

22.106 Neutron Interactions and Applications

Problem Set 6

Due SES #24

Question 1

Solve the steady-state mono-energetic transport equation in the infinite slab system illustrated in figure 1 to obtain the angular flux. Consider that all slabs are purely absorbing and that the boundary conditions are vacuum on each side.

Slab 1	Slab 2	Slab 3	Slab 4
$\Delta_1 = 1$	$\Delta_2 = 2$	$\Delta_3 = 2$	$\Delta_4 = 1$
$\Sigma_1 = 1$	$\Sigma_2 = 1.5$	$\Sigma_3 = 0.5$	$\Sigma_4 = 1$
$S_1 = 2$	$S_2 = 0$	$S_3 = 1$	$S_4 = 0$

Figure 1

Question 2

- a) Using the CPM method, evaluate the scalar flux in each slab illustrated in figure 2. Assume the problem to be steady-state, mono-energetic with vacuum boundary conditions on each side. The following link can be used to compute the exponential integrals (<http://keisan.casio.com/> select Special Functions / Exponential Integral)

Slab 1	Slab 2	Slab 3
$\Delta_1 = 1$	$\Delta_2 = 2$	$\Delta_3 = 2$
$\Sigma_{s1} = 0.5$	$\Sigma_{s2} = 1.0$	$\Sigma_{s3} = 0.2$
$\Sigma_{t1} = 1$	$\Sigma_{t2} = 1.5$	$\Sigma_{t3} = 0.5$
$S_1 = 1$	$S_2 = 0$	$S_3 = 4$

Figure 2

- Explain in words the meaning of P_{21} and state the main assumptions made in defining this term.
- What is the probability that a neutron born isotropically and uniformly in slab 1 will leak out of the system without making a collision.

Question 3

Starting from the 1D steady-state integro-differential form of the transport equation with isotropic scattering, derive the integral form of the transport equation

Question 4

- Write the one-dimensional mono-energetic steady-state S_2 equations with an isotropic source and scattering.
- Explain why odd order S_N equations are avoided.
- Solve the S_2 equations for the following purely absorbing slab with reflective boundary conditions and an isotropic uniform unit source.

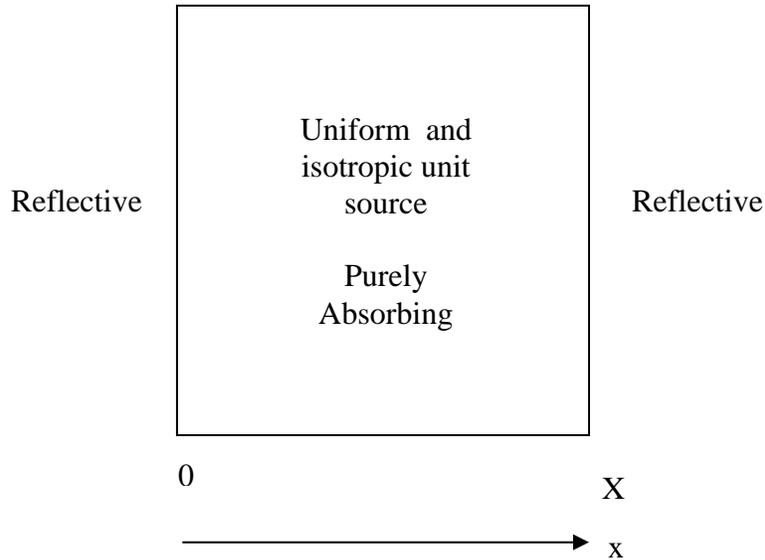


Figure 3

Question 5

Prove the equivalence of the S_2 and P_1 equations for a steady-state mono-energetic problem with isotropic source and scattering.

Question 6

The Chebyshev polynomials of the first kind are a set of orthogonal polynomials defined as the solutions to the Chebyshev differential equation and denoted $T_n(\mu)$. As they are orthogonal over the -1 to 1 range, they are suitable for expanding the angular dependence of the angular flux in slab geometry. These polynomials also obey the two following relations:

- orthogonality relation

$$\int_{-1}^1 \frac{T_n(\mu)T_m(\mu)}{\sqrt{1-\mu^2}} d\mu = \begin{cases} \pi, & m = n = 0, \\ \pi/2, & m = n \neq 0, \\ 0, & m \neq n. \end{cases}$$

- recurrence relation

$$2\mu T_n(\mu) = T_{n+1}(\mu) + T_{n-1}(\mu).$$

- Starting from the time-independent mono-energetic transport equation in a slab with isotropic scattering and isotropic source, derive the set of equations obtained when expanding the angular flux in terms of Chebyshev polynomials in accordance with the angular flux expression below

$$\varphi(x, \mu) = \frac{\phi_0(x)}{\pi} T_0(\mu) + \frac{2}{\pi} \sum_{n=1}^{\infty} \phi_n(x) T_n(\mu)$$

- b) Discuss the preference of odd versus even orders in the T_N expansion
- c) From the equations in a), derive a diffusion-like equation from the T_l equations.

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