

22.058, Principles of Medical Imaging
Fall 2002
Homework #2

1. If $\text{FT}\{g(x)\} = G(k)$, show that $\text{FT}\{g(x-a)\} = \exp(-ika) G(k)$.
2. Calculate the Fourier Transform of $A \cos(k_0 x + a)$, for $a/k_0 = \{0, \pi/4, \pi/2, \pi\}$. Plot the result.
3. If $\text{FT}\{g(x)\} = G(k)$, show that $\text{FT}\{g(ax)\} = (1/a) G(k/a)$.
4. Calculate the Fourier Transform of $(A) \text{TopHat}(x/A)$, for $A = \{1, 2, 4\}$. Plot the result.
5. Approximate the shape of a TopHat function from the Fourier Transform given in 4. Use 4, 8, 16, and 32 Fourier components in your approximation. Plot the result. You can calculate this in Matlab or Mathematica.
6. Calculate in real space the convolution of $\text{TopHat}(x)$ with $\text{TopHat}(x/2)$.
7. Repeat the calculation in 6 using Fourier convolution.
8. Calculate the convolution of $\text{TopHat}(x/a)$ with $\text{Comb}(x/b)$ for $2a < b$, $2a > b$, $2a = b$.
9. Define the Instrument Response Function and the Point Spread Function and explain their differences.
10. What is the relationship between the modulation transfer function, the optical transfer function and the point spread function? How is the system resolution defined?