

22.058, Principles of Medical Imaging
Fall 2002
Homework #1 Solutions

1. Define the instrument response function (IRF) and the point spread function (PSF), and explain their relation to the image resolution. Why is it convenient to assume that the PSF is spatially invariant?

The instrument response function describes the true mapping of every point in the source (x_s, y_s) to every point in the detector (x_d, y_d) , $IRF(x_d, y_d | x_s, y_s)$.

The point spread function describes the average broadening of any point in the source when mapped onto the detector, $P(x_d - x_s, y_d - y_s)$.

The definition of a point spread function permits the imaging process to be described as a convolution, $Image = Object \otimes PSF + noise$.

2. What is meant by stating that edges in an image correspond to high spatial frequencies?

An edge is a rapid change in intensity over space. Since the wave number, k , reports this rate of change, edges correspond to high values of k .

3. Describe a pin-hole camera as prototype imaging system. Show both figures and equations.

QuickTime™ and a
None decompressor
are needed to see this picture.

- a. Magnification = $-b/a$

b. Blurring due to finite sized pin-hole = $\frac{(a+b)r}{a}$

c. Leakage through width = $e^{-\mu w}$

d. Critical angle beyond which no photo reach the detector = $\theta_C = \cos^{-1}\left(\frac{w}{r}\right)$

QuickTime™ and a
None decompressor
are needed to see this picture.

e. Oblique angle effect of pin-hole spot size.

<i>longer path</i>	$\alpha \cos^2(\theta)$
<i>oblique to detector</i>	$\alpha \cos(\theta)$
<i>reduced pinhole</i>	$\frac{\alpha \cos(\theta)}{\alpha \cos^4(\theta)}$

f. Detector resolution (assume a 2D array detector like the CCD).

QuickTime™ and a
None decompressor
are needed to see this picture.

\therefore a set of points $\int_{-\infty}^{\infty} I(x) \text{TopHat}\left(\frac{2x}{d} - n(D+d)\right) dx$

4. Define the following in terms of the pin-hole camera and suggest a means of measuring them:
 - a. Resolution – full width at half height at the Point Spread Function. Limited mainly by the finite pinhole. The smaller, the better.
 - b. Sensitivity – absolute intensity of image. Limited by the finite pinhole. The larger, the better.
 - c. Contrast – intensity difference for defined features. The leakage through the screen limits this.
 - d. Contrast to noise ratio – contrast as above divided by RMS noise. Noise is essentially dark contrast of the detector.
 - e. Distortions (sources and what are the effects).
5. Given the above non-idealities, suggest a linear model that captures most of the physics and defend it. Experimentally how would you explore if your model was adequate?

For a finite field of view include the blurring of the pin-hole and an average attenuation. The smaller the field of view, the better will be the approximation.

6. Write a program that simulates the action of a true pin-hole camera in 1 dimension.