CONSERVATION EQUATIONS

Lumped-parameter formulation

$$\frac{\partial M_{cv}}{\partial t} = \sum_i \dot{m}_i$$ (mass)

$$\frac{\partial (M\vec{V})_{cv}}{\partial t} = \sum_i (\dot{m}\vec{V})_i + \sum_j \vec{F}_j$$ (momentum)

$$\frac{\partial E_{cv}}{\partial t} = \dot{Q} - \dot{W} + \sum_i \left[\dot{m} \left(h + \frac{V^2}{2} + gz \right) \right]_i$$ (energy)

$$\frac{\partial S_{cv}}{\partial t} = \sum_j \left(\frac{\dot{Q}}{T} \right)_j + \sum_i (\dot{m}s)_i + \dot{S}_{gen}$$ (entropy)

1D formulation

$$\frac{\partial \rho}{\partial t} = -\frac{\partial G}{\partial z}$$ (mass)

$$\frac{\partial G}{\partial t} = -\frac{\partial}{\partial z}\left[\frac{G^2}{\rho} \right] - \frac{\partial P}{\partial z} - \frac{\tau_w p_w}{A} - \rho g \cos\theta$$ (momentum)

$$\rho \frac{\partial h}{\partial t} = -G \frac{\partial h}{\partial z} + \frac{q'' p_h}{A} + \left[\frac{\partial P}{\partial t} + \frac{G}{\rho}\left(\frac{\partial P}{\partial z} + \frac{\tau_w p_w}{A} \right) \right]$$ (energy)

Differential (3D) formulation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$$ (mass)

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} \right] = -\nabla P + \mu \nabla^2 \vec{V} + \rho \vec{g}$$ (momentum, for incompressible fluid)

$$\rho c_p \left[\frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T \right] = -\nabla \cdot \vec{q}'' + q''' + \beta T \frac{DP}{Dt} + \phi$$ (energy)

Symbols:

A	Flow Area
c	Specific Heat
E	Internal Energy
F	Force
g	Gravitational Acceleration
G	Mass Flux
h	Enthalpy
$\dot{m}$	Mass Flow Rate
M	Mass
p	Perimeter
P	Pressure
$\dot{Q}$	Rate of Heat Transfer
S, s	Entropy
t	Time
T	Temperature
V	Velocity
$\dot{W}$	Rate of Energy Transfer as Work
z	Elevation

Subscripts:

cv	Control Volume
gen	Generation
h	heated
P	Pressure
r	Radial
w	Wall or Wetted

Greek Symbols

β	Therm. Expansion Coeff.
ϕ	Dissipation function
μ	Viscosity
ρ	Density
τ	Shear Stress

REACTOR THERMAL PERFORMANCE PARAMETERS

Parameter	Name	Typical values PWR	BWR	Units
$\dot{q}$	Power of fuel rod	67	77	kW (BTU/hr)
q'	Linear heat generation rate (or linear power)	18	20	kW/m (BTU/hr-ft)
q''	Heat flux	600	530	kW/m^2 ($BTU/hr\text{-}ft^2$)
q'''	Volumetric heat generation rate	350	240	MW/m^3 ($BTU/hr\text{-}ft^3$)
$\dot{Q}$	Core power	*	*	MW

* It varies much from plant to plant

For a fuel rod operating at steady-state conditions, the parameters are related as follows:

$$\dot{q} = q'L = q''2\pi\, R_{co} L = q'''\pi\, R_f^2 L = \dot{Q} / N$$

Where R_f is the fuel pellet radius, R_{co} is the fuel rod outer radius, L is the fuel rod active (heated) length and N is the total number of fuel rods in the core.

22.06 Engineering of Nuclear Systems
Fall 2010