22.058, Principles of Medical Imaging

Fall 2002

Homework #4

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1. Write a complete system description for the instrument function of a planar x-ray imager (assume scanned fan beam). Include:

- Finite size source
- Heal effect on source intensity and energy spectrum
- Oblique angle effects
- Depth dependent magnification
- Quantum efficiency and PSF for the scintillator/photographic plate.

$$I(x,y)=\frac{1}{4\pi d^2}\iint S(x',y')\,e^{-\int \mu_o\left(\frac{x-mx'}{M},\frac{y-my'}{M},z\right)dz}\,dx'dy'$$

where

$d \equiv$ source−to−detector distance

$S(x',y') \equiv$ source

$\mu_o \equiv$ linear attenuation coefficient

$M = d/z$

$m = -(d-z)/z$

- To include energy effects, make:
  i. $S(x',y') \Rightarrow S(x',y',E)$
  ii. is a function of E
  iii. add integration of E to both integrals
- Quantum efficiency degrades I by η, a uniform effect
- $I(x_D,y_D) = I(x,y) \otimes PSF_{detector}$

2. For a cylindrical object (long axis perpendicular to the beam) calculate the profile of X-ray intensity in a fan beam geometry, assuming that the beam is mono-energetic.

QuickTime™ and a
None decompressor
are needed to see this picture.

$$\mu(x,z)=\begin{cases}1, & \text{if } (z-a)^2+x^2\le r\\ 0, & \text{otherwise}\end{cases}$$

$$I(x)=\frac{1}{4\pi d^2}\int e^{-\int \mu\left(\frac{x-mx'}{M},z\right)dz}dx'$$

$$M=\frac{d}{z}\ \text{and}\ m=\frac{-(d-z)}{z}$$

$$\therefore\ \frac{x-mx'}{M}=\frac{z\left(x+\left(\frac{d-z}{z}\right)x'\right)}{d}=\frac{zx}{d}+x'-\frac{z}{d}x'$$

$$I(x)=\frac{1}{4\pi d^2}\int e^{-\int \mu\left(\frac{z}{d}(x-x')+x'\right)dz}dx'$$

3. Calculate the effect of beam hardening on the CT image of a disk.

The center is less attenuating than it should be, therefore the image is:

QuickTime™ and a
None decompressor
are needed to see this picture.

4. For the following sample, show (a) the projections and (b) the filtered projections.

QuickTime™ and a
None decompressor
are needed to see this picture.

See Appendix A.

5. A sinusoidally modulated x-ray image is recorded by a one-sided screen film system as shown below. Find the recorded S/N as a function of frequency, where the signal is the sinusoidal component and the noise is the average background. On average the screen produces l photons per x-ray photon, t of which are transmitted to the emulsion where r is recorded. The pixel area of the film is much smaller than the system resolution. Neglect any critical angle effect between the screen and the film.

X-ray photon number as a function of z = $n_0 (1+\cos(2 \pi k z))$.

QuickTime™ and a
None decompressor
are needed to see this picture.

$$PSF = \int_0^d e^{-\mu x} \frac{M}{r^2} dx$$

or in 1 – D

$$PSF = \int_0^d e^{-\mu x} \frac{M}{z^2} dx$$

6. Write a program that calculates the Radon transform of an object function, then Fourier filters the projects, and finally reconstructs an image via back projection.

See Appendix A.

APPENDIX A: Mathematica File (Projection2.nb)

Projection reconstruction and the Radon Transform 2

The Radon Transform

The forward Radon transform is to convert a 2-D object into a set of projects within the plane.

```
Radon[object_, n_, fov_] :=
Table
[
      Integrate
      [
            object DiracDelta
            [
                  m - x Cos[\[Theta]] - y Sin[\[Theta]]
            ],
            {x, -fov, fov},
            {y, -fov, fov}
      ],
      {m, -2 fov/(n - 1),
      +2 fov/(n - 1), 2 fov/n},
      {\[Theta], 0, Pi, Pi/(n - 1)}
];
```

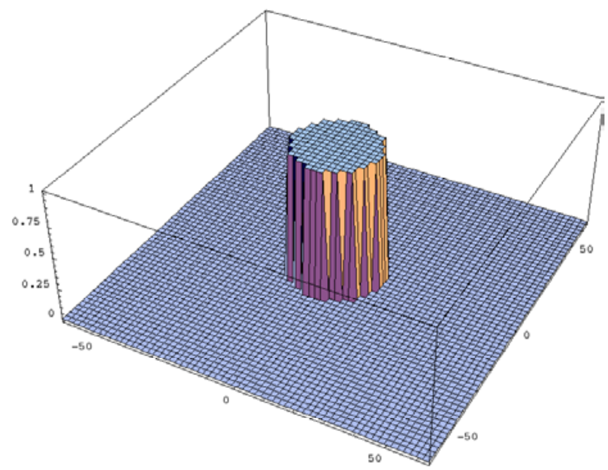
The double integral on the previous page is very slow to evaluate, and so we reduce it to a line integral along the line defined by the delta function.

```
Radon2[object_, x_, y_, n_, fov_] :=
Table
[
      Nintegrate
      [
            object[x, y],
            {yp, -fov, fov},
            {PrecisionGoal -> 4}
      ],
      {xp, -fov, fov, 2*fov/(n - 1)},
      {\[Theta], 0, Pi, Pi/(n - 1)}
] // N
```

Define a simple test object

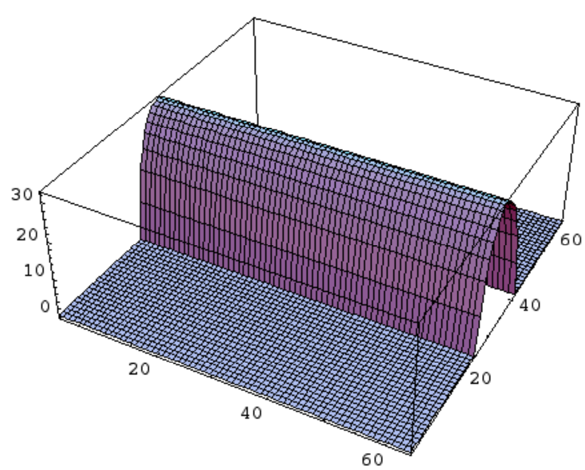
```
object1[x_, y_] := If[x^2 + y^2 < 256, 1, 10^(-6)] // N;
```

```
Plot3D
[
      object1[x, y],
      {x, -64, 64},
      {y, -64, 64},
      {PlotRange -> All, PlotPoints -> {64, 64}}
]
```




```
Robject1 =
Radon2
[
      object1,
      xp Cos[\[Theta]] - yp Sin[\[Theta]],
      yp Cos[\[Theta]] + xp Sin[\[Theta]], 64, 64
];
```

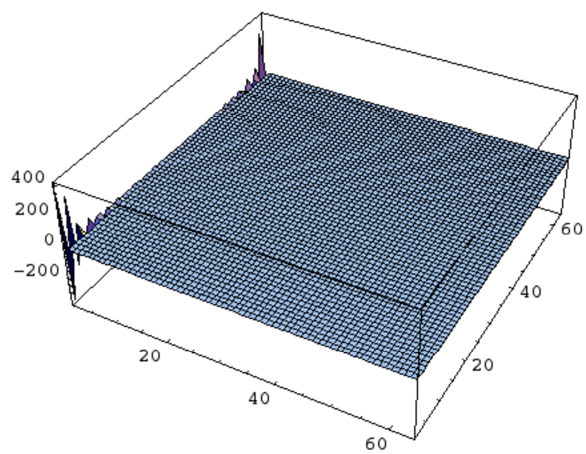
```
ListPlot3D[Robject1]
```



Filtered Back Projection

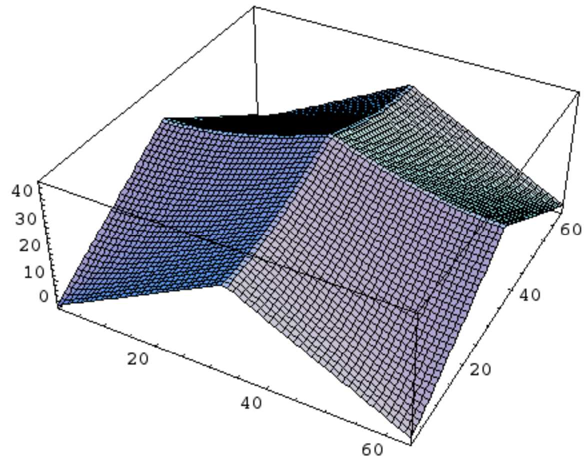
```
Fdata = Fourier[Robject1];
```

```
ListPlot3D[Re[Fdata], {PlotRange -> All}]
```



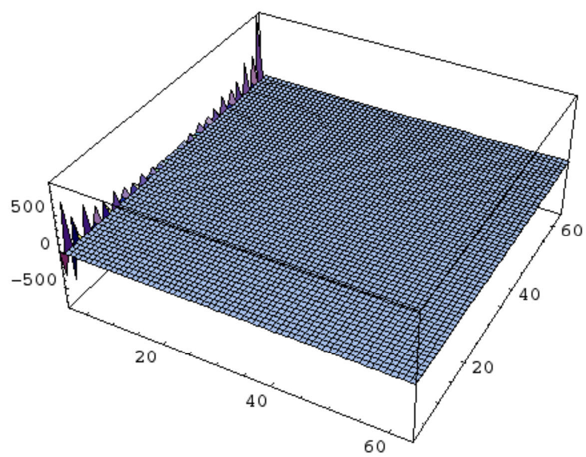

```
Filt =
Table
[
      If
      [
            x <= 32 && y <= 32,
            Sqrt[x^2 + y^2],
            If
            [
                  x <= 32 && y > 32,
                  Sqrt[x^2 + (65 - y)^2],
                  If
                  [
                        x > 32 && y <= 32,
                        Sqrt[(65 - x)^2 + y^2],
                        If
                        [
                              x > 32 && y > 32,
                              Sqrt[(65 - x)^2 + (65 - y)^2]
                        ]
                  ]
            ]
      ],
      {x, 0, 63},
      {y, 0, 63}
];
```

```
ListPlot3D[Filt]
```



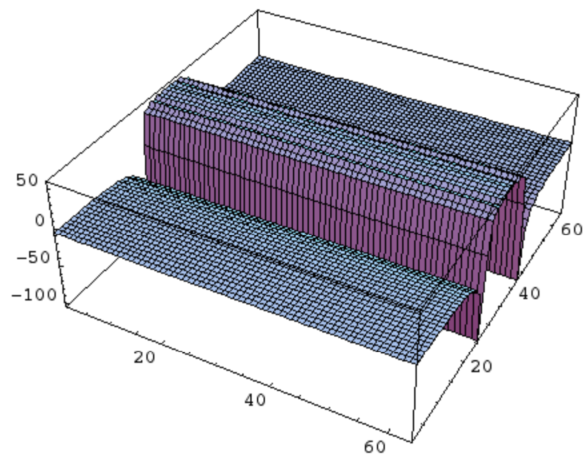

```
FiltFdata = Fdata*Filt;
```

```
ListPlot3D[Re[FiltFdata], {PlotRange -> All}]
```




```
Filtdata = Fourier[FiltFdata];
```

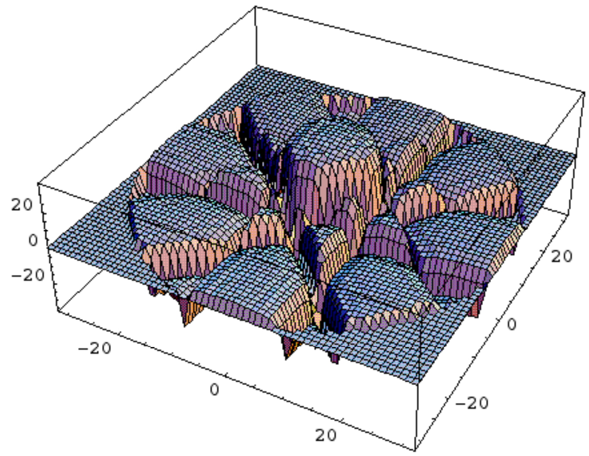
```
ListPlot3D[Re[Filtdata], {PlotRange -> All}]
```



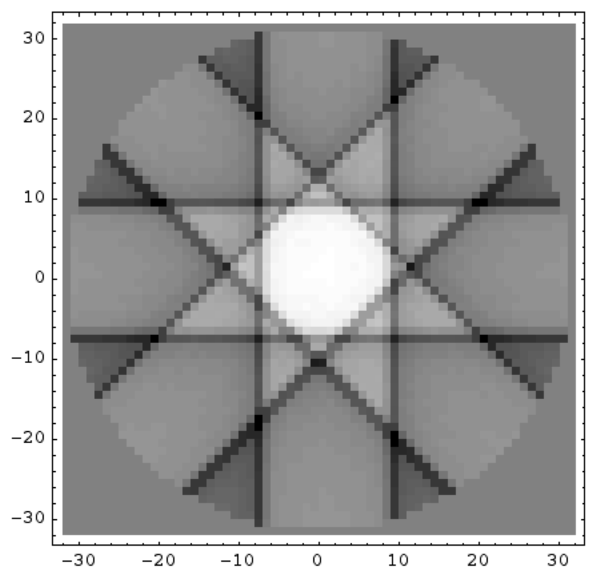
Back Projection of Filtered

```
Bflimited[x_, y_, n_] :=
If
[
      x^2 + y^2 > 32^2,
      0,
      1/(2 Pi)
      Sum
      [
            Transpose
            [
                  Re[Filtdata]
            ]
            [[m*64 + 1]]
            [[Floor[x Cos[m*Pi] + y Sin[m Pi]] + 33]],
            {m, 0, 1 - 1/n, 1/n}
      ]
];


Plot3D
[
      BFlimited[x, y, 4],
      {x, -32, 32},
      {y, -32, 32},
      {PlotRange -> All, PlotPoints -> {64, 64}}
]
```

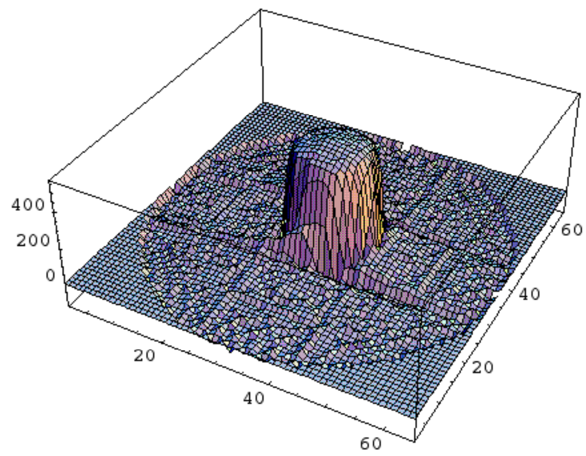



```
DensityPlot
[
      BFlimited[x, y, 4],
      {x, -32, 32},
      {y, -32, 32},
      {PlotRange -> All, PlotPoints -> {64, 64}, Mesh -> False}
]
```




```
Image =
Table
[
      BFlimited[x, y, 64],
      {x, -32, 32},
      {y, -32, 32}
];


ListPlot3D[Image, {PlotRange -> All}]
```



ListDensityPlot[Image, {PlotRange -> {0, 500}, Mesh -> False}]