1. **Give the functional form and sketch both the real and imaginary components of the Fourier transformation (with respect to x) of the following functions. Identify the spatial frequencies, wave-numbers, and features in the spectra.**

   k_o **is a constant real number** **(25 points)**

   a. $\cos(k_o x) + i\sin(k_o x)$ **(5 points)**

   b. $TopHat\left(\frac{x}{4}\right)$ **(5 Points)**

   c. $\sin(8k_o x)\cos(k_o x)$ **(5 points)**

   d. $\sum_{n=-2}^{2} \delta(x-n) = \sum_{n=-\infty}^{\infty} \delta(x-n) \bullet TopHat\left(\frac{x}{2}\right)$ **(5 Points)**

   e. $TopHat(x-4)$ **(5 Points)**

2. **Show that the k=0 point of F (k) is equal to the area f(x), where f(x) ⇔ F (k).**

   (10 points)

3. **Show that the** $k_x = 0$ **point of** $F(k_x, k_y)$ **is equal to the projection of** $f(x,y)$ **onto the y-axis where** $f(x,y) \Leftrightarrow F(k_x, k_y)$**.** **(5 points)**

4. **A simple way of characterizing the spatial distribution of a radiation source is to image it with a pin-hole imager.** **(40 points)**

   a. Draw the experimental geometry, and explain why this is a useful experiment. What is the form of the image, $I(x,y)$**, in terms of the source distribution,** $S(x,y)$**, assuming a perfect pin-hole camera? Place the a distance,** a**, from the source and the screen (detectors) and a distance,** b**, from the pin-hole. Include the magnification in your answer.** **(20 points)**

b. **Assume the pin-hole is of a finite size (radius = r), and therefore a true representation of the source is not observed. In linear imaging terms, explicitly describe the detected signal,** $I(x,y)$ **in both the source distribution and the pin-hole size. (10 points)**

c. **There is an oblique effect if the source extends far from the central line through the pin-hole. Without calculating the geometry of this effect, describe how it arises. (10 points)**

5. **The Nyquist condition states that to correctly measure a frequency the signal must be sampled at least twice a period. (20 points)**
   a. **Let** f **be the Nyquist frequency, show that the signals,** $\cos[2\pi(f+\Delta f)t]$ *and* $\cos[2\pi(f-\Delta f)t]$**, lead to the exact same data points when sampled at times** $t(n) = n/2f$**. (10 points)**
   b. **Explain aliasing in terms of the above result. (10 points)**