E.M. INTERACTION WITH MATTER

SCATTERING of E.M. RADIATION

- Type of scattering depends on the e.m. energy, $\hbar\omega$ wrt the electron energy:
  - Ionization energy E_I
  - Rest-mass energy $m_e c^2$

Rayleigh/Raman Scattering	Photoelectric Absorption	Thomson Scattering	Compton Scattering	Pair Production
$\hbar\omega < E_I$	$\hbar\omega \geq E_I$	$\hbar\omega \gg E_I$	$\hbar\omega \sim m_e c^2$	$\hbar\omega > 2m_e c^2$
$\sim$eV	$\sim$keV	$\sim$keV	$\sim$MeV	$\geq$MeV
Visible	X-rays	X-rays	γ-rays	hard γ-rays

CLASSICAL MODEL

- E.M. wave interacting with an oscillating electron
  - Frequency of oscillation is given by Coulomb energy: $\omega_0^2 = k_C/m_e$
  - e.m. field adds a driving force at e.m. frequency ω, $\quad F = -eE_0 \sin(\omega t)$
  - Radiated power is related to the electron acceleration, a

$$P = \frac{2}{3}\frac{e^2}{c^3}a^2 = \frac{1}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{\omega^4}{(\omega_0^2 - \omega^2)^2} c E_0^2$$

- The cross-section is then $\quad \sigma = P/I_0, \quad I_0 = \dfrac{cE_0^2}{8\pi}$

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \left(\frac{\omega^2}{\omega_0^2 - \omega^2}\right)^2$$

RAYLEIGH SCATTERING

- At very low energy $\omega \ll \omega_0$ cross-section becomes:

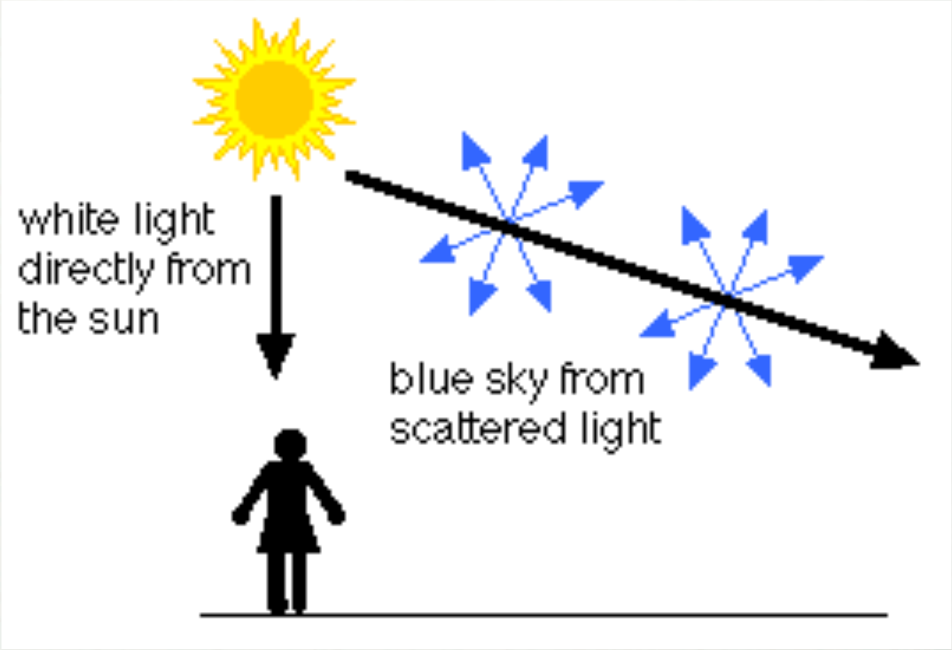
$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \left(\frac{\omega}{\omega_0}\right)^4$$

- Very strong depence on the frequency/wavelength
  - shorter wavelengths are scattered most
  - longer wavelengths travel in (almost) straight lines
- Rayleigh scattering experiment?

RAYLEIGH SCATTERING

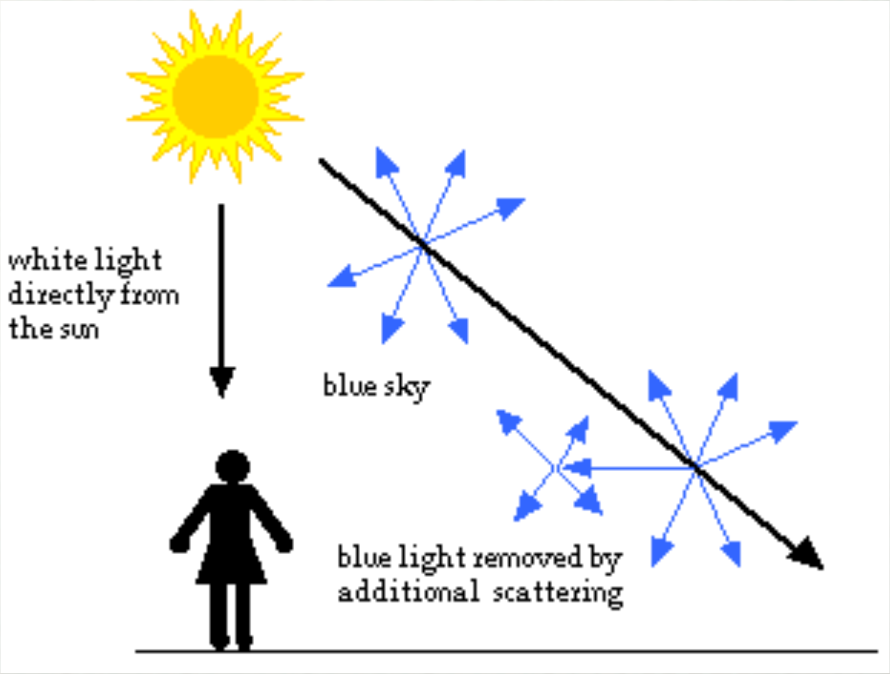
- ## Why is the sky blue?

The blue color of the sky is due to Rayleigh scattering. As light moves through the atmosphere, most of the longer wavelengths pass straight through. Little of the red, orange and yellow light is affected by the air. However, much of the shorter wavelength light is absorbed by the gas molecules. The absorbed blue light is then radiated in different directions. It gets scattered all around the sky. Whichever direction you look, some of this scattered blue light reaches you. Since you see the blue light from everywhere overhead, the sky looks blue.



- ## Why is it paler close to the horizon?

As you look closer to the horizon, the sky appears much paler in color. To reach you, the scattered blue light must pass through more air. Some of it gets scattered away again in other directions. Less blue light reaches your eyes. The color of the sky near the horizon appears paler or white.



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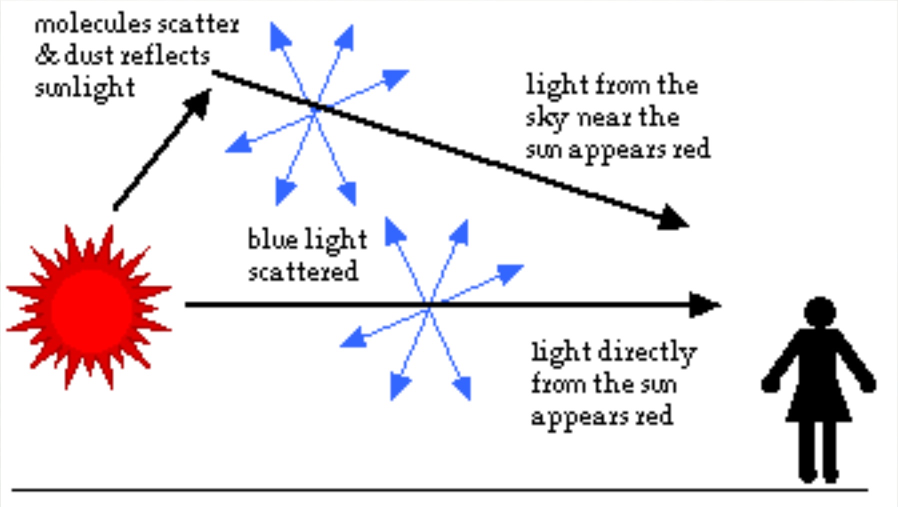
RAYLEIGH SCATTERING

- **Why is the sky red at sunset?**

  As the sun begins to set, the light must travel farther through the atmosphere before it gets to you. More of the light is reflected and scattered. As less reaches you directly, the sun appears less bright. The color of the sun itself appears to change, first to orange and then to red. This is because even more of the short wavelength blues and greens are now scattered. Only the longer wavelengths are left in the direct beam that reaches your eyes.



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- **Why are clouds white?**

  The water droplets that make up the cloud are much larger than the molecules of the air and the scattering from them is almost independent of wavelength in the visible range.

THOMSON SCATTERING

- Intermediate energy:
  - Higher than ionization energy (electron appears to be “free”) but not enough to impart relativistic speed:

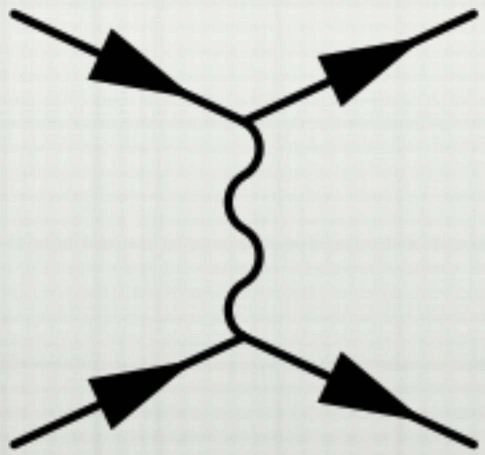
$$\hbar\omega_0 \ll \hbar\omega \ll m_e c^2$$

  - Simplify the cross-section: $\frac{\omega^2}{\omega_0^2 - \omega^2} \approx -1$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2 \approx 2.3 \text{ barns}$$

HIGH-ENERGY

- The classical picture fails at high energy (> ionization energy)
- e.m. radiation can now be considered a "particle" (photon)
  - scattering = photon-electron collision
- QM scattering theory
  - Quantized electromagnetic field $\left(a_k^\dagger a_h\right)$
  - Feynman Diagrams



HIGH-ENERGY

- The classical picture fails at high energy (> ionization energy)
- e.m. radiation can now be considered a “particle” (photon)
  - scattering = photon-electron collision
- Photoelectric effect: photon absorbed, electron emitted
- Compton Scattering $$\Delta\lambda = \frac{2\pi\hbar}{m_e c}(1 - \cos\vartheta)$$
- Pair production (E>1.022 MeV)