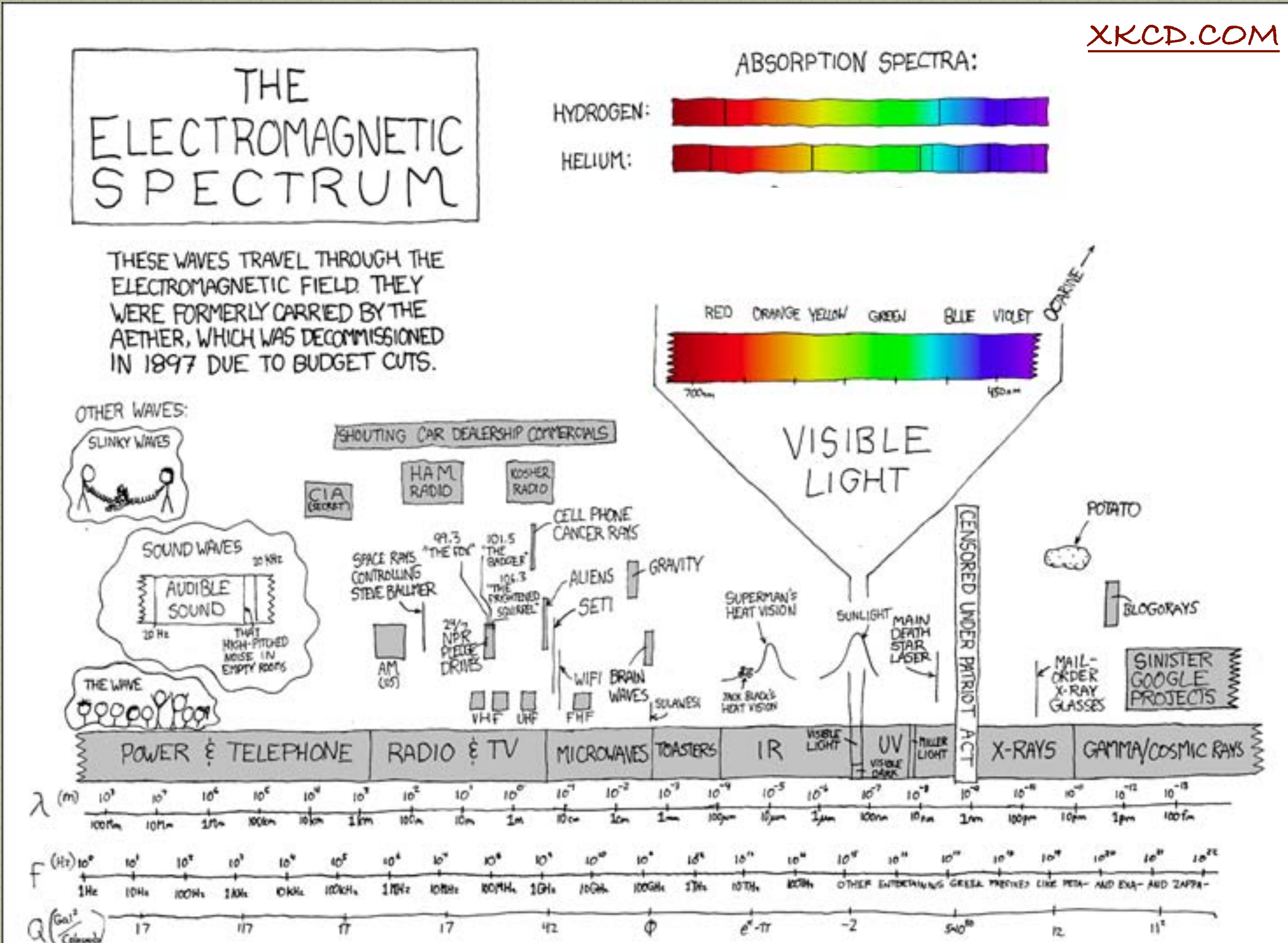
GAMMA DECAY



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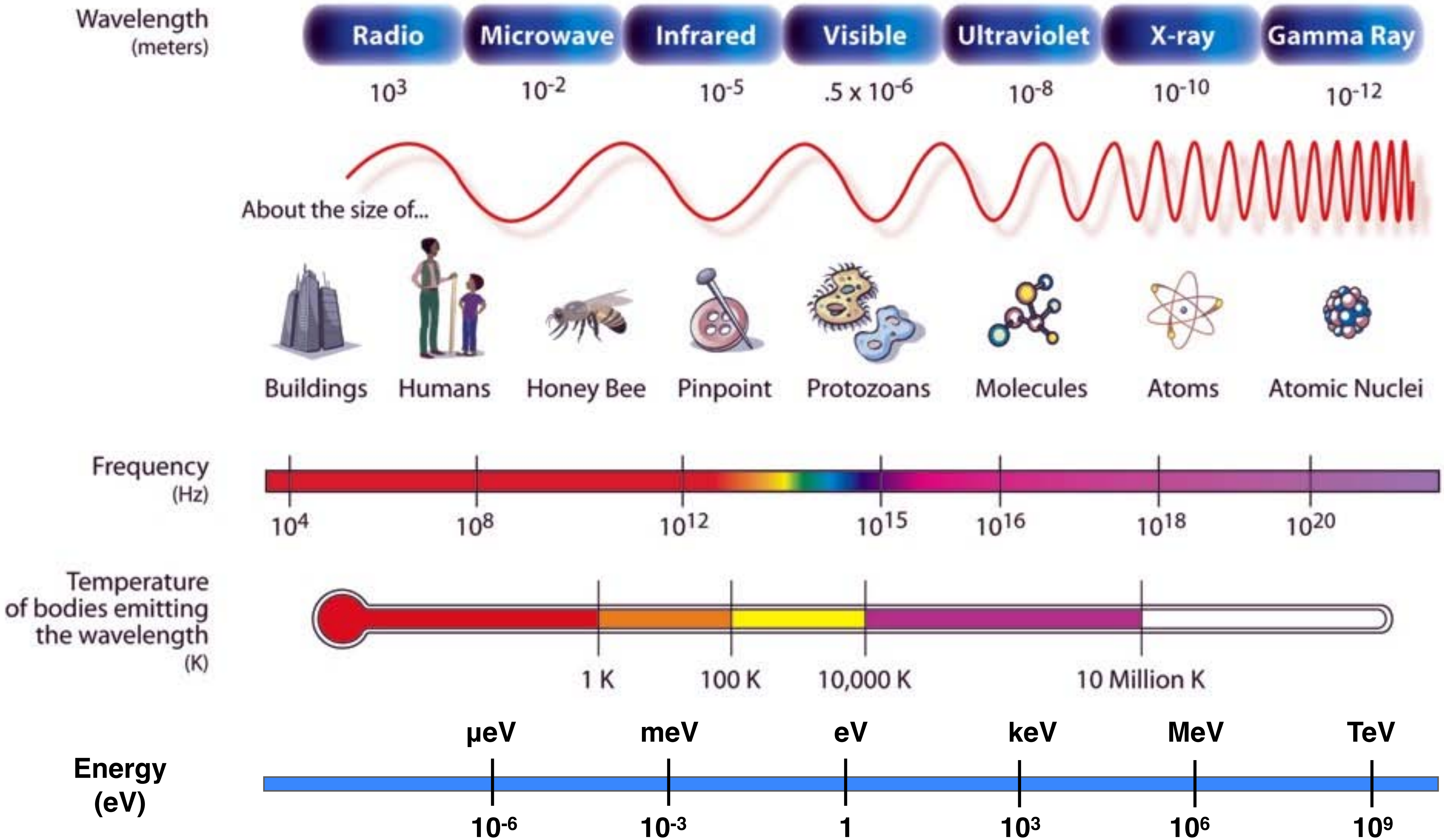
ELECTROMAGNETIC SPECTRUM



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DIPOLE RADIATION

- Rate from Fermi's Golden Rule + Density of states:

$$W = \frac{2\pi}{\hbar} |\langle \psi_f | \hat{V} | \psi_i \rangle|^2 \rho(E_f) = \frac{\omega^3}{2\pi c^3 \hbar} |\langle \hat{r} \rangle|^2 \sin^2\theta \, d\Omega$$

- Integrating over angles:

$$\lambda(E1) = \frac{4}{3} \frac{e^2 \omega^3}{\hbar c^3} |\langle r \rangle|^2$$

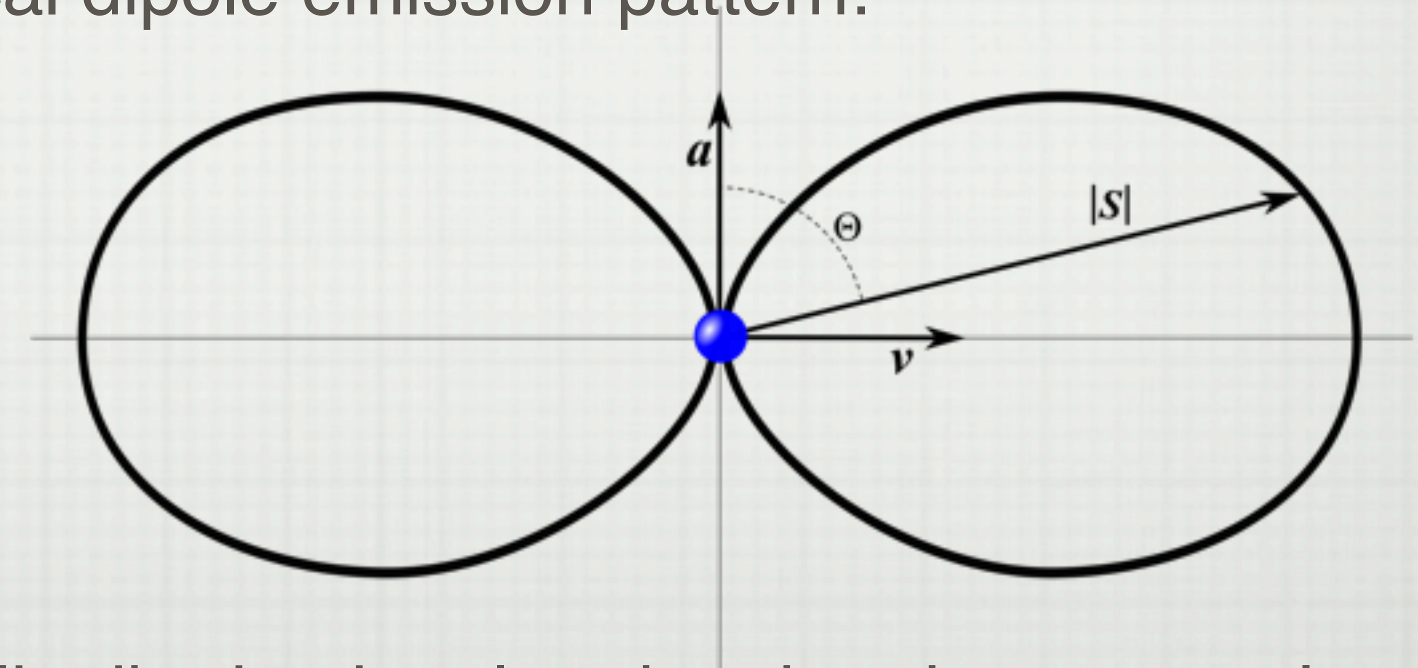
DIPOLE APPROXIMATION

- In deriving the dipole emission formula we only kept lowest order expansion:

$$\vec{A} \propto a_k^\dagger e^{i\vec{k}\cdot\vec{r}}\vec{\epsilon}_k \approx a_k^\dagger(1 + i\vec{k}\cdot\vec{r} + \ldots)\vec{\epsilon}_k \rightarrow a_k^\dagger\vec{\epsilon}_k$$

- This yields the typical dipole emission pattern:

$$W \propto \sin\theta^2 d\Omega$$



- In QM the angular distribution is related to the photon angular momentum

BEYOND THE DIPOLE

- Higher order terms in the expansion give rise to gamma emission
  - with different angular-dependence pattern
  - and higher angular momentum for the gamma photon emitted

$$\vec{A} \propto a_k^\dagger e^{i\vec{k}\cdot\vec{r}}\vec{\epsilon}_k \approx a_k^\dagger \sum_\ell \frac{(i\vec{k}\cdot\vec{r})^\ell}{\ell!}\vec{\epsilon}_k$$

- Each ℓ term contributes to a different decay rate.

MULTIPOLE RADIATION

- Electric multipole

$$\lambda(E\ell) = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2}\frac{e^2}{\hbar c}\left(\frac{E}{\hbar c}\right)^{2\ell+1}\left(\frac{3}{\ell+3}\right)^2 c\left\langle|\hat{\vec{r}}|\right\rangle^{2\ell}$$

$$\left\langle|\hat{\vec{r}}|\right\rangle \approx R_0 A^{1/3}$$

- Rates:

$$\lambda(E1) = 1.0\times 10^{14} A^{2/3} E^3$$

$$\lambda(E2) = 7.3\times 10^{7} A^{4/3} E^5$$

$$\lambda(E3) = 34 A^{2} E^7$$

$$\lambda(E4) = 1.1\times 10^{-5} A^{8/3} E^9$$

MULTIPOLE RADIATION

- Magnetic multipole

$$\lambda(M\ell) = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \frac{e^2}{\hbar c} \frac{E}{\hbar c}^{2\ell+1} \left(\frac{3}{\ell+3}\right)^2 c \left\langle|\hat{\vec{r}}|\right\rangle^{2\ell-2} \left[\frac{\hbar}{m_p c}\left(\mu_p - \frac{1}{\ell+1}\right)\right]$$

- Rates:

$$\lambda(M1) = 5.6 \times 10^{13} E^3$$

$$\lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5$$

$$\lambda(M3) = 16 A^{4/3} E^7$$

$$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$$

WHICH TRANSITION?

- The lowest multipole dominates:
  - Lower multipoles decay faster (higher rates)
  - Electric multipoles are faster than magnetic multipoles

➡ Why don't we always only observe electric dipole (E1) radiation?

SELECTION RULES

- The multipole ℓ is related to the gamma angular momentum
  - the angular momentum must be conserved in gamma decay
- Possible ℓ: $|I_f - I_i| \leq \ell_\gamma \leq I_f + I_i$
- Parity: $(-1)^{\ell}$ for Electric and $(-1)^{\ell-1}$ for Magnetic: parity must be conserved, $\Pi_\gamma = \Pi_i \Pi_f$

Multipolarity	Angular Momentum l	Parity Π	Multipolarity	Angular Momentum l	Parity Π
M1	1	+	E1	1	-
M2	2	-	E2	2	+
M3	3	+	E3	3	-
M4	4	-	E4	4	+
M5	5	+	E5	5	-

WHICH TRANSITION?

- The lowest permitted multipole dominates
- Electric multipoles are more probable than the same magnetic multipole by a factor 100

$$\frac{\lambda(El)}{\lambda(Ml)} \approx 10^2$$

- Emission from the multipole ℓ+1 is 10^{-5} times less probable than the ℓ-multipole emission

$$\frac{\lambda(E, l+1)}{\lambda(El)} \approx 10^{-5}, \qquad \frac{\lambda(M, l+1)}{\lambda(Ml)} \approx 10^{-5}$$

WHICH TRANSITION?

- Combining the two rules:

$$\frac{\lambda(E, l+1)}{\lambda(Ml)} \approx 10^{-3}, \qquad \frac{\lambda(M, l+1)}{\lambda(El)} \approx 10^{-7}$$

- Thus E2 competes with M1
- But M2 does not compete with E1

INTERNAL CONVERSION

- In some cases energy is not released in the form of gamma photons, but carried away by an electron:

$$ {}^{A}_{Z}X^{*} \rightarrow \ {}^{A}_{Z}X^{+} + e^{-} $$

- This process is called Internal Conversion
- It is the only process possible, when selection rules do not allow any of the multipole transitions:
  - e.g. even-even nuclides, decay from a 0^{+} level
  - the photon cannot have zero angular momentum.