22.01 Fall 2015, Quiz 2 Study Sheet

November 14, 2015

Familiarize yourself with all these concepts from an *intuitive* and a *quantitative* point of view. Keep in mind that the material we covered in class, especially in Friday's recitation, will be particularly relevant.

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1 Photon Interactions with Matter

- Three main mechanisms of interaction
  - Photoelectric Effect
  - Compton Scattering
  - Pair Production
- Mechanisms and Energetics
  - Photoelectric Effect
    * Photon is absorbed by an electron, causes an ejection
    * $T_{e^-} = \hbar\omega - E_{binding}$
    * Usually an inner shell electron
      · Level 1 electrons about 80% of the time
    * $\sigma_\tau \propto Z^5 \left(\frac{1}{\alpha}\right)^{\frac{7}{2}}$
      · Strongly more likely with increasing atomic number
      · Strongly less likely with increasing photon energy
  - Compton Scattering
    * Photon gets scattered by an electron, leaves at a lower energy and at a different angle
    * Key relationships: See Equations 10.6 - 10.9 in Yip
      · Equation 10.6 - Shows the *increase* in photon wavelength
      · Equation 10.7 - Relates the change in energy to photon energy & angle
      · Equation 10.8 - Gives the energy of the Compton electron
      · Equation 10.9 - Relates the exit angle of the scattered photon and the Compton electron
    * σ_C: See Equation 10.17 (Klein-Nishina cross section)
      · **Know what this means in terms of forward scattering bias**
    * Understand the *Compton Edge* - Backscattered photon is more likely (see Klein Nishina cross section), they impart the most energy to the Compton electron
    * It is the Compton *electrons* that are counted in detectors, along with any other ionized electrons from any process
  - Pair Production
    * Photon above 1.022MeV gets near an electron cloud
    * Spontaneously changes into an electron/positron pair

    * Electron energy is counted by an ionization cascade
    * Positron moves some distance in a material, annihilates with another electron
    * Gives off two 511keV gammas in opposite directions
    * Single escape peak - One gets out of the detector, one is reabsorbed (medium-sized detectors)
    * Double escape peak - Both get out of the detector (small-sized detectors)
    * No escape peaks - All photons get absorbed (large detectors)
    * 511keV peak - Pair production happened somewhere *outside* the detector, then a 511keV photon enters the detector and undergoes photoelectric or Compton scattering
  - See Equations 10.43 - 10.45 for cross section comparisons
  - Mass attenuation coefficients - density-normalized photon interaction probabilities
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$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} \tag{1}$$
  - **Know how to interpret photon spectra from detectors** - see Figure 10.18 for an example

- Ion/Electron Interactions
  - Four methods: elastic & inelastic collisions, with other electrons and with nuclei
    * Elastic collisions with electrons are the main mechanism of energy loss
  - Derived using the hollow cylinder approach (see pp. 225-226) as a Coulomb force balance & integration
  - $p_{e^-} = \int F_y(t)\,dt = m \int a(t)\,dt = mv$
  - Stopping power formula: $-\frac{dT}{dx} = \frac{4\pi N Z_1^2 Z_2 e_c^4}{m_e v^2} ln\left(\frac{2m_e v^2}{\bar{I}}\right) = \frac{4\pi N Z_1^2 Z_2 e_c^4}{E_i}\frac{M}{m_e} ln\left(\frac{\gamma_e E_i}{\bar{I}}\right);\ \gamma_s = \frac{4m_e M}{(M+m_e)^2}$
  - Relativistic correction: $-\frac{dT}{dx} = \frac{4\pi N Z_1^2 Z_2 e_c^4}{m_e v^2}\left[ln\left(\frac{2m_e v^2}{\bar{I}}\right) - ln\left(1-\beta^2\right) - \beta^2\right]$
  - Stopping power curve shape: follows 1/E times ln(E), see & understand Figure 11.4
  - Number of ion pairs produced: $i = \frac{1}{W}\left(-\frac{dT}{dx}\right)$
  - Cross section comparison - see Equations 11.18 - 11.21 (p. 233)
  - Know when ionization loss is high (low energy) and radiation loss is high (very high energy)
  - Range: When the ions come to rest - see p. 235 for full explanation

- Neutron interactions
  - No charge, almost all nuclear elastic collisions
    * Never forget the Q-equation! See p. 143 to refresh your memory
    * Assumptions:
      · Q=0 (elastic scattering)
      · M1=M3=1 amu, M2=M4=A amu
      · A neutron can lose at most $(1-\alpha)$ of its energy in an elastic collision, $\alpha = \left(\frac{A-1}{A+1}\right)^2$
      · We assume that scattering is isotropic in angle, this is less true with increasing energy
  - Inelastic Scattering
    * One neutron goes in, *compound nucleus* formed, different neutron comes out
    * See p. 244 for energy level diagram, p. 246 for diagram with resonance with nuclear energy levels
    * Often Q<0, cross section is zero until E=Q
  - Fission

    * Also a compound nucleus formation, but this time the "liquid drop" splits into two uneven sized droplets
    * First, the neutron rich fission products shed 1-2 neutrons each
    * Then, beta decay reduces their assymetry further
    * See p. 255 (Figure 12.11) for a time diagram of what happens

- Neutron Transport
  - Balance of neutron gains & losses from a location (dV), in a certain energy range (dE), going in a certain solid angle direction (dΩ)
  - Gains
    * Fission
    * External sources
    * Scattering down into our energy group
  - Losses
    * Scattering down out of our energy group
    * Absorption
    * Leakage
  - Terminology
    * $\phi(E)$: Angular flux, $\Phi(E)$: Total flux
    * $\Sigma_t = \Sigma_a + \Sigma_s$ (total = absorption + fission)
    * $\Sigma_a = \Sigma_f + \Sigma_\gamma$ (absorption = fission + capture)
    * $\Sigma_s = \Sigma_{s,el} + \Sigma_{s,non-el}$ (scattering = elastic + inelastic)
    * E is our energy, E' is some other energy
    * Ω is our angle, Ω' is some other angle
  - Equational form:

$$\frac{dn(\mathbf{r}, E, \mathbf{\Omega}, t)}{dt} = \frac{\nu\chi(E)}{4\pi}\int\limits_V\int\limits_E\int\limits_{\Omega'} d^3r dE' d\Omega' \Sigma_f(E')\,\phi(\mathbf{r}, E', \mathbf{\Omega}, t) \tag{2}$$

$$+S_0(\mathbf{r}, E, \mathbf{\Omega}, t) + \int\limits_V\int\limits_E\int\limits_{\Omega'} d^3r dE d\Omega \Sigma_s(E')\,\phi(\mathbf{r}, E, \mathbf{\Omega'}, t)\, F(E' \to E, \Omega' \to \Omega)$$

$$-\int\limits_V d^3r dE d\Omega \Sigma_t(E')\,\phi(\mathbf{r}, E, \mathbf{\Omega}, t) - \int\limits_V d^3r dE d\Omega \Omega \cdot \nabla\phi(\mathbf{r}, E, \mathbf{\Omega}, t)$$

  - Terms: change = fission + external + scattering in - all collisions - flow outwards
  - Simplifications: make everything isotropic (eliminate angular dependence), any volume element is the same (eliminate r-dependence)
  - Big simplification: Assume all neutrons are in one energy group, homogeneous (perfectly mixed) reactor with one average material
  - New equation: gains = losses

$$\Sigma_a\Phi - \nabla D \cdot \nabla\Phi = \frac{\nu\Sigma_f\Phi}{k_{eff}} \tag{3}$$

  - Solution takes form of $\frac{-\nabla^2\Phi}{\Phi} = constants;\quad sin\,\&\,cos$
  - Eliminate sin due to symmetry concerns
  - Solution is $\frac{-\nabla^2\Phi}{\Phi} = B^2$; B is the "buckling"

- Material buckling & criticality:

$$\Sigma_a \Phi - \nabla D \cdot \nabla \Phi = \frac{\nu \Sigma_f \Phi}{k_{eff}} \implies \Sigma_a \Phi - DB^2 \Phi = \frac{\nu \Sigma_f \Phi}{k_{eff}} \implies k_{eff} = \frac{\nu \Sigma_f}{\Sigma_a + DB^2} \quad (4)$$

- k_eff is a balance between neutron production & destruction
- If k_eff = 1, the reactor is critical (perfect balance between production & loss)
- Then, using Equation 4, we can give a criticality condition based on the materials in a reacto:

$$B^2_{material} = \frac{\nu \Sigma_f - \Sigma_a}{D} \quad (5)$$

- and its geometry:

$$\frac{-\nabla^2 \Phi}{\Phi} = B^2_{geometry}; \quad , \Phi = A \, cos \, (B_g x) \quad (6)$$

- The reactor is perfectly critical when

$$B_g^2 = B_m^2 \quad (7)$$

– Another criticality condition: the six-factor formula

$$k_{eff} = \eta f p \epsilon P_{FNL} P_{TNL} \quad (8)$$

- Assumes two groups (fast & thermal) of neutron energies
- η - average number of neutrons per fission
- f - thermal utilization factor - probability that an absorption takes place in the fuel and not elsewhere
- p - resonance escape probability, the probability that a neutron slows down through and out of the resonance region of cross sections into the thermal energies
- ϵ - fast fission factor (not all fissions are thermal)
- P_{FNL} - probability that a given fast neutron stays in the reactor (not that high)
- P_{TNL} - probability that a given thermal neutron stays in the reactor (pretty high)