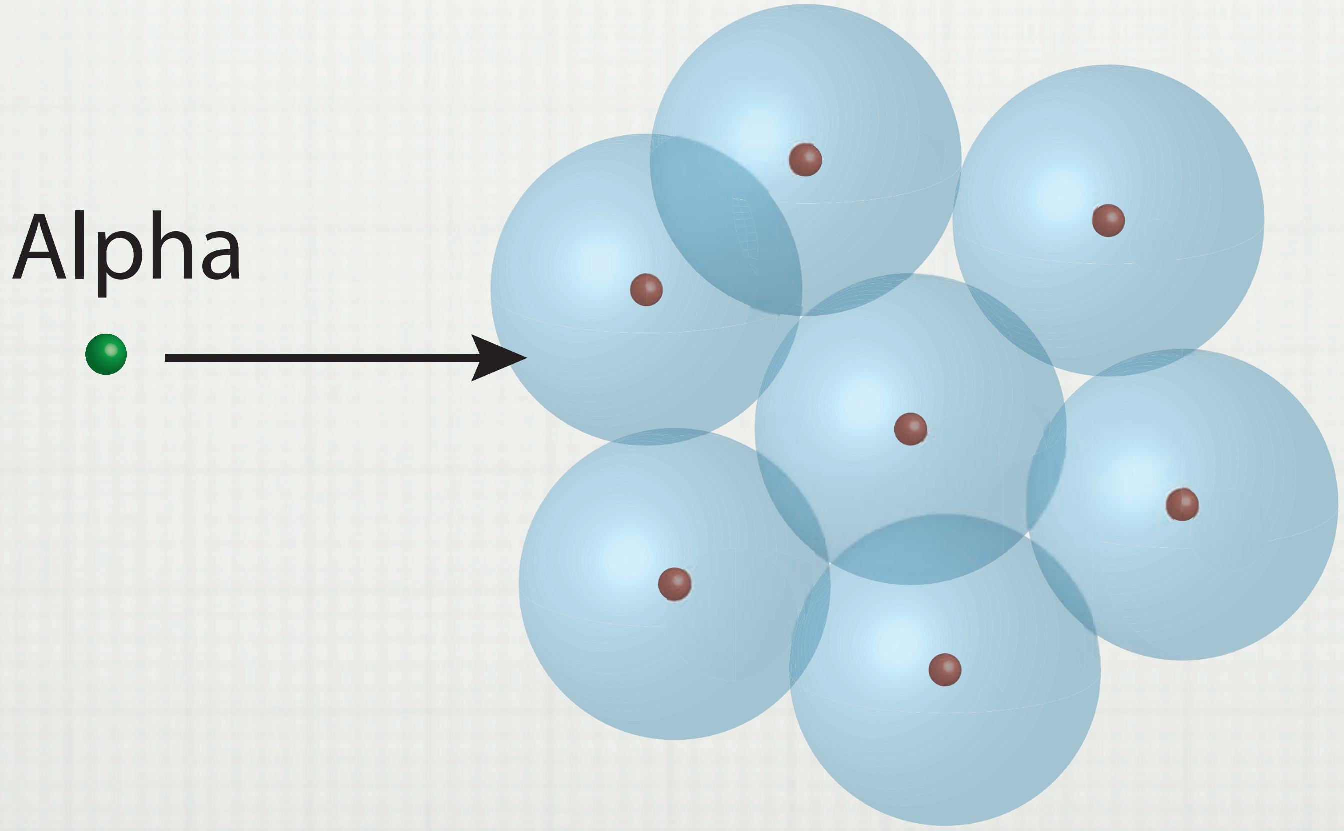
SCATTERING

INTERACTION OF RADIATION WITH MATTER

CHARGED PARTICLES IN MATTER

- Charged particles interact mostly with the electronic cloud



- Small energy loss, but very frequent collisions

CHARGED PARTICLES IN MATTER

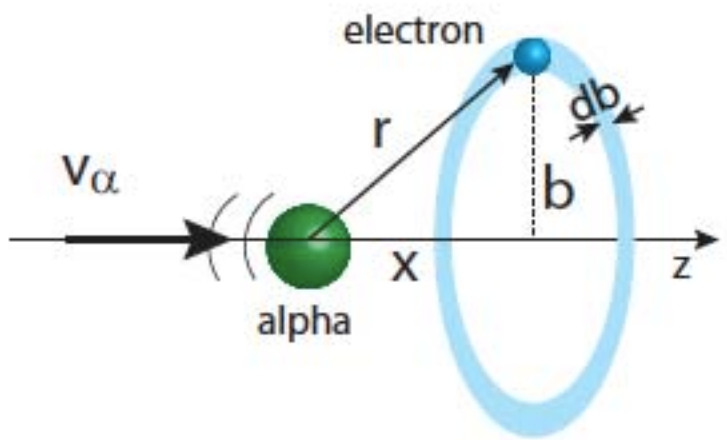
- Classical, non-relativistic collisions of charged particles with electrons

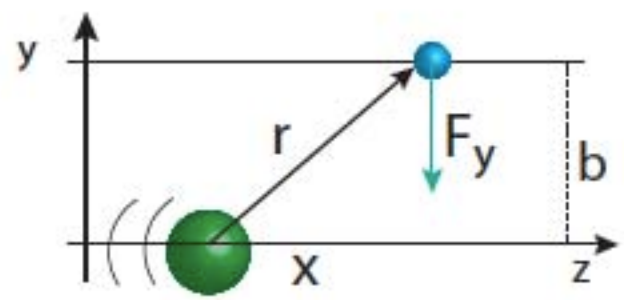
- Conservation of energy and momentum

- Charged particle looses a tiny fraction of its original energy

$$\Delta E = \frac{1}{2} m_e v_e^2 = \frac{1}{2} m_e (2v_\alpha)^2 = 4\frac{m_e}{m_\alpha} E_\alpha \quad \rightarrow \quad \frac{\Delta E}{E_\alpha} = 4\frac{m_e}{m_\alpha} \ll 1$$

COULOMB INTERACTION





$$F_y = \frac{eQ}{4\pi\epsilon_0}\frac{b}{(x^2+b^2)^{3/2}}$$

$$\Delta p_= \int_0^\infty F_y dt = \int_{-\infty}^{\infty} \frac{dx}{v_\alpha}\frac{e^2 Z_\alpha}{4\pi\epsilon_0}\frac{b}{(x^2+b^2)^{3/2}}$$

$$\Delta p = p_e = \frac{e^2 Z}{4\pi\epsilon_0 v b}\int \frac{d\xi}{(1+\xi^2)^{3/2}} = 2\frac{e^2 Z_\alpha}{4\pi\epsilon_0 v_\alpha b}$$

$$\boxed{\Delta E = \frac{p_e^2}{2m} = 2\frac{e^4 Z_\alpha^2}{(4\pi\epsilon_0)^2 m_e v^2 b^2}}$$

STOPPING POWER

- Integrate over all impact parameters b

$$-dE = 2\pi dx \int n_e \Delta E b db \;\rightarrow\; \frac{dE}{dx} = -2\pi \int n_e \Delta E(b) b db = -\frac{4\pi e^4 Z_\alpha^2 n_e}{(4\pi\epsilon_0)^2 m_e v_\alpha^2} \int \frac{db}{b}$$

- Lower bound (closest approach max energy lost):

$$b_{min} \sim \frac{1}{4\pi\epsilon_0} \frac{e^2}{2m_e v_\alpha^2}$$

- Upper bound: Bohr radius (from ionization energy)

$$\frac{b_{max}}{b_{min}} = \frac{2m_e v_\alpha^2}{Z_\alpha E_I}$$

$$-\frac{dE}{dx} = \frac{4\pi e^4 Z_\alpha^2 n_e}{(4\pi\epsilon_0)^2 m_e v_\alpha^2} \ln\left(\frac{b_{max}}{b_{min}}\right) = \frac{4\pi e^4 Z_\alpha^2 n_e}{(4\pi\epsilon_0)^2 m_e v_\alpha^2} \ln \Lambda$$

STOPPING RANGE

1. Thousands of events (collisions) are needed to effectively slow down and stop the alpha particle

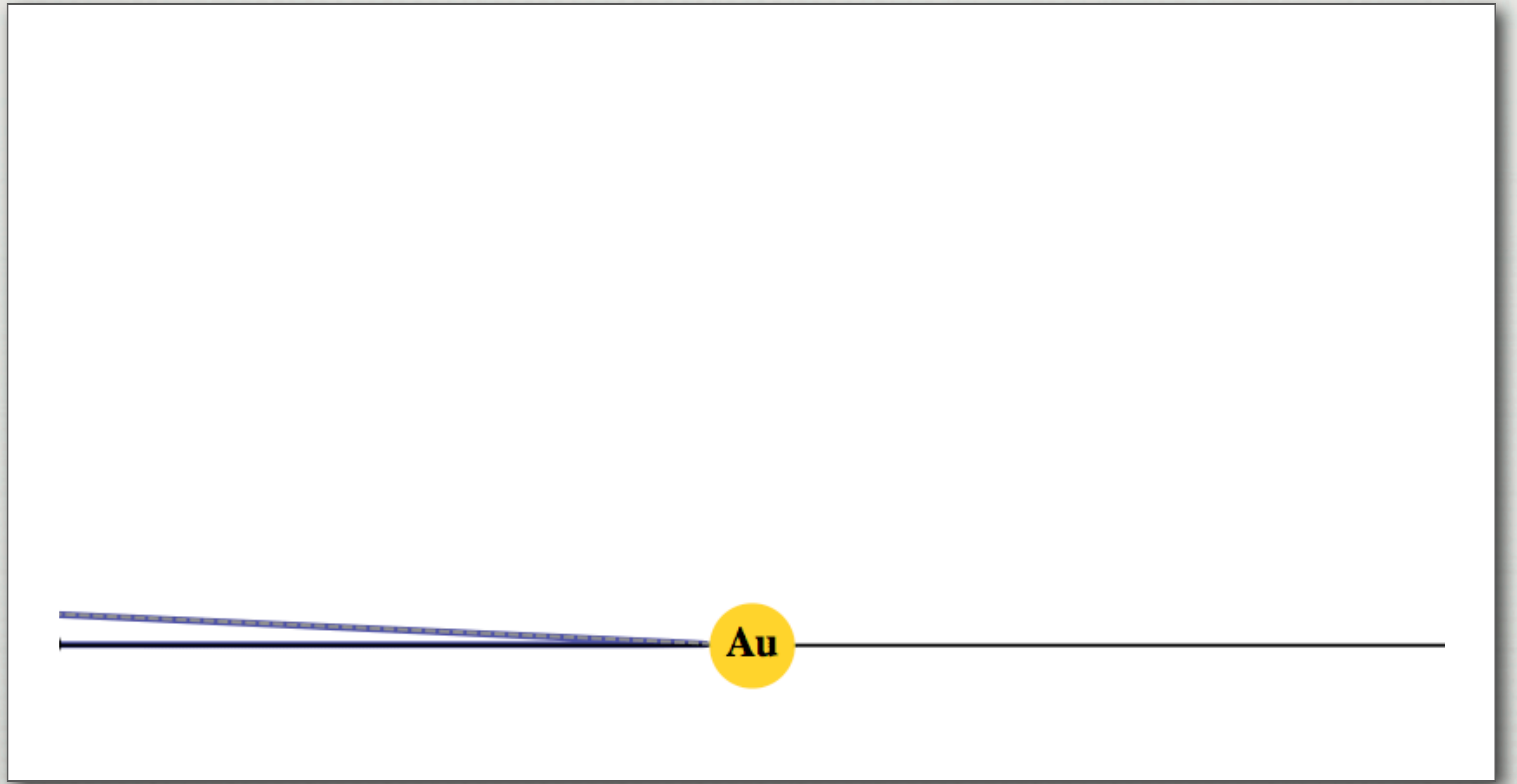
2. As the alpha particle is barely perturbed by individual collisions, the particle travels in a straight line.

3. The collisions are due to Coulomb interaction, which is an infinite-range interaction. Then, the alpha particle interacts simultaneously with many electrons, yielding a continuous slowing down a certain stopping range.

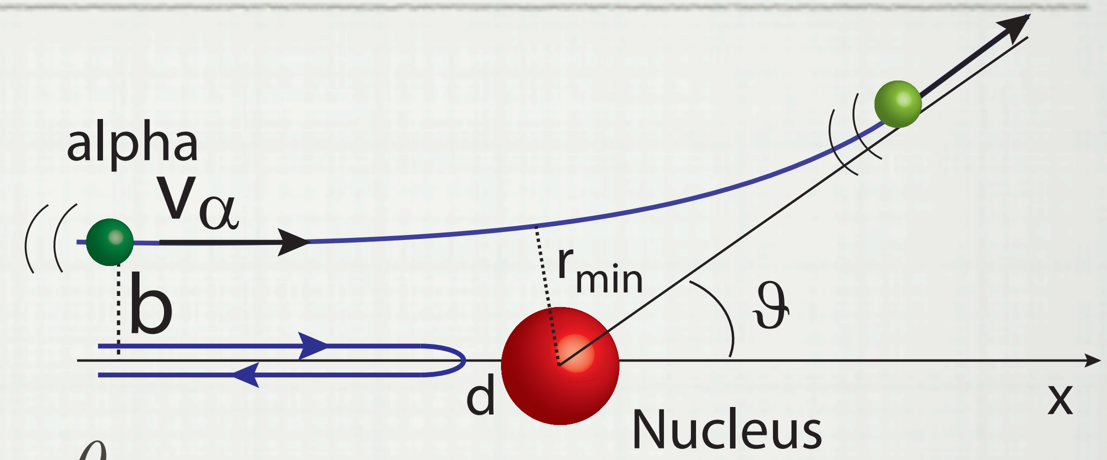
4. The electrons which are the collision targets get ionized, thus they lead to a visible trail (e.g. in cloud chambers)

RUTHERFORD SCATTERING

ANIMATION



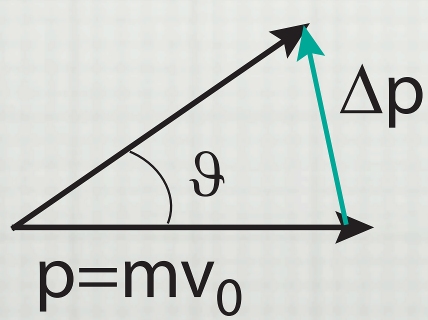
RUTHERFORD SCATTERING

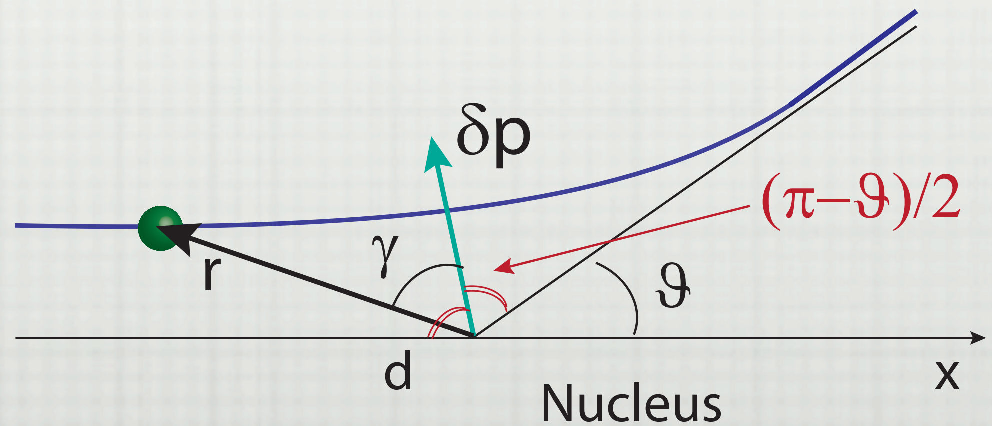


- Classical x-section:

$$\frac{d\,\sigma}{d\,\Omega} = \frac{2\pi b db}{d\Omega}$$

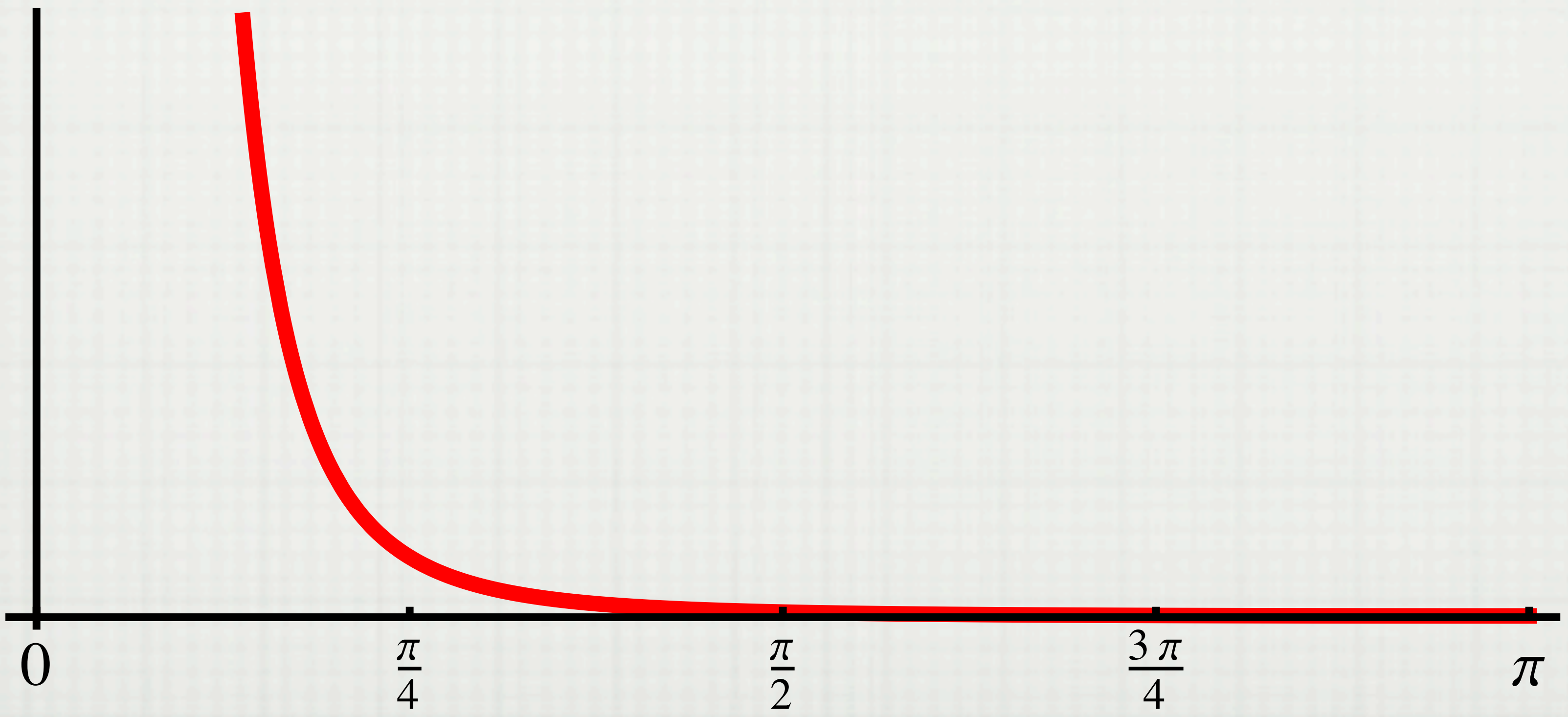
- Momentum change: $\Delta p = 2mv_0 \sin\frac{\theta}{2}$
- Conservation of angular momentum
- Coulomb interaction: $d\vec{p} = \vec{F}dt = \frac{zZe^2\hat{r}}{4\pi\epsilon_0|r^2|}dt$





COULOMB SCATTERING

- Cross-section: $$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{4\pi\epsilon_0}\right)^2 (4T_a)^{-2} \sin^{-4}\left(\frac{\theta}{2}\right)$$



- Why Rutherford used Gold in the experiment?