{dSv}{dt}\right._{per\,dV} D^2 dD d\theta \tag{11}$$

$$Dose\,rate = 2\pi\int_0^{r_{cutoff}} \left.\frac{dSv}{dt}\right._{per\,dV} D^2 dD \tag{12}$$

Now we define a cutoff radius, which attenuates let's say down to one thousandth of the incoming gamma rays:

$$\frac{I}{I_0} = \frac{1}{1000} = e^{-\left(\frac{\mu}{\rho}\right)_{ar}\rho_{air}r_{cutoff}} = e^{-0.01225r_{cutoff}}; \quad r_{cutoff} = 564\,m \tag{13}$$

We therefore have:

$$Dose\,rate = 2\pi\int_0^{564\,m} \left.\frac{dSv}{dt}\right._{per\,dV} D^2 dD = 5.9 \cdot 10^{-16}\,\frac{Sv}{sec} = 0.0177\frac{\mu Sv}{yr} \tag{14}$$

3. Calculate the *aerial activity density* of radon in the lower 1 km of atmosphere. This is expressed in $\frac{Ci}{m^2}$.
To get the aerial activity density of the lowest 1 km of air in the atmosphere, we take the specific activity of air at sea level $\left(0.004\,\frac{pCi}{m^3}\right)$ and multiply by the density function of air as a function of height:

$$\rho_{air}(h) = \frac{PM}{RT}; \quad T(h) = T_0 - Lh; \quad P(h) = P_0\left(1 - \frac{Lh}{T_0}\right)^{\frac{gM}{RL}} \tag{15}$$

where ρ is the density in $\frac{kg}{m^3}$, h is the height in meters, P is the pressure in Pa, M is the molar mass of dry air ($0.0289644\frac{kg}{mol}$), R is the ideal gas constant $\left(8.31447\frac{J}{mol-K}\right)$, T is

the temperature in Kelvin, L is the temperature lapse rate $\left(0.0065\frac{K}{m}\right)$**,** P_0 **is the absolute pressure at sea level** $(101,325\,Pa)$**,** T_0 **is the temperature at sea level** $(288.15\,K,\ or\ 15C)$**, and** g **is the gravitational constant** $\left(9.81\,\frac{m^2}{s}\right)$**. When graphed, this function is almost exactly a straight line between the values of** $\rho(0) = 1.225\,\frac{kg}{m^3}$ **and** $\rho(1000) = 1.112\,\frac{kg}{m^3}$**, so we will approximate the integral from sea level to an altitude of 1 km as:**

$$\int_0^{1,000} \rho(h)\,dh \approx 1,000\left(\frac{1.225 + 1.112}{2}\right) = 1,168.5\,\frac{kg}{m^2} \tag{16}$$

We also assume that the relative fraction of radon in the air remains constant as a function of height, to make the calculation easier. Converting this aerial air density to an activity density, we get:

$$\left(0.004\,\frac{pCi}{m^3}\right)\left(\frac{1\,m^3}{1.225\,kg\,air}\right) = 0.0033\,\frac{pCi}{kg} \tag{17}$$

Multiplying equations 16 and 17, we get the aerial activity density of air as 3.856 $\frac{pCi}{m^2}$**.**

4. Assume that the radon all washes down to your body level, and approximate yourself as a point source. Repeat (a-b), but this time integrating a disc of air surrounding you, and accounting for a rain volume fraction of 20% during the storm.

   Let's stick with our analogy of a human, but this time approximate it as a cylinder with radius $r = 0.75\,m$**. Three main things differ in this calculation: (1) The activity density of the air surrounding the person, and (2) the partial shielding of the gamma rays by the 20% water in the air during a storm, (3) we only have to integrate over a disc instead of a hemisphere. The first one is easy to deal with: We start with our original dose rate equation:**

$$\frac{dSv}{dt}_{per\,dV} = dV\left(6.44\cdot 10^{-18}\right)\left[\left(1 - cos\left(2\,sin^{-1}\frac{r}{D}\right)\right)e^{-0.01225D}\right]\left(1 - e^{-9.85r}\right) \tag{18}$$

Next we modify it by multiplying by the ratio between our two activity densities, with and without the rain storm:

$$\frac{dSv}{dt}_{per\,dV} = dV\cancelto{6.208\cdot 10^{-15}}{\left(6.44\cdot 10^{-18}\right)}\left[\left(1 - cos\left(2\,sin^{-1}\frac{r}{D}\right)\right)e^{-0.01225D}\right]\left(1 - e^{-9.85r}\right) \tag{19}$$

Next, we compute the effective mass attenuation coefficient and total air/water density by their volume fractions:

$$\left(\frac{\mu}{\rho}\right)_{air/water} = 0.8\left(\frac{\mu}{\rho}\right)_{water} + 0.2\left(\frac{\mu}{\rho}\right)_{water} = 0.01\,\frac{m^2}{kg} \tag{20}$$

This one was easy, since the mass attenuation coefficients for air and water are almost exactly the same. Now for the density:

$$\rho_{air/water} = 0.8\rho_{air} + 0.2\rho_{water} = 201\,\frac{kg}{m^3} \tag{21}$$

Modifying our dose rate equation further:

$$\frac{dSv}{dt}_{per\,dV} = dV\cancelto{6.208\cdot 10^{-15}}{\left(6.44\cdot 10^{-18}\right)}\left[\left(1 - cos\left(2\,sin^{-1}\frac{r}{D}\right)\right)e^{\cancelto{2.01}{-0.01225D}}\right]\left(1 - e^{-9.85r}\right) \tag{22}$$

Finally, our dV and our integral change because we're in cylindrical coordinates now:

$$Dose\,rate = \int_0^{2\pi}\int_0^{r_{cutoff}} \frac{dSv}{dt}_{per\,dV} D\,dD\,d\theta \tag{23}$$

We recalculate the cutoff radius, accounting for the denser air/water mixture:

$$\frac{I}{I_0} = \frac{1}{1000} = e^{-\left(\frac{\mu}{\rho}\right)_{ar}\rho_{air}r_{cutoff}} = e^{-2.01 r_{cutoff}}; \quad r_{cutoff} = 3.4\,m \tag{24}$$

Note how much shielding the air provides!!! Taking the same numerical integral as before for Equation 23, we arrive at:

$$Dose\,rate = 2\pi \int_0^{3.4\,m} \frac{dSv}{dt}_{per\,dV} DdD = 6.89 \cdot 10^{-16} \frac{Sv}{sec} = 0.0207 \frac{\mu Sv}{yr} \tag{25}$$

It turns out that the greatly higher radon dose is greatly shielded by the water in the air. Thank god!

__For the internal exposure approach__, estimate the fraction of radon taken in with each breath (continuing a calculation that we started in class), both normally without rain and when the rain washes all the radon down to ground level. Make any assumptions about how the radon concentration increases during rain as you need, and try to calculate the amount of radon in each breath that will decay into daughter products while in the lungs. Keep in mind the tissue weighting factor for lungs.

For the internal exposure approach, we start by symbolically defining everything we need:

$$Dose\,rate = \frac{A_{Rn\,in\,air} V_{breath} W_{lungs} Q_\alpha E_\alpha}{m_{lungs}} \tag{26}$$

All the quantities except the last one are easy to look up:

- $\mathrm{A_{Rn\,in\,air}} = 400\,\frac{\mathrm{pCi}}{\mathrm{m^3}} = 14.8\,\frac{Bq}{m^3}$
- $\mathrm{V_{breath}} \approx 0.0005\,\mathrm{m^3}$
- $\mathrm{W_{lungs}} = 0.12$ **(lecture notes, tissue weighting factor)**
- $\mathrm{m_{endothelial\,cells\,inlungs}} \approx \frac{1.3\,\mathrm{kg}}{1000} = 0.0013\,kg$
- $\mathrm{Q_\alpha} = 20$ **(lecture notes for alpha particles)**

$$\begin{aligned} \mathrm{E_\alpha} &= 5.590\,\mathrm{MeV}\,\left(^{222}\mathrm{Rn}\right) + 6.115\,\mathrm{MeV}\,\left(^{218}\mathrm{Po}\right) \\ &+ \sim 0.5\,MeV\,\beta^-\left(^{214}Pb\right) * \frac{1}{20}\,(Q=1) + \sim 1.6\,MeV\,\beta^-\left(^{214}Bi\right) * \frac{1}{20}\,(Q=1) \\ &+ 7.833\,MeV\,\left(^{2142}Po\right) = 19.643\,\frac{MeV\ equiv}{decay} = 3.14 \cdot 10^{-12} \frac{J\ equiv.}{decay} \end{aligned} \tag{27}$$

This takes into account all the daughter products which decay much, much more quickly than the radon itself, ending with Pb-210 with a half life of 22 years. Let's stop the chain there. Finally we assume that the lungs are, on average, half full of fresh air with the normal radon concentration. Multiplying through, we get $4.3 \cdot 10^{-11}\,\frac{Sv}{sec} = 1.29\,mSv/yr$**, which is a very reasonable estimate given that the average background dose is 3-4 mSv/yr, and radon typically constitutes 50% of that. If we assume that the 967x radon from the upper atmosphere washes down to ground level, then the background dose to the endothelial cells in the lungs should jump to 1.25 Sv/yr, which is ridiculous. Chances are this isn't what actually happens!**

3 Radiation Units

1. Which type of radiation dose unit (Roentgen, Rad, Gray, Rem, Sievert) do you think your Geiger counter is best for directly measuring, and why?
   A Geiger counter could easily measure Roentgens, Gray, or Rad, but not Rem or Sieverts. This is because Gy and Rad are units of pure energy deposition into a fixed mass, therefore the count rates from a Geiger counter can be easily converted into an energy absorption rate that would be absorbed by soft tissue (humans). To measure Rem or Sieverts, the Geiger counter (or the person) would have to know the type and possibly the energy of the radiation, to convert it from dose to equivalent dose.

2. Calculate the radiation energy absorbed for a dose of 1Gy to the following organs: skin, eyes, thyroid, brain.
   This is a trick question! The units of Gy are in $\frac{J}{kg}$, so all that changes per each organ is its weight. Assuming a 75kg human's skin weighs about 5kg, that gives a radiation energy absorbed of 5J. Ditto for eyes (0.015 kg, 0.015 J), thyroid (same as eyes, about) and brain (3 kg, 3 J).

3. A 1Ci source of ^{137}Cs is dropped in NW13 during a 22.09 lab (note, this happened once!!!).

   (a) Estimate the full body dose equivalent in mSv to a student, assuming they ran at a speed of 5 $\frac{m}{s}$ from the source once it was spilled.
   First, we use the KAERI table of nuclides to see that Cs-137 is a beta emitter, though almost all of the time it is accompanied by a 662 keV gamma ray. The beta will be stopped by a little bit of air, while the gamma will be the thing really giving dose to the student. We approach this problem just like the external exposure problem above, by approximating a spherical, 75 kg student of radius 0.75 m. Let's start with our original dose rate equation, noting that even the mass attenuation coefficients are the same in this problem!!! (Air and water). Let's also use a simpler expression for the solid angle of the human:

$$\Omega = \frac{A_{object}}{D^2}; \quad fractional\, flux = \frac{\pi r^2}{4\pi D^2} \tag{28}$$

$$\frac{dSv}{dt} = \frac{\left(\cancel{5.1}^{\,6.62} \cdot 10^5\, eV\right)\left(1.6 \cdot 10^{-19} \frac{J}{eV}\right)}{75\, kg} \underbrace{\left[\cancel{(0.0008)}^{\,inapplicable} \left(\cancel{7.4\, Bq}^{\,3.7 \cdot 10^{10} \frac{Bq}{1 Ci}}\right) \cancel{\left(1 - cos\left(2 sin^{-1} \frac{r}{D}\right)\right)}^{\,\frac{\pi r^2}{4\pi D^2}} e^{-0.01225D} \right]}_{Gamma\, flux\, reaching\, human\, per\, dV} \underbrace{\left(1 - e^{-9.85r}\right)}_{Fraction\, interacting\, in\, h} \tag{29}$$

Now we just have to develop an expression for the distance of the student to the source (D) as a function of time:

$$D = 0.375 + 5t \tag{30}$$

and substitute this into our equation, and integrate over all time:

$$\frac{dSv}{dt} = \frac{\left(6.62 \cdot 10^5\, eV\right)\left(1.6 \cdot 10^{-19} \frac{J}{eV}\right)}{75\, kg} \underbrace{\left[\left(3.7 \cdot 10^{10}\right) \frac{\pi r^2}{4\pi D^2} e^{-0.01225D}\right]}_{Gamma\, flux\, reaching\, human\, per\, dV} \underbrace{\left(1 - e^{-9.85r}\right)}_{Fraction\, interacting\, in\, human} \tag{31}$$

$$\frac{dSv}{dt} = \underbrace{\left[\left(5.23 \cdot 10^{-6}\right) \frac{\pi r^2}{4\pi D^2} e^{-0.01225D}\right]}_{Gamma\, dose\, reaching\, human\, per\, dV} \underbrace{\left(1 - e^{-9.85r}\right)}_{Fraction\, interacting\, in\, human} \tag{32}$$

$$Dose = \int_0^\infty \frac{dSv}{dt}_{per\,dV} dt \tag{33}$$

Numerically evaluating this integral, we get a total dose (assuming tissue and quality factors of 1, and assuming a similar mass attenuation coefficient) of $0.76\ \mu Sv$.

(b) Estimate the total specific energy absorption in Roentgens from this accident.
That's an invalid question, as human bodies don't hold a charge.

(c) Does this constitute a significant radiation exposure? Why or why not?
This does not constitute a significant exposure. It constitutes and extra 1/14th of the normal amount of radiation one would expect to receive in a given day.

4 BONUS 25 Point Question

Estimate the additional dose incurred by spooning (see Figure 1) while sleeping, compared to sleeping alone.